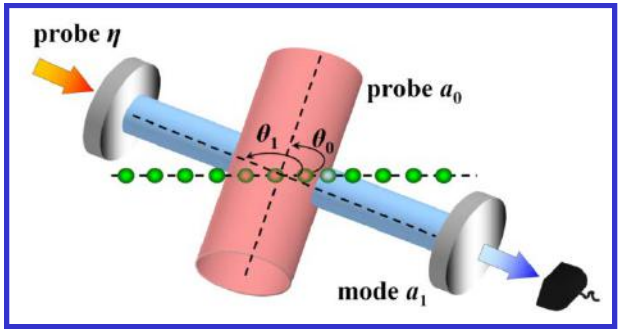
Quantum optics of many-body systems



Igor Mekhov

Université Paris-Saclay (SPEC CEA)
University of Oxford, St. Petersburg State University

Lecture 4

universite PARIS-SACLAY

Previous lectures

light
matter

Classical optics
light waves
material devices

Atom optics
matter waves
light forces

Light–matter interaction

Quantum optics
classical (hot) atoms ☹
quantum light states ☺

Quantum atom optics
☺ quantum atomic states
☹ classical light

Quantum measurements

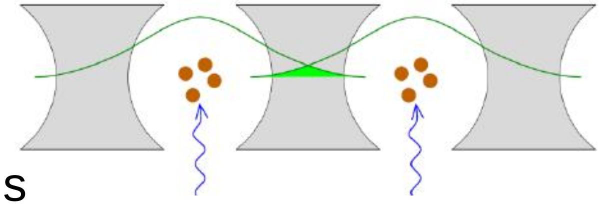
☺ **Quantum optics of quantum gases** ☺

and of other systems

Further plan

- **Synthetic many-body systems with short-range interactions**

  - Arrays of coupled cavities

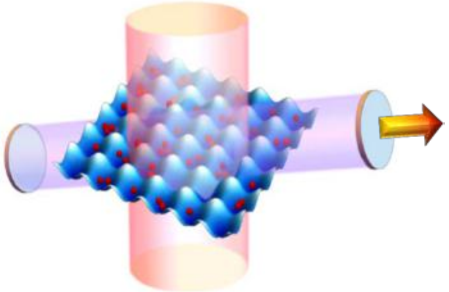


  - Bose-Hubbard and Jaynes-Cummings-Hubbard models

  - Strongly interacting photons in free space (without a cavity)

- **Many-body cavity QED with collective long-range interactions**

  - Dicke phase transition

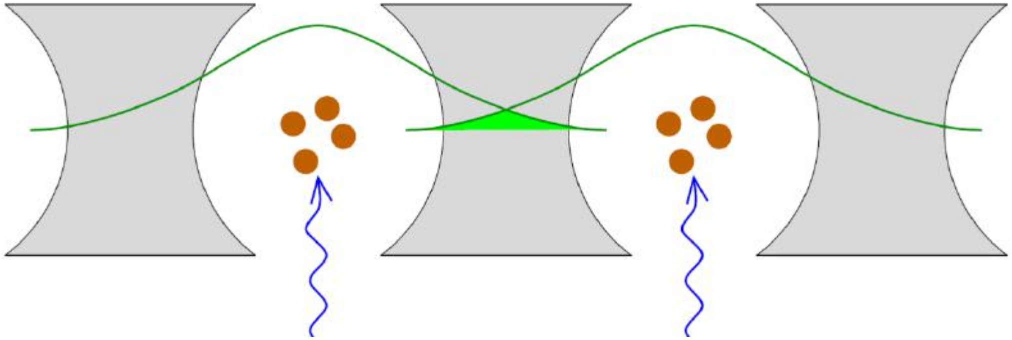


  - From classical optical lattices to *dynamical* and *quantum optical lattices*

  - **Quantum measurement-induces** phenomena and control methods for many-body states

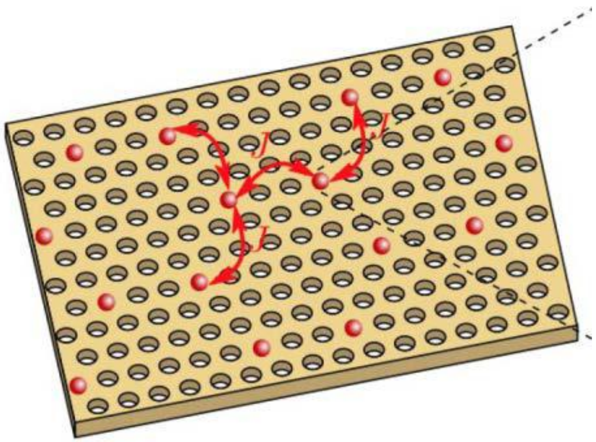
Arrays of coupled cavities

Building an artificial lattice of strongly interacting particles:
on-site interaction and tunnelling

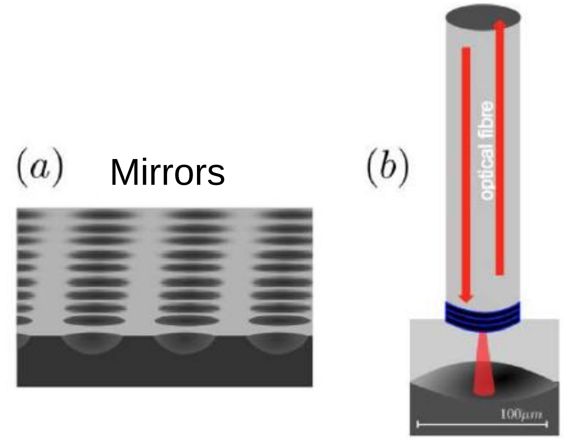


- Tunnelling of photons between neighbouring cavities
- On-site nonlinearity due to the nonlinear “atoms”
- Open system: dissipative and driven

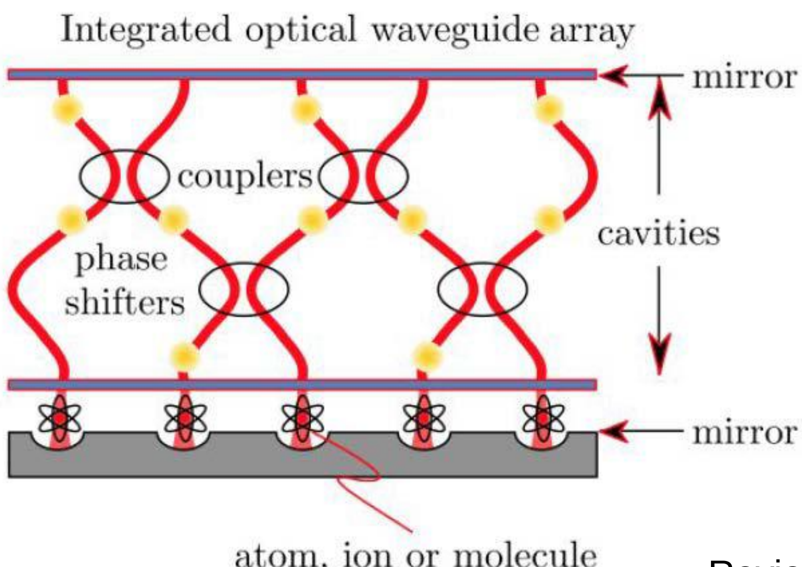
Proposals with various systems



Photonic crystals with quantum dots

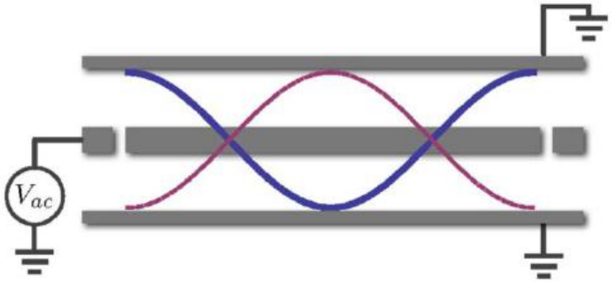


Array of fibres

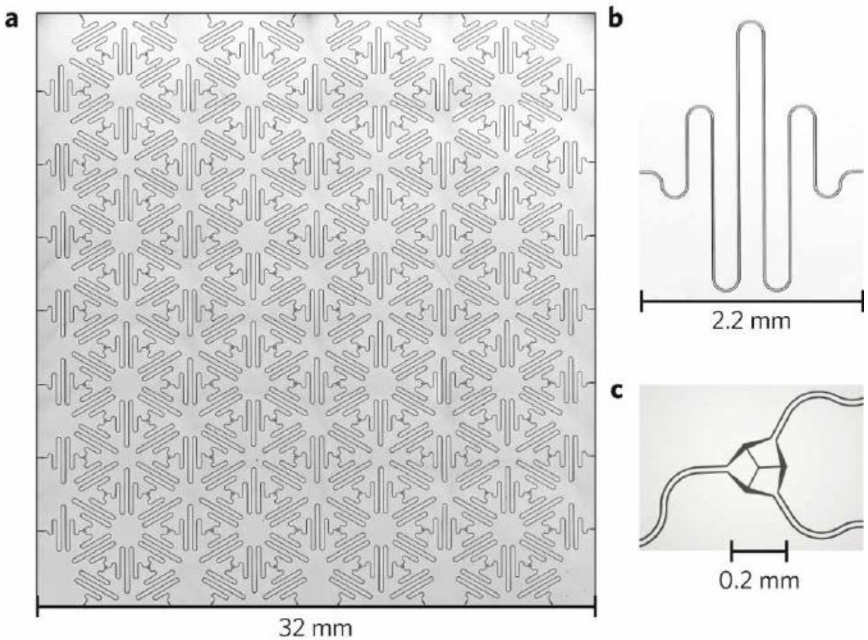


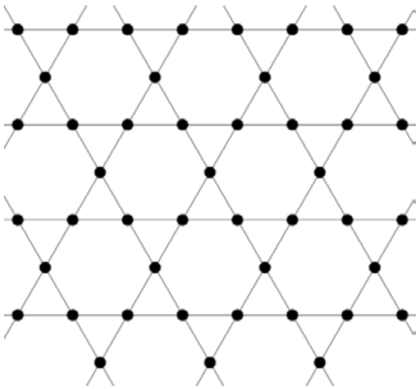
Reviews: M. Hartmann; D. Jaksch; D. Angelakis

Circuit QED systems: microwave resonators with superconducting qubits



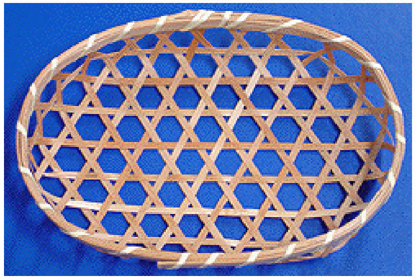
Various lattice geometries:





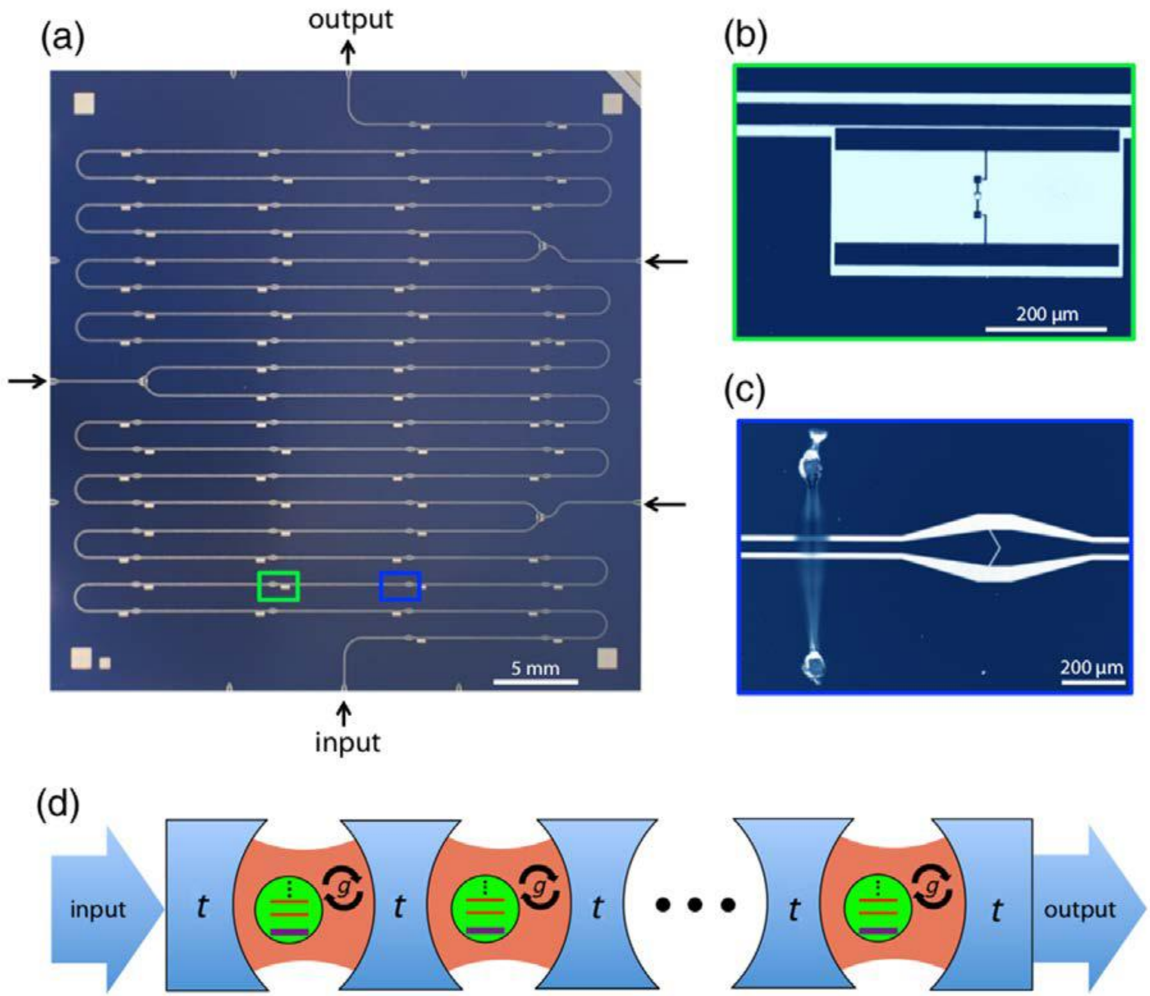
kagome lattice

(no Prof. Kagome!)



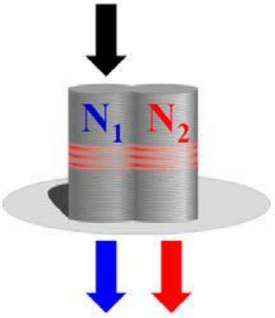
Reviews: M. Hartmann; D. Jaksch; D. Angelakis

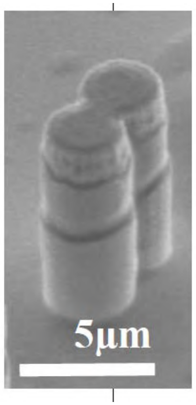
A chain of 72 microwave cavities with qubits

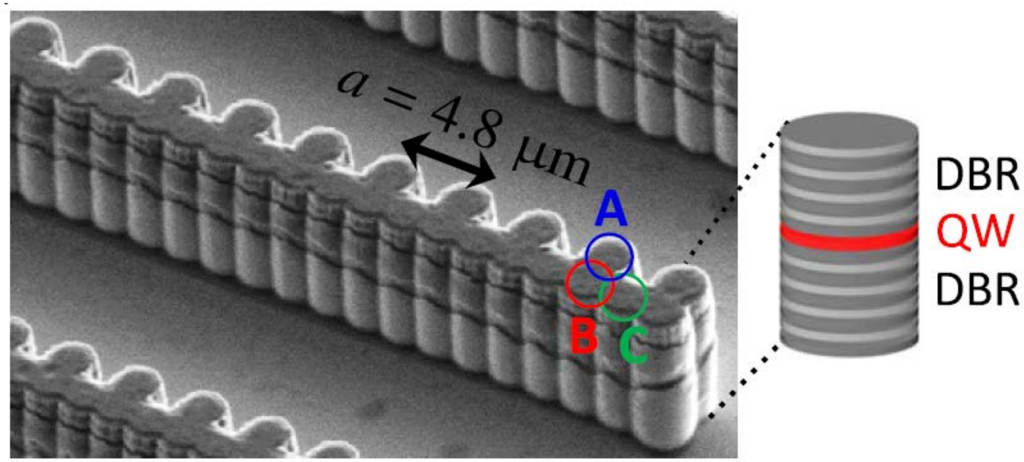


A. Houck group, Phys. Rev. X. (2017)

Coupled semiconductor cavities with exciton-polaritons







J. Bloch's group, Phys. Rev. Lett. (2016)

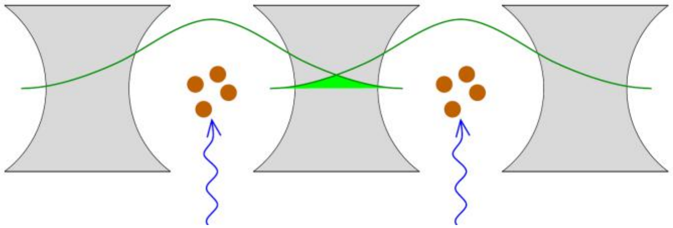
Various systems: the goal is to achieve the strong-light matter coupling regime

$$g \gg \kappa, \gamma$$

Parameter	Symbol	Photonic crystals	Integrated optics	Superconducting
Resonance frequency	$\omega_c/2\pi$	325 THz	380 THz	10 GHz
JC parameter	$g/2\pi$, g/ω_c	20 GHz	33 MHz	200 MHz
Cavity decay rate	$\gamma/2\pi$	1 GHz	10 MHz	100 kHz
Atom decay rate	$\kappa/2\pi$	8 GHz	10 MHz	2 kHz
Cooperativity	$g/\gamma\kappa$	2.5	10	$\gg 1$
Resonator coupling	$J/2\pi$	100 GHz	2 GHz	10 MHz

Reviews: M. Hartmann; D. Jaksch; D. Angelakis

Targeted models:



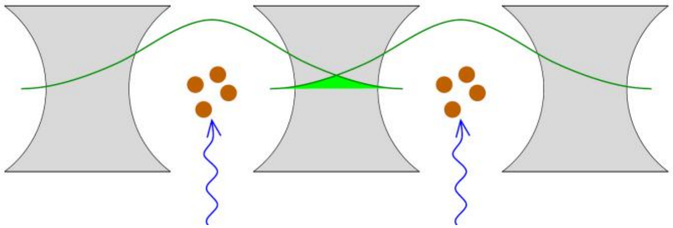
Bose-Hubbard model:

$$\hat{H}_{\mathrm{BH}} = -J \sum_{\langle i,j \rangle} \left(\hat{a}_i^\dagger \hat{a}_j + \mathrm{h.c.} \right) + \frac{U}{2} \sum_i \hat{a}_i^\dagger \hat{a}_i^\dagger \hat{a}_i \hat{a}_i + \omega_c \sum_i \hat{a}_i^\dagger \hat{a}_i$$

Jaynes-Cummings-Hubbard model

$$\hat{H}_{\mathrm{JCH}} = -J \sum_{\langle i,j \rangle} \left(\hat{a}_i^\dagger \hat{a}_j + \mathrm{h.c.} \right) + g \sum_i \left(\hat{a}_i^\dagger \hat{\sigma}_i^- + \mathrm{h.c.} \right)$$
$$+\omega_c \sum_i \left(\hat{a}_i^\dagger \hat{a}_i + \hat{\sigma}_i^+ \hat{\sigma}_i^- \right) - \Delta \sum_i \hat{\sigma}_i^+ \hat{\sigma}_i^-$$

Open systems: dissipation and external drive



Master equation:

$$d\rho/dt = -i\left[\hat{H}_\alpha, \rho\right] + \sum_j \frac{\gamma_j}{2}\left(2\hat{a}_j\rho\hat{a}_j^\dagger - \hat{a}_j^\dagger\hat{a}_j\rho - \rho\hat{a}_j^\dagger\hat{a}_j\right)$$

Hamiltonian evolution and dissipation (non-Hermitian evolution + quantum jumps)

Atomic spontaneous emission:

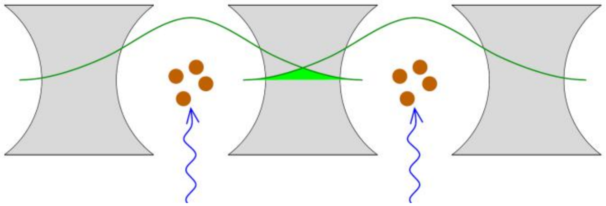
$$\mathscr{L}_{\text{decay}}\{\rho\} = \sum_j \frac{\kappa_j}{2}\left(2\hat{\sigma}_j^-\rho\hat{\sigma}_j^+ - \hat{\sigma}_j^+\hat{\sigma}_j^-\rho - \rho\hat{\sigma}_j^+\hat{\sigma}_j^-\right)$$

Coherent drive (pump):

$$\hat{H}_{\text{drive}} = \sum_j \Omega_j(t)\hat{a}_j^\dagger + \Omega_j^*(t)\hat{a}_j$$

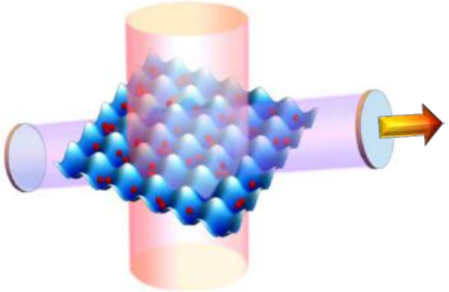
Local vs global dissipation

Master equation with ***local*** dissipation



$$d\rho/dt = -i\left[\hat{H}_\alpha, \rho\right] + \sum_j \frac{\gamma_j}{2}\left(2\hat{a}_j\rho\hat{a}_j^\dagger - \hat{a}_j^\dagger\hat{a}_j\rho - \rho\hat{a}_j^\dagger\hat{a}_j\right)$$

Global jump operators (***global*** dissipation and measurements):



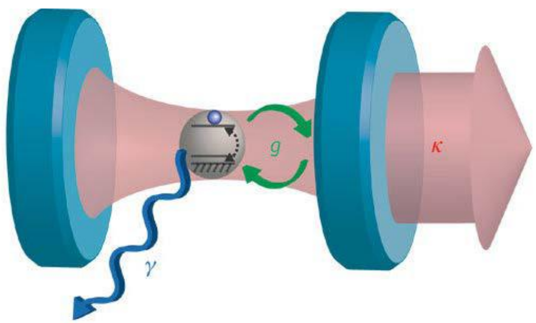
$$\hat{A} = \sum_j c_j \hat{a}_j$$

$$d\rho/dt = -i[\hat{H}_\alpha, \rho] + \frac{\Gamma}{2}\left(2\hat{A}\rho\hat{A}^\dagger - \hat{A}^\dagger\hat{A}\rho - \rho\hat{A}^\dagger\hat{A}\right)$$

Global projection (dissipation) *does not destroy the quantum coherence* (superposition) in many-body systems:

- Preserving and creating the *long-range entanglement*
- Quantum measurement-induced *preparation and control* of many-body states

Cavity QED reminder (lecture 2)



$|e\rangle$

$|g\rangle$

$$|e, n\rangle \longleftrightarrow |g, n+1\rangle$$

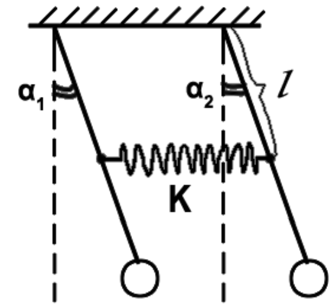
$$\mathcal{H} = \hbar\omega a^\dagger a + \frac{1}{2}\hbar\omega_a\sigma_z + \hbar g(\sigma_+ a + a^\dagger\sigma_-)$$

$$\sigma_+ = |e\rangle\langle g|$$

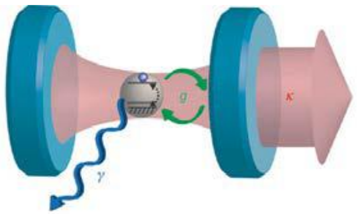
Dynamics: vacuum Rabi oscillations (e.g. of photon number and population difference)

$$W(t) = \cos(\Omega_0 t) \qquad \Omega_0 = 2g$$

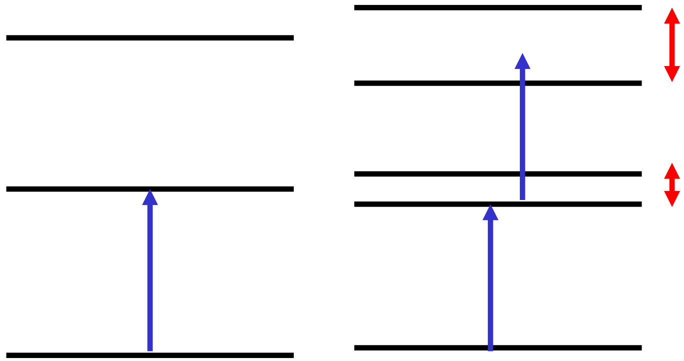
Vacuum Rabi frequency



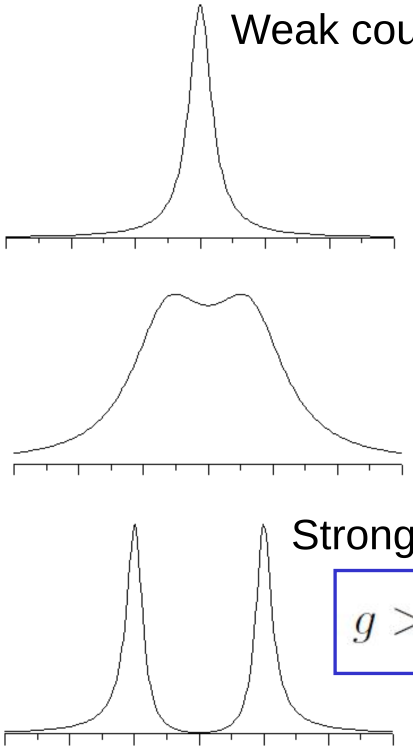
Dressed states, photon blokade



$$\mathcal{H}\Psi_n^{\pm} = \hbar\left(\omega(n-\frac{1}{2}) \pm \frac{\Omega_n}{2}\right)\Psi_n^{\pm}$$



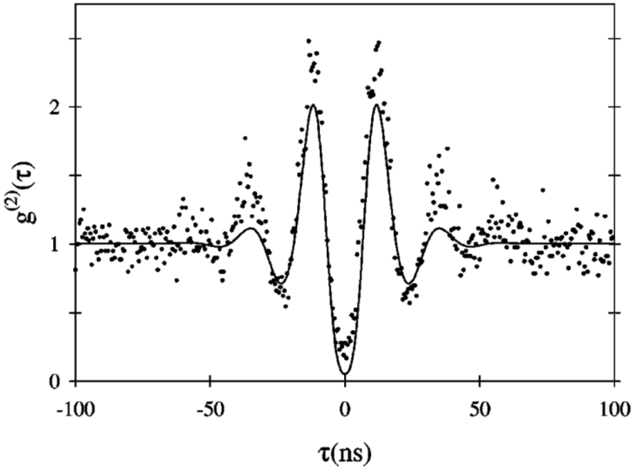
$$\Omega_n = 2g\sqrt{n+1}$$



$$g > \kappa, \gamma$$

Antibunching
(*photon blockade*)

$$g^{(2)}(0) = \frac{\langle \hat{a}^{\dagger}\hat{a}^{\dagger}\hat{a}\hat{a}\rangle}{\langle \hat{a}^{\dagger}\hat{a}\rangle^2}$$



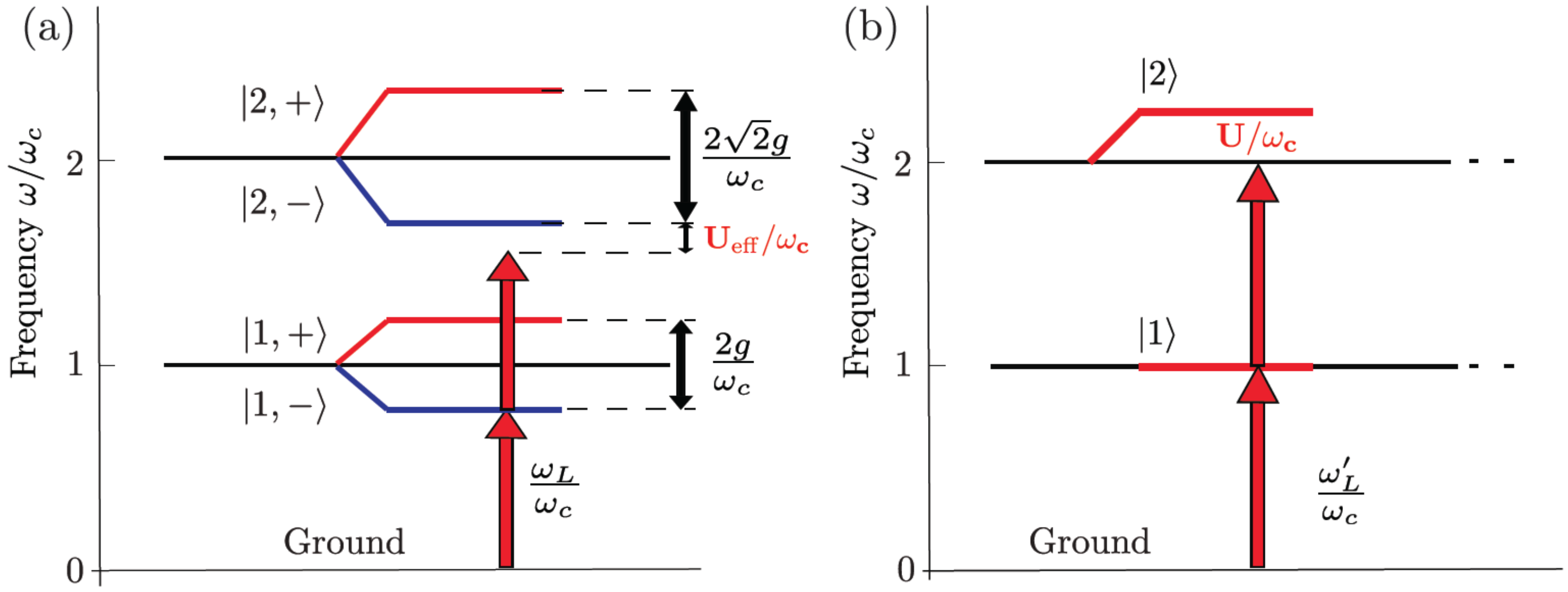
$$g^{(2)}(0) = 1 + \frac{\sigma^2 - \langle \hat{n}\rangle}{\langle \hat{n}\rangle^2} \qquad \sigma^2 = \langle \hat{n}^2\rangle - \langle \hat{n}\rangle^2$$

Interaction energy in the BH and JCH models: nonlinearity of interaction

$$\hat{H} = -J \sum_{<i,j>} \hat{a}_i^\dagger \hat{a}_j + \sum_i (\epsilon_i - \mu)\, \hat{n}_i + \sum_i \frac{1}{2} U\, \hat{n}_i(\hat{n}_i - 1)$$

JCH: photon blockade leads to the photon-photon repulsion

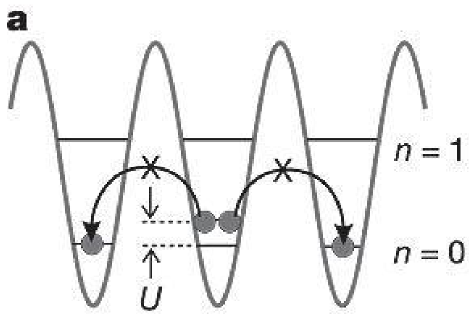
$$U_{\rm eff} = g(2 - \sqrt{2})$$



Reviews: M. Hartmann; D. Jaksch; D. Angelakis

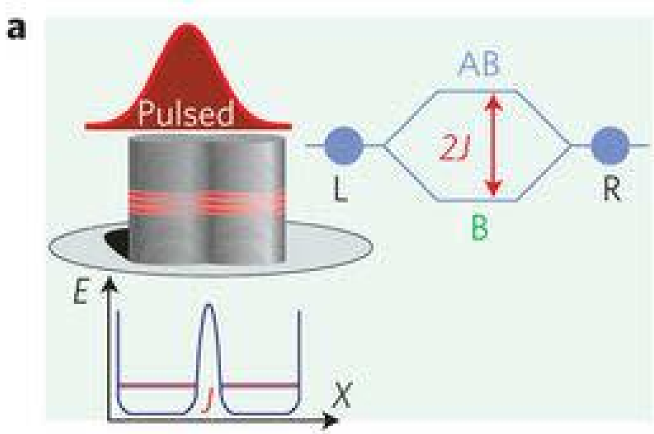
Self-trapping, localization-delocalization transition

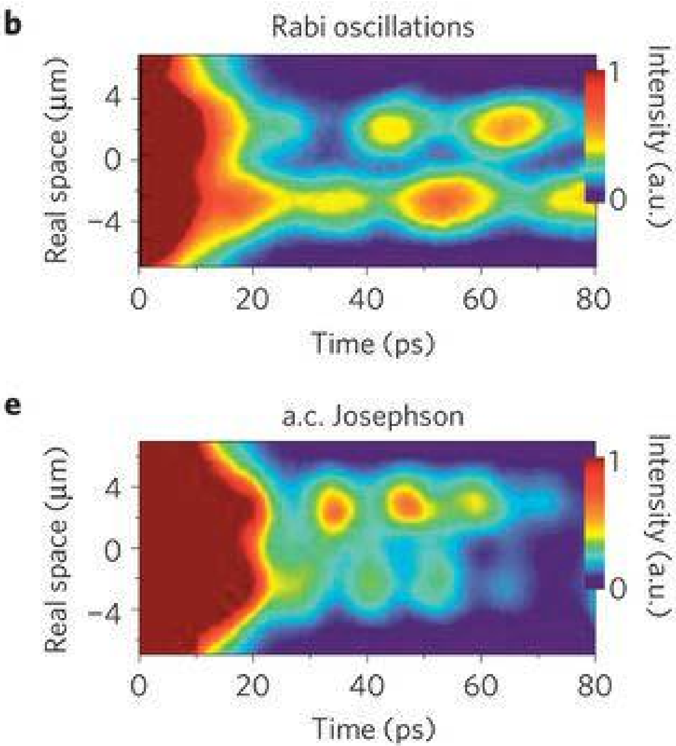
(related to the repulsively bound pairs in optical lattices)



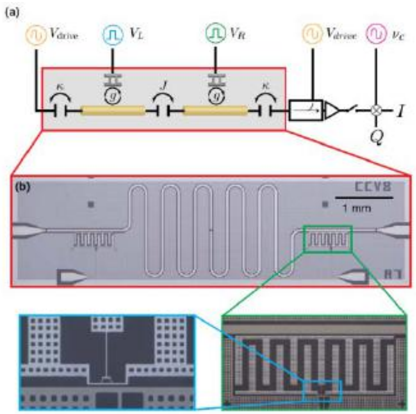
No analogy in standard condensed matter physics
(interaction + isolation form the environment)

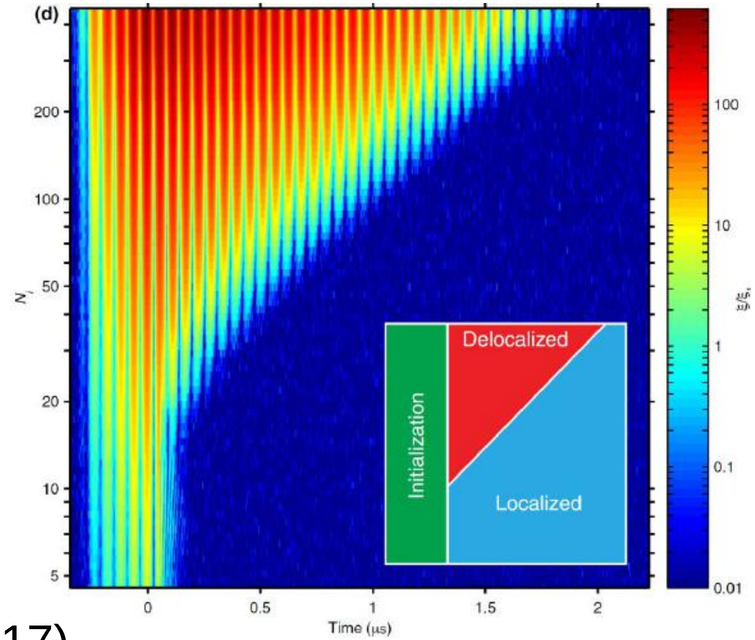
Exciton-polaritons (J. Bloch's group)



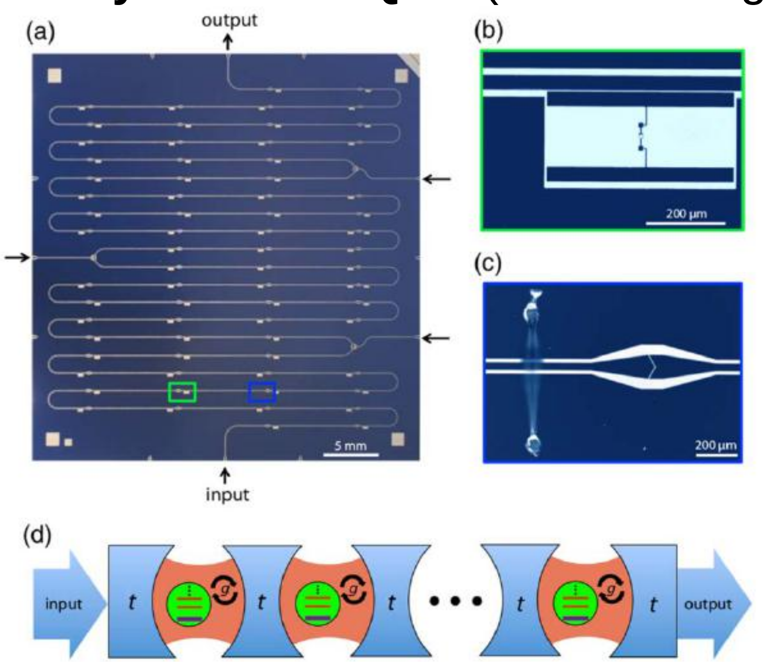


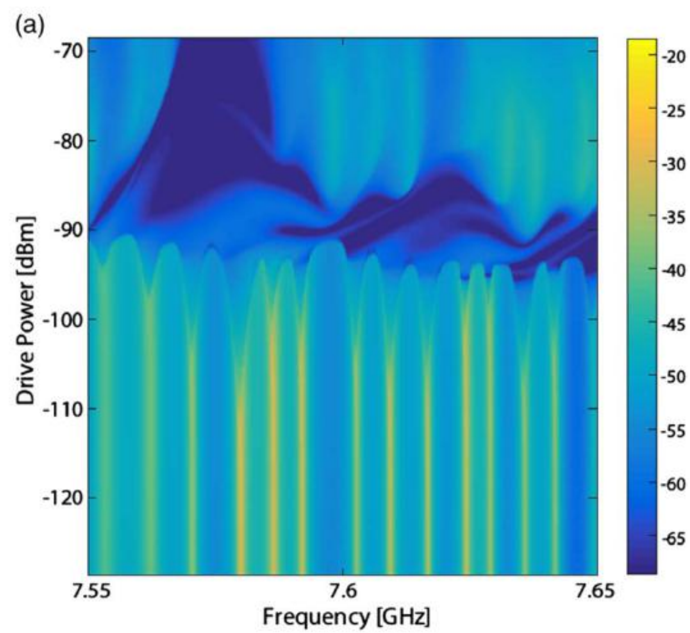
Circuit QED (A. Houck's group, 2014)





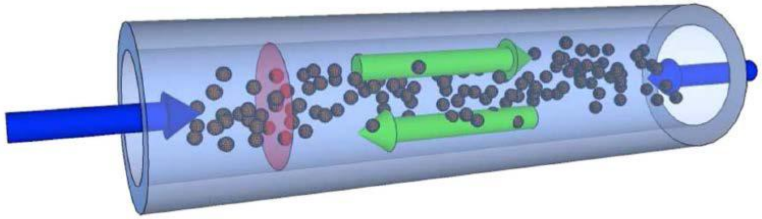
Bistability in circuit QED (A. Houck's group, 2017)



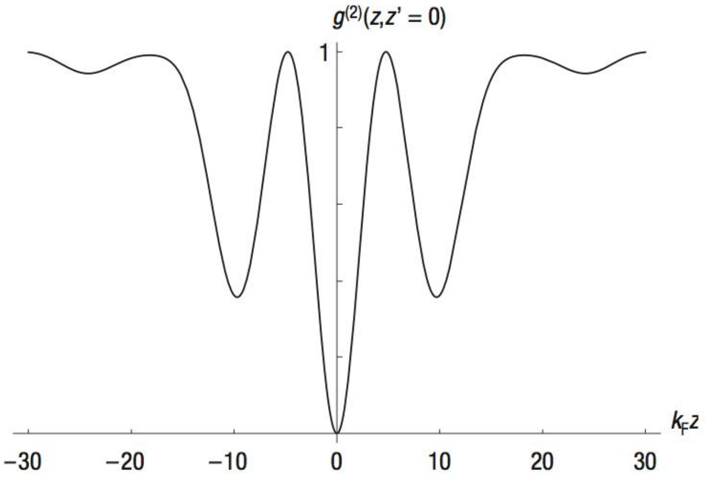


Strongly interacting photons in 1D

Atoms in fibers, Tonks-Girardeau gas



D. Chang, V. Gritsev, G. Morigi, V. Vuletic, M. Lukin, E. Demler



Lieb-Liniger model: fermionization of strongly repulsive bosons

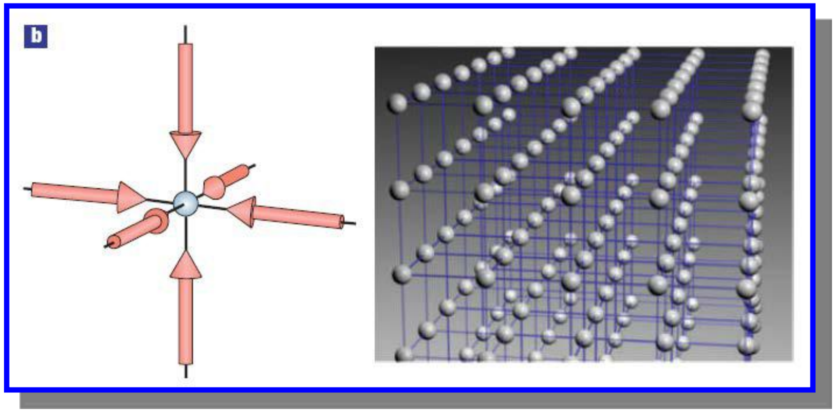
$$H_{LL} = \int_0^L dz \left[\frac{\hbar}{2m_{\text{eff}}} (\partial_z \psi^\dagger)(\partial_z \psi) + \frac{\tilde{g}}{2} (\psi^\dagger)^2 \psi^2 \right]$$

Fermionization: antibunching in space (Friedel oscillations)

$$g^{(2)}(z, z') = \frac{\langle \psi^\dagger(z) \psi^\dagger(z') \psi(z) \psi(z') \rangle}{\langle \hat{n}(z) \rangle \langle \hat{n}(z') \rangle}$$

Quantum gases in cavities

Classical optical lattices



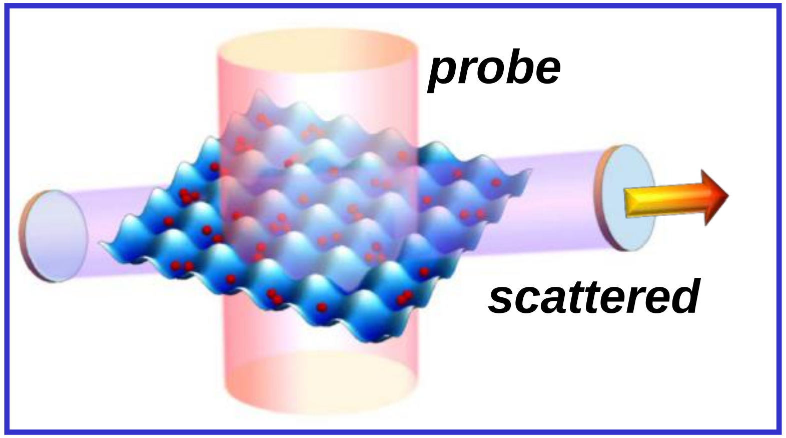
The light is *not only nonclassical*,
It is *not even quantized.*

No light fluctuation
No light-matter entanglement

Optical lattices with quantized light:

Key *new opportunities* and *physical effects*

Reviews: I. Mekhov & H. Ritsch, J. Phys. B (2012), H. Ritsch et. al., Rev. Mod. Phys. (2013)



- **Nondestructive (QND) probing of many-body atomic states**
- **State preparation and control by the quantum measurement backaction**
- **Quantum optical lattices (quantum trapping potentials)**

Towards quantum optical lattices

- **Dynamical optical lattices** (self-consistent solution for atomic and light states)

  Current experimental state of the art.

  T. Esslinger (ETH Zurich), A. Hemmerich (Hamburg), C. Zimmermann (Tubingen), B. Lev (Stanford), Ph. Bouyer & A. Bertoldi (IOGS)

- **Quantum optical lattices**

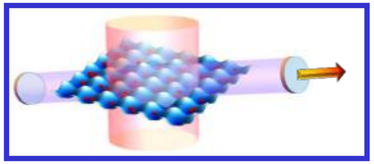
  Using the quantum fluctuations of light and light-matter correlations/ entanglement

Quantum measurements by light

- **Quantum nondemolition (QND) measurements (expectation values)**
- **Preparation of many-body states**
- **Quantum weak measurements as a novel source of competitions in many-body systems. Non-Hermitian dynamics (beyond standard dissipation)**
- **Feedback control of many-body states (beyond dissipative phase transitions)**

Quantum and dynamical lattices

Many-body Hamiltonian (second quantisation form):



$$H = H_{\text{field}} + \int d^3\mathbf{r}\Psi^\dagger(\mathbf{r})H_{a1}\Psi(\mathbf{r}) + \frac{2\pi a_s\hbar^2}{m}\int d^3\mathbf{r}\Psi^\dagger(\mathbf{r})\Psi^\dagger(\mathbf{r})\Psi(\mathbf{r})\Psi(\mathbf{r})$$

Single-atom Hamiltonian (Jaynes-Cummings model with motion):

$$H_{a1} = \frac{\mathbf{p}^2}{2m_a} + \frac{\hbar\omega_a}{2}\sigma_z - i\hbar\sum_l [\sigma^+ g_l a_l u_l(\mathbf{r}) - \text{h.c.}]$$

Adiabatic elimination of the atom polarization (or excited state):

$$H_{a1} = \frac{\mathbf{p}^2}{2m_a} + V_{\text{cl}}(\mathbf{r}) + \frac{\hbar}{\Delta_a}\sum_{l,m} u_l^*(\mathbf{r})u_m(\mathbf{r})g_l g_m a_l^\dagger a_m$$

Expansion in localised Wannier functions:

$$\Psi(\mathbf{r}) = \sum_{k=1}^{M} b_k w(\mathbf{r}-\mathbf{r}_k), \quad \hat{n}_i = b_i^\dagger b_i, \quad \hat{N}_K = \sum_{i=1}^{K}\hat{n}_i, \quad \hat{N}_M = \hat{N}$$

Generalized Bose-Hubbard model

$$H = H_f + \sum_{i,j=1}^{M} J_{i,j}^{\mathrm{cl}} b_i^\dagger b_j + \hbar g_0^2 \sum_{l,m} \frac{a_l^\dagger a_m}{\Delta_{ma}} \left(\sum_{i,j=1}^{K} J_{i,j}^{lm} b_i^\dagger b_j \right) + \frac{U}{2} \sum_{i=1}^{M} \hat{n}_i (\hat{n}_i - 1)$$

Origin of the long-range interaction:

$$a_l \sim \sum_{<i,j>}^{K} J_{i,j}^{lm} b_i^\dagger b_j$$

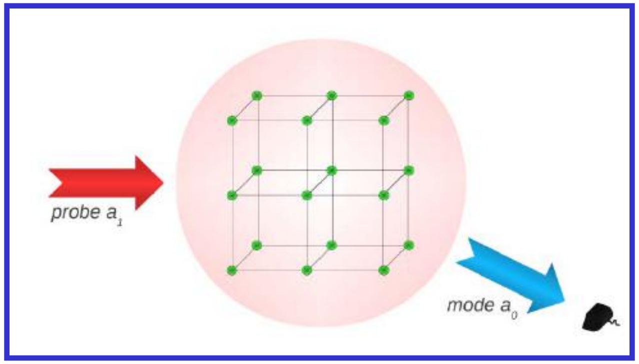
$$H \sim \sum_{i,k} X_{i,k} b_i^\dagger b_{i+1} b_k^\dagger b_{k+1}$$

Enrichment of the physical picture due to the light quantization:

- ✓ **Tunneling coefficients dynamically depend on the atomic state**
- ✓ **Long-range cavity-mediated interaction**
- ✓ **Joint light and atom quantum fluctuations (entanglement)**

→ **Broader quantum simulations, quantum control of many-body states**

Light scattering model



Light depends on the density *operator:*

$$a_{\text{out}} \sim \int u^*_{\text{out}}(\mathbf{r}) u_{\text{in}}(\mathbf{r}) \hat{n}(\mathbf{r}) \mathrm{d}\mathbf{r}$$

$$\hat{n}(\mathbf{r}) = \hat{\Psi}^\dagger(\mathbf{r})\hat{\Psi}(\mathbf{r}) \qquad \hat{\Psi}(\mathbf{r}) = \sum_i b_i w(\mathbf{r} - \mathbf{r}_i)$$

Diffraction *beyond Bragg peaks*

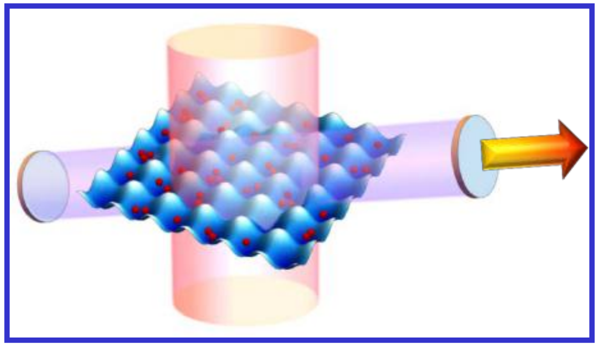
$$a_{\text{out}} \sim \hat{D} + \hat{B} \qquad \hat{D} = \sum_{i=1}^{K} J_{i,i}\hat{n}_i \qquad \hat{B} = \sum_{\langle i,j \rangle}^{K} J_{i,j} b_i^\dagger b_j$$

On-site densities and inter-site matter-field interference (phase, tunnelling)

Diffraction maxima (Bragg peaks) are classical.
Diffraction minima show the quantum fluctuation of atomic state.

Scientific Rep. 7, 42597 (2017)

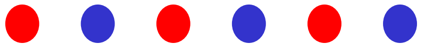
Generation of multiple matter modes



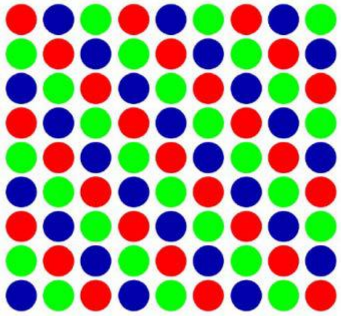
For angles at <u>*diffraction minima*</u>, groups of atoms scatter light with equal phases.

Thus, they are
indistinguishable for light scattering
(*no which-path information*)

<u>Ex.</u> *R=2* modes: $\hat{D} = \sum_m (-1)^m \hat{n}_m = \hat{N}_{\text{even}} - \hat{N}_{\text{odd}}$ (odd and even sites)



$\hat{D} = \sum_m e^{im\delta} \hat{n}_m$ $\hat{D} = \sum_{l=1}^{R} \hat{N}_l e^{i2\pi l/R}$



(sum of *smaller* number of *macroscopic* modes)

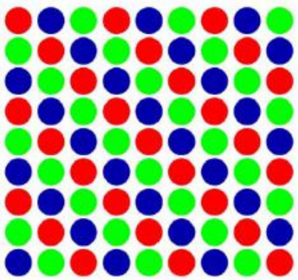
PRL 114, 113604 (2015)

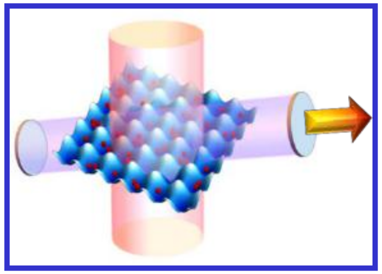
Spatial structure of scattering enables the competition between global and short-range processes

Interaction of global modes

$$\hat{F} = \sum_{\varphi} \hat{D}_{\varphi} + \sum_{\varphi'} \hat{B}_{\varphi'}$$

(**density** and **bond** modes)





PRL 115, 243604 (2015)
PRA 93, 063632 (2016)

$$\hat{D}_{\varphi} = J_{D,\varphi}\hat{N}_{\varphi}, \text{ with } \hat{N}_{\varphi} = \sum_{i\in\varphi} \hat{n}_i,$$

$$\hat{B}_{\varphi'} = J_{B,\varphi'}\hat{S}_{\varphi'}, \text{ with } \hat{S}_{\varphi'} = \sum_{\langle i,j\rangle\in\varphi'} (\hat{b}_i^\dagger \hat{b}_j + h.c.)$$

Total Hamiltonian: $\mathcal{H} = \mathcal{H}^b + \mathcal{H}^a + \mathcal{H}^{ab}$

Contribution from quantum / dynamical potential (light scattering):

$$\mathcal{H}^{ab} = g_2 \hat{a} \hat{F}^\dagger + g_2^* \hat{a}^\dagger \hat{F}$$

$$a \sim \hat{F} = \hat{D} + \hat{B}$$

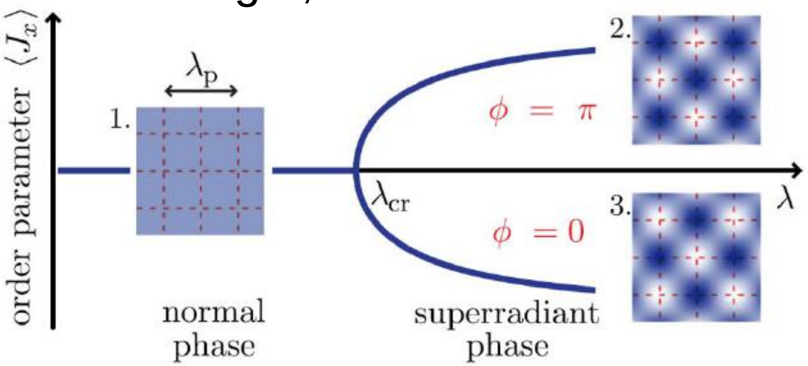
$$\mathcal{H}^b_{\text{eff}} = \mathcal{H}^b + \frac{g_{\text{eff}}}{2}(\hat{F}^\dagger \hat{F} + \hat{F}\hat{F}^\dagger)$$

Effective *interaction* between atomic modes

Two-mode example: phase transition

Phase transition: homogeneous gas turns into a checkerboard pattern: self-organization, Dicke phase transition, supersolid state.

T. Esslinger, P. Domokos



$$\mathcal{H}^b_{\text{eff}} = \mathcal{H}^b + \frac{g_{\text{eff}}}{2}(\hat{F}^\dagger \hat{F} + \hat{F}\hat{F}^\dagger)$$

In the ground state, light scattering tends to be *maximized* (or *minimized*)

Typical case: *B*=0
$$\hat{D} = \sum_m (-1)^m \hat{n}_m = \hat{N}_{\text{even}} - \hat{N}_{\text{odd}}$$

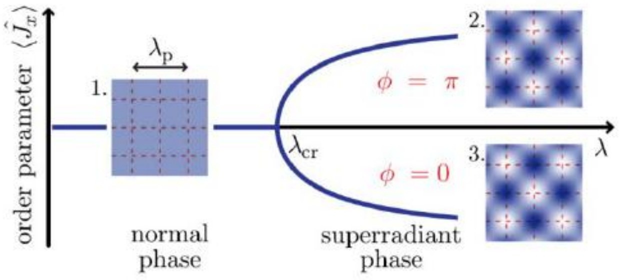
$$\hat{D}^\dagger \hat{D} = (\hat{N}_{\text{even}} - \hat{N}_{\text{odd}})^2 = \hat{N}^2_{\text{even}} + \hat{N}^2_{\text{odd}} - \underline{2\hat{N}_{\text{even}}\hat{N}_{\text{odd}}}$$

Interaction between two nonlocal modes (supersolid-like state)

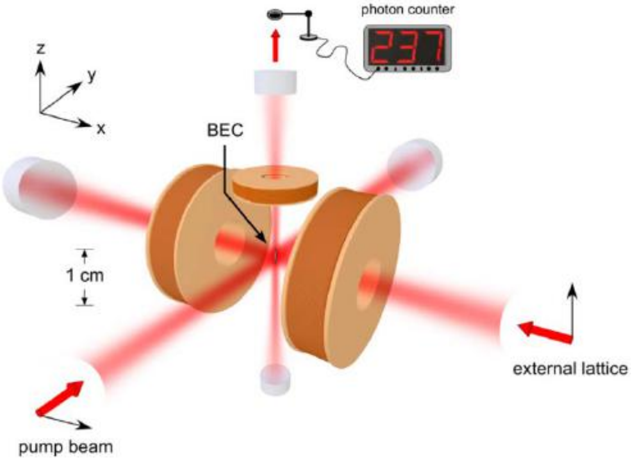
standard supersolid appears due to the density interaction at neighbouring sites: $\sum_i \hat{n}_i \hat{n}_{i+1}$

Two-mode example: phase transition

Dicke phase transition (BEC, without a lattice): T. Esslinger, ***Nature (2010)***

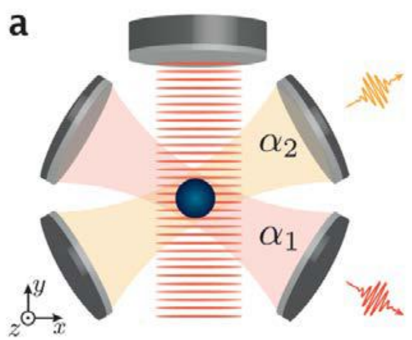


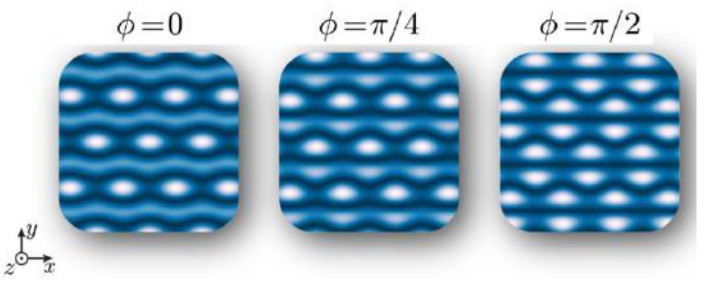
Lattice supersolid (with a lattices):
T. Esslinger, ***Nature (2016)***,
A. Hemmerich, ***PRL (2015)***



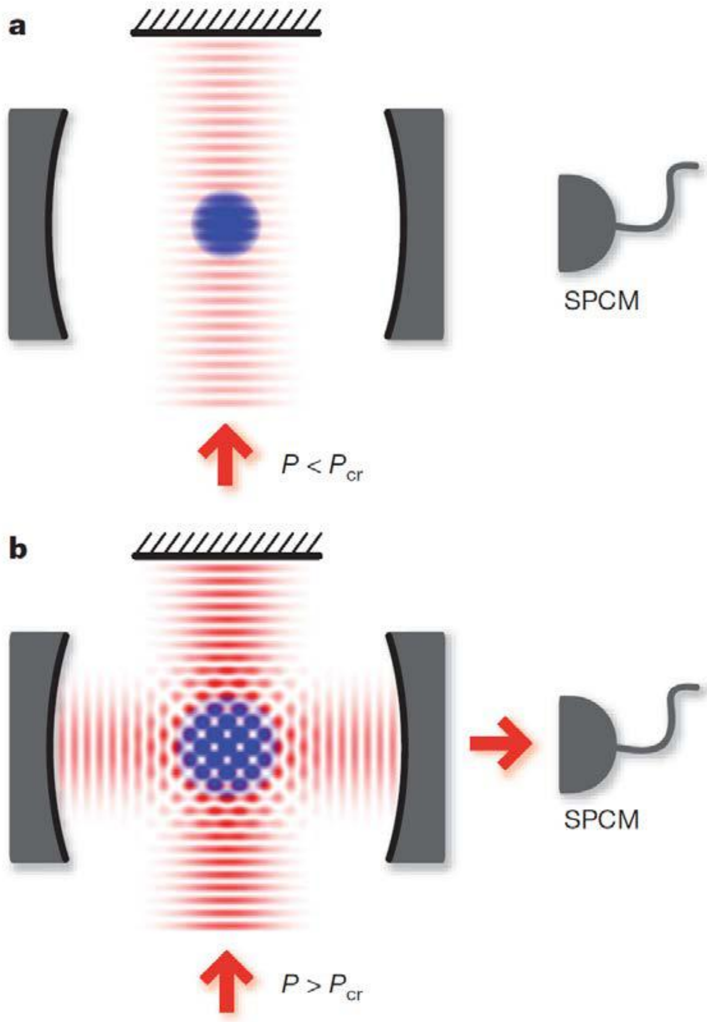
4 phases:
MI, SF,
SS, DW

Supersolid in two crossed cavities: T. Donner & T. Esslinger, ***Nature (2017)***





Superradiant Dicke phase transition



Dicke Hamiltonian:

$$\frac{\hbar\lambda}{\sqrt{N}}(\hat{a}^\dagger + \hat{a})(\hat{J}_+ + \hat{J}_-)$$

$$\lambda = \eta\sqrt{N}/2$$

Coupling: the pump strength

Motional levels:

$$\hat{J}_+ = \hat{J}_-^\dagger = \sum_i |\pm\hbar k, \pm\hbar k\rangle_{ii}\langle 0, 0|$$

Interference of matter waves in a standing wave cavity

T. Esslinger, ***Nature (2010)***

Novel phases: properties of both long- and short-range models

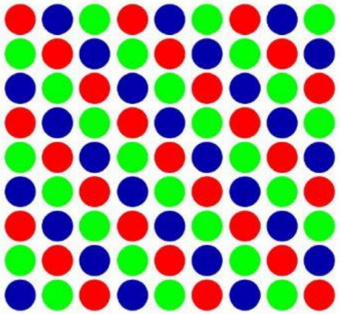
$$\hat{F}^{\dagger}\hat{F}+\hat{F}\hat{F}^{\dagger}=\sum_{\varphi,\varphi'}[\gamma^{D,D}_{\varphi,\varphi'}\hat{N}_{\varphi}\hat{N}_{\varphi'}+\gamma^{B,B}_{\varphi,\varphi'}\hat{S}_{\varphi}\hat{S}_{\varphi'}+\gamma^{D,B}_{\varphi,\varphi'}(\hat{N}_{\varphi}\hat{S}_{\varphi'}+\hat{S}_{\varphi}\hat{N}_{\varphi'})]$$

Structured global scattering *mimics* the "long-range" interactions
(much "longer" than for Rydberg atoms, dipolar molecules, or spins)

The mode interaction "length" can be tuned

Density coupling: multimode density waves (DW)
multimode supersolids (SS)

Bond coupling: superfluid dimers (SFD), trimers, etc.
supersolid dimers (SSD), trimers, etc.



PRL 115, 243604 (2015); NJP 17, 123023 (2015); PRA 93, 063632 (2016)

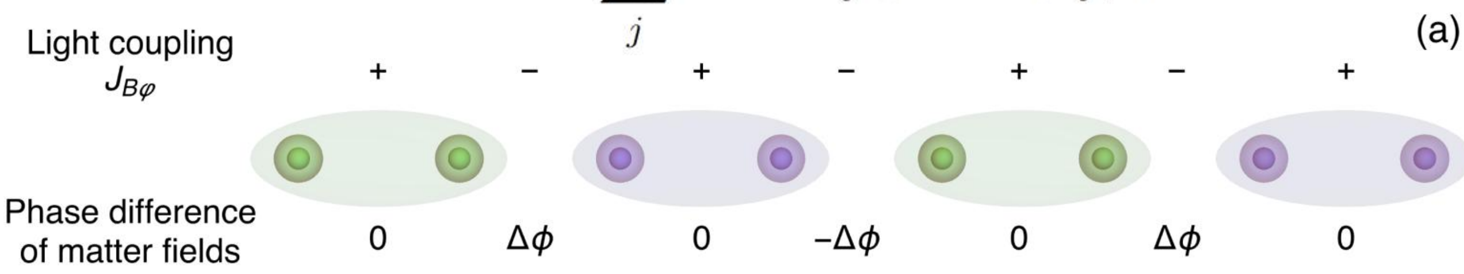
Self-organization of quantum matter waves

Phase transition: matter-field phase alternates for different sites.

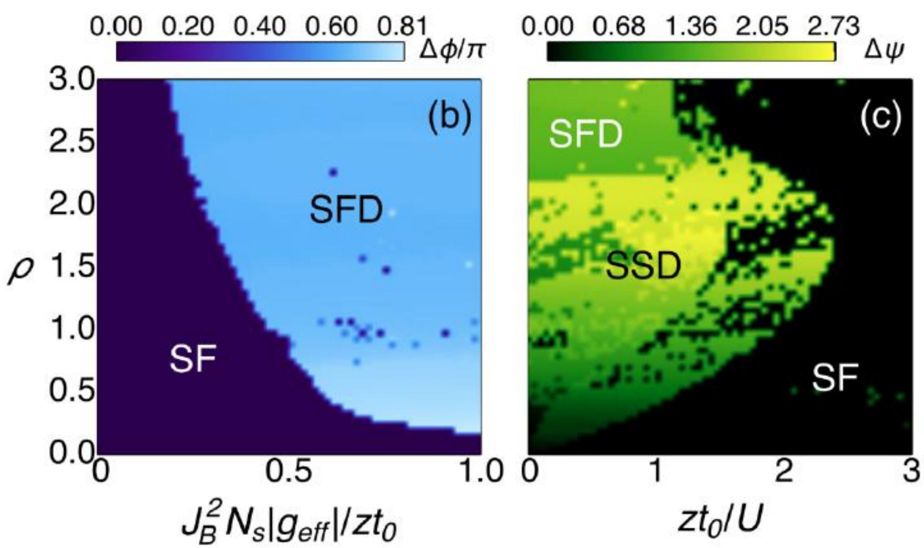
PRL 115, 243604 (2015)

Matter-field phases *compete* with the imposed light-field phases.

$$\hat{B} = \sum_j (-1)^j (b_j^\dagger b_{j+1} + b_j b_{j+1}^\dagger)$$



Competition between the kinetic energy and light-matter interaction.



Superfluid dimers

Supersolid dimers

Quantum vs dynamical optical lattice

$$\mathcal{H}^b_{\text{eff}} = \mathcal{H}^b + \frac{g_{\text{eff}}}{2}(\hat{F}^\dagger \hat{F} + \hat{F}\hat{F}^\dagger) \qquad g_{\text{eff}} = \Delta_p |g_2|^2/(\Delta_p^2 + \kappa^2)$$

Semiclassical approximation (similar to optics): $\hat{a}\hat{F}^\dagger = \langle \hat{a} \rangle \hat{F}^\dagger$

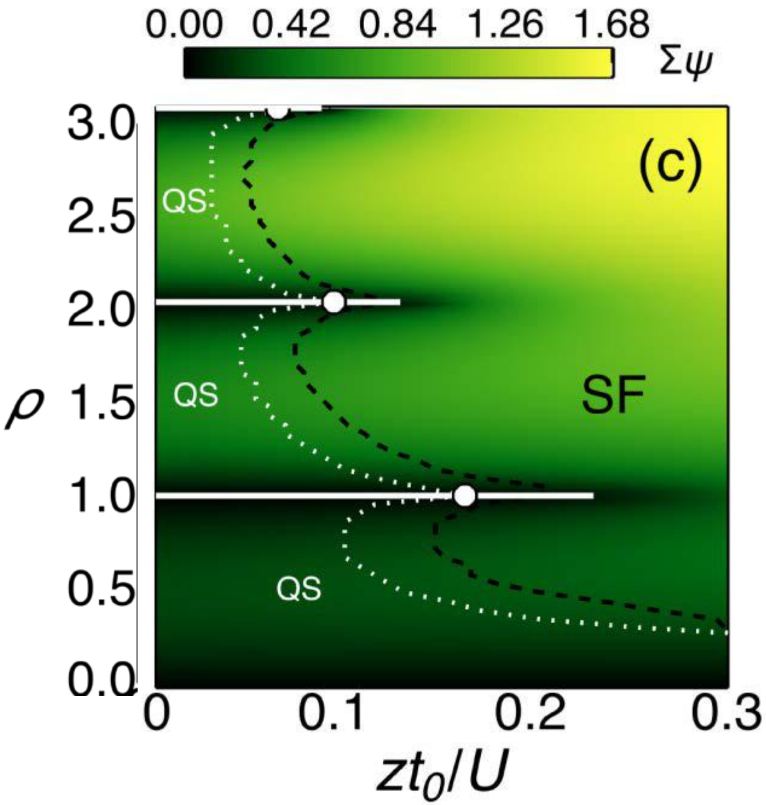
light is classical (and dynamical)
atoms are quantum (here, the motion)

$$\hat{F}^\dagger \hat{F} + \hat{F}\hat{F}^\dagger = \langle \hat{F}^\dagger \rangle \hat{F} + \langle \hat{F} \rangle \hat{F}^\dagger + \delta \hat{F}^\dagger \hat{F}$$

$\Delta_p < 0$ Ground state for *maximized* scattering, build-up of strong light, quantum fluctuations can be neglected
Dynamical, but NOT quantum OL (self-organization)

$\Delta_p > 0$ Ground state for minimized scattering, NO strong light, *quantum fluctuations* play the leading role:
Quantum OL

Density coupling: quantum OL



No light build-up: fluctuations define physics

Renormalization of the interaction strength:

$$U \quad = U + 2g_{\mathrm{eff}} J_D^2$$

Shift of the MI-SF transition point:

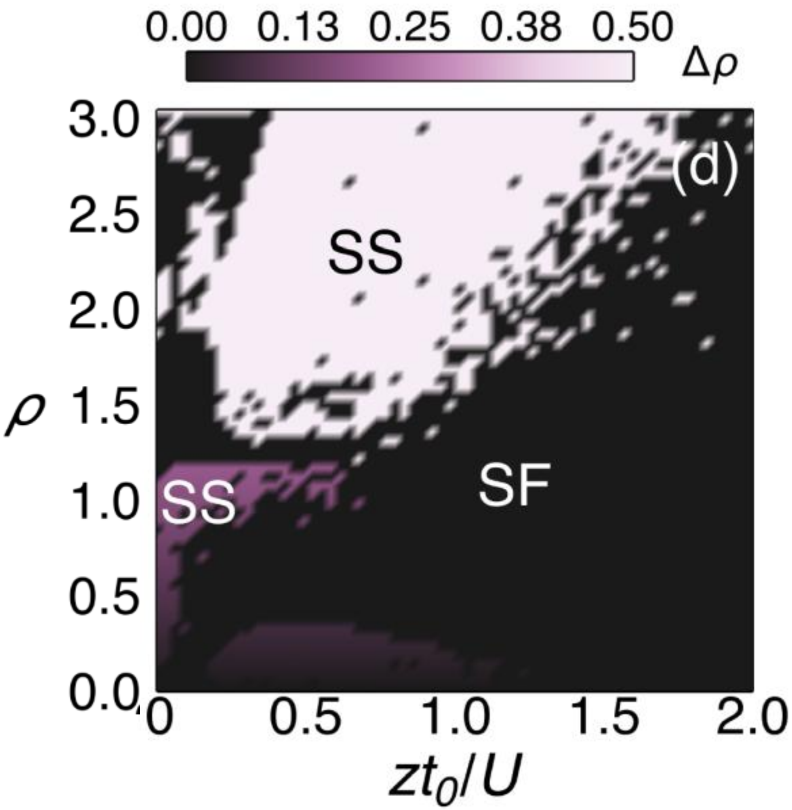
Suppression of light fluctuation leads to suppression of atomic fluctuations

Fluctuations in SF can be suppressed as well (QS)

Purely light-induced effective on-site interaction (beyond BH model)

PRL 115, 243604 (2015)

Quantum OL for bond modes



PRL 115, 243604 (2015)

No light build-up: fluctuations define physics

New terms (not present in BH model) lead to the supersolid state

Solely due to the quantum light-matter correlations

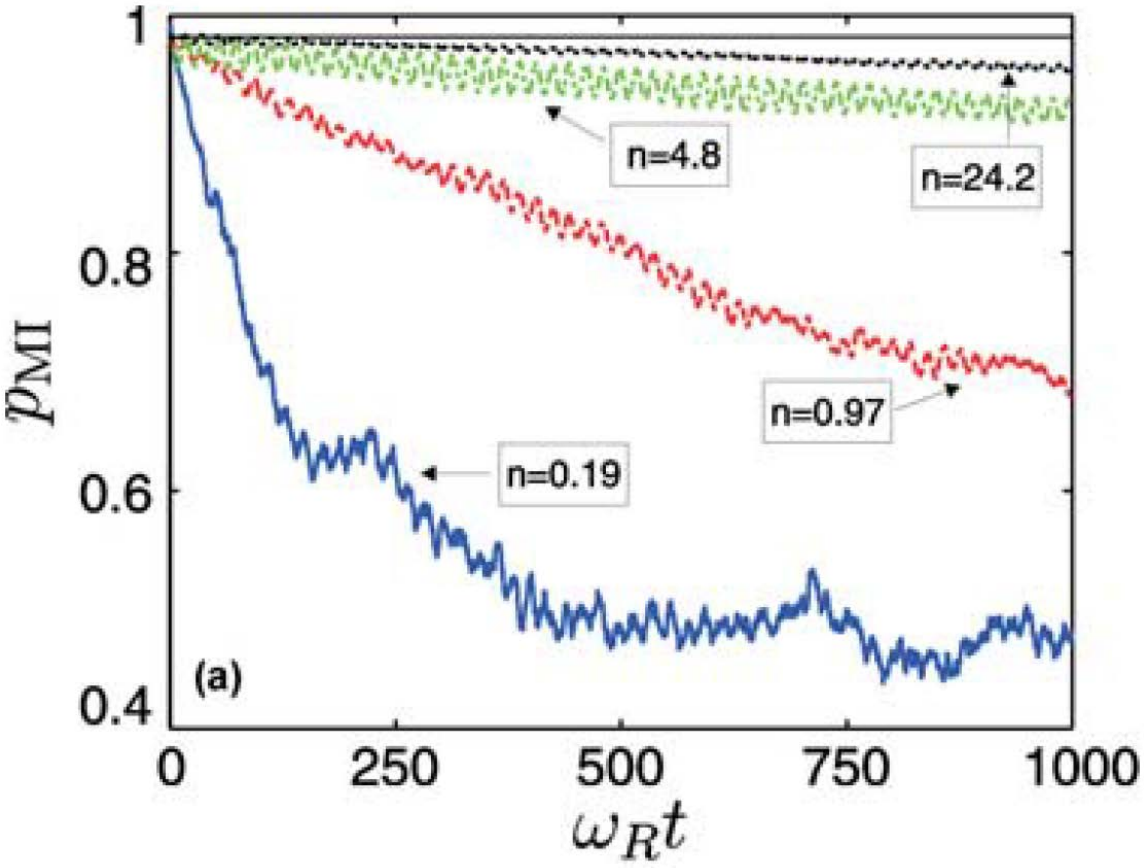
$$b_i^\dagger b_{i+1} b_k^\dagger b_{k+1} \qquad \sum_i \hat{n}_i \hat{n}_{i+1}$$

Supersolid without a cavity: W. Ketterle, Nature (2017)

Fully quantum trapping potential

Leaving the MI ground state (quantum optical lattices)

average photon number in a cavity < 1



Dynamics:
light fluctuations drive the atomic fluctuations

Superposition of

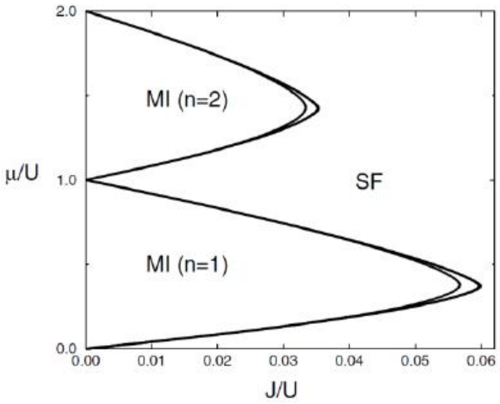
MI + SF + free particle

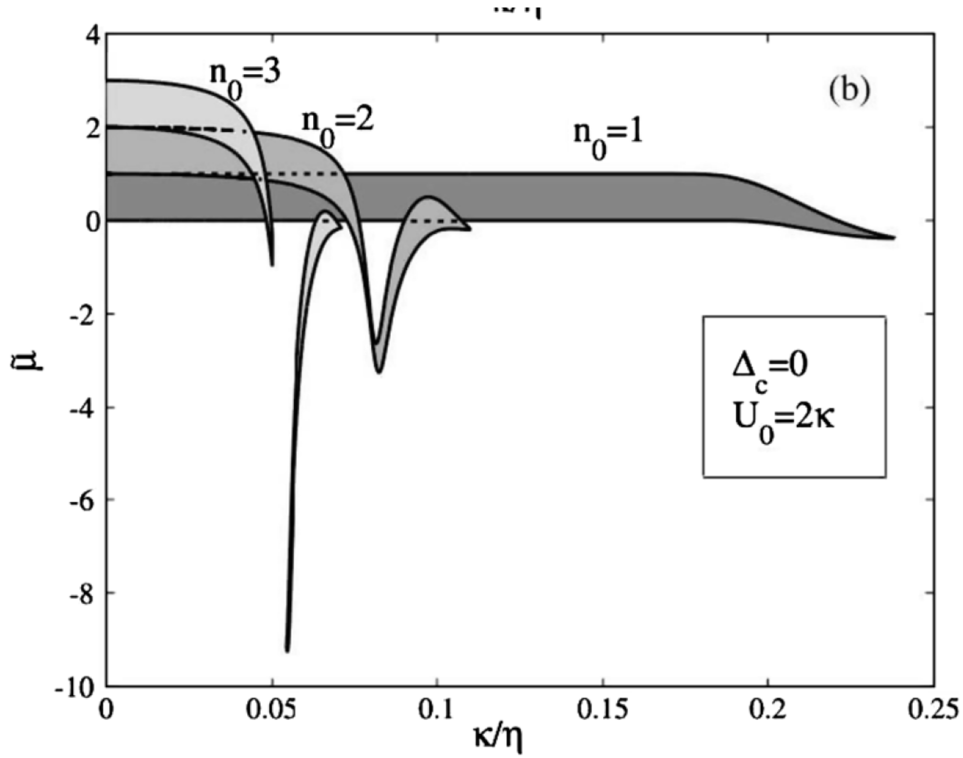
???

EPJD 46, 545 (2008)

Other many-body states

Overlap of the Mott insulator lobes (dynamical lattice)





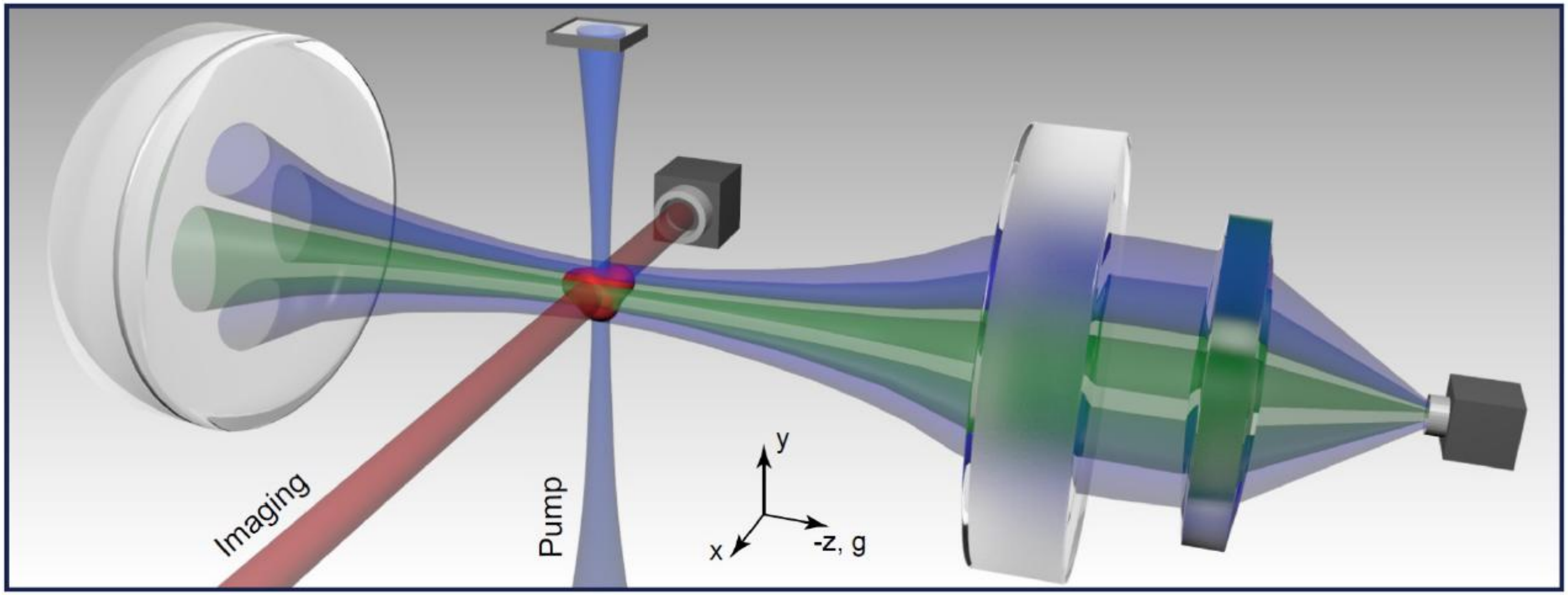
J. Larson, B. Damski, G. De Chiara, G. Morigi, M. Lewenstein, …

Other many-body states

Bose and spin glasses (disorder):
G. Morigi, S. Sachdev, Ph. Strack, S. Diehl, …

Fermions in a cavity: J. Keeling, F. Piazza, …

Multimode cavities: B. Lev, S. Gopalakrishnan, …

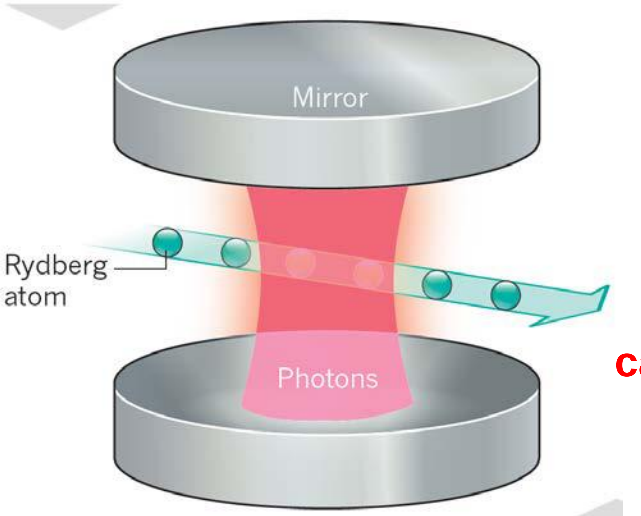


Quantum measurement

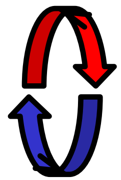
Cavity QED

S. Haroche setup

Probing light state by atoms



light

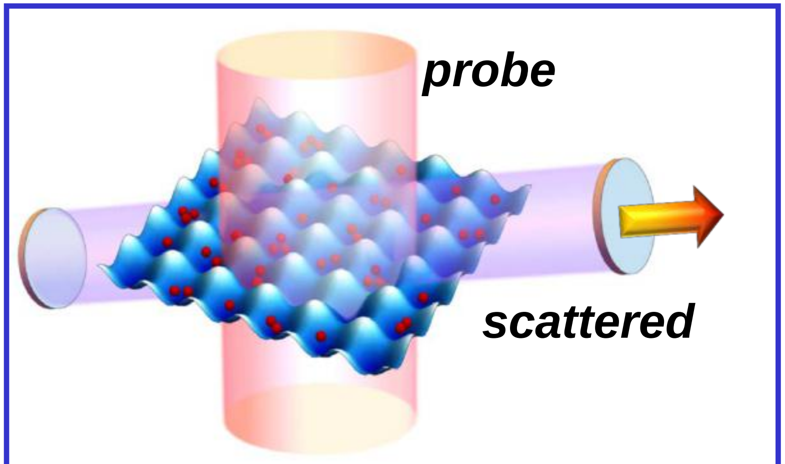


matter

cavity ~ lattice site

Many-body cavity QED

Probing atomic state by light



Analogy: strongly correlated states of ***thousands*** of “cavities”

Matter waves: natural nonclassical states, interacting, can be fermions

Quantum measurements

- Quantum nondemolition (QND) measurements (expectation values)
- Preparation of many-body states
- Quantum weak measurements as a novel source of competitions in many-body systems. Non-Hermitian dynamics (beyond standard dissipation)
- Feedback control of many-body states (beyond dissipative phase transitions)

QND measurements of *expectation values*

$$H = \hbar\left(\omega_1 + \frac{g^2}{\Delta_a}\hat{D}_{11}\right)a_1^\dagger a_1 + \frac{g^2}{\Delta_a}\left(a_0^\dagger \hat{D}_{10}^* a_1 + a_0 \hat{D}_{10} a_1^\dagger\right) - i\hbar(\eta^* a_1 - \eta a_1^\dagger)$$

QND Hamiltonian for negligible tunneling

Various QND variables $$\hat{D}_{lm} \equiv \sum_{i=1}^{K} u_l^*(\mathbf{r}_i)u_m(\mathbf{r}_i)\hat{n}_i$$

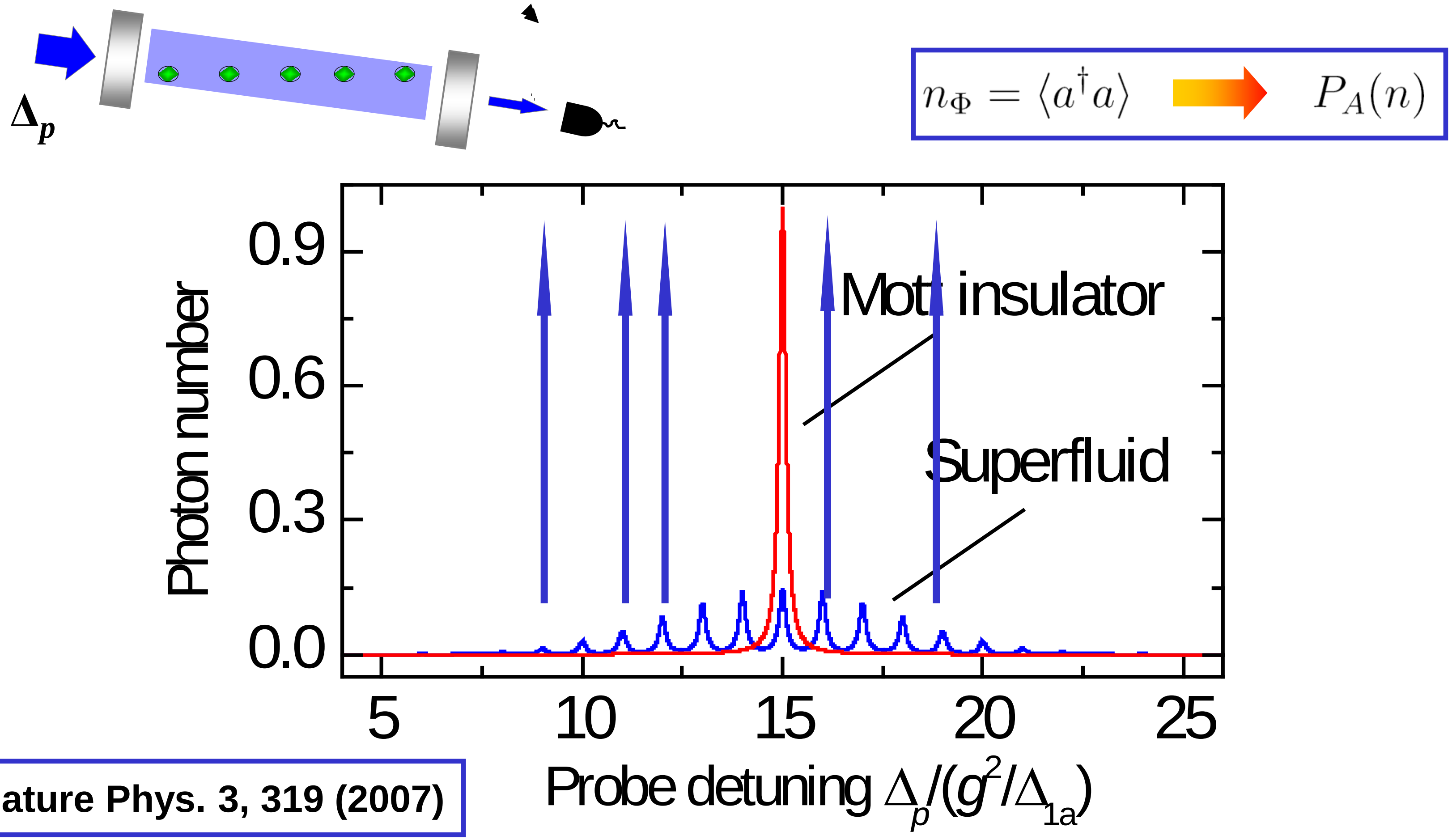
The conjugate variable (matter phase) is destroyed, but does not affect dynamics)

Photon number: density and tunnelling correlations: $\langle \hat{n}_i \hat{n}_j \rangle$, $\langle b_i^\dagger b_{i+1} b_j^\dagger b_{j+1} \rangle$

Photon number variance: 4- and 8-point correlations: $\langle \hat{n}_i \hat{n}_j \hat{n}_k \hat{n}_l \rangle$,

$\langle b_i^\dagger b_{i+1} b_j^\dagger b_{j+1} b_k^\dagger b_{k+1} b_l^\dagger b_{l+1} \rangle$

Spectrum: number distribution mapping



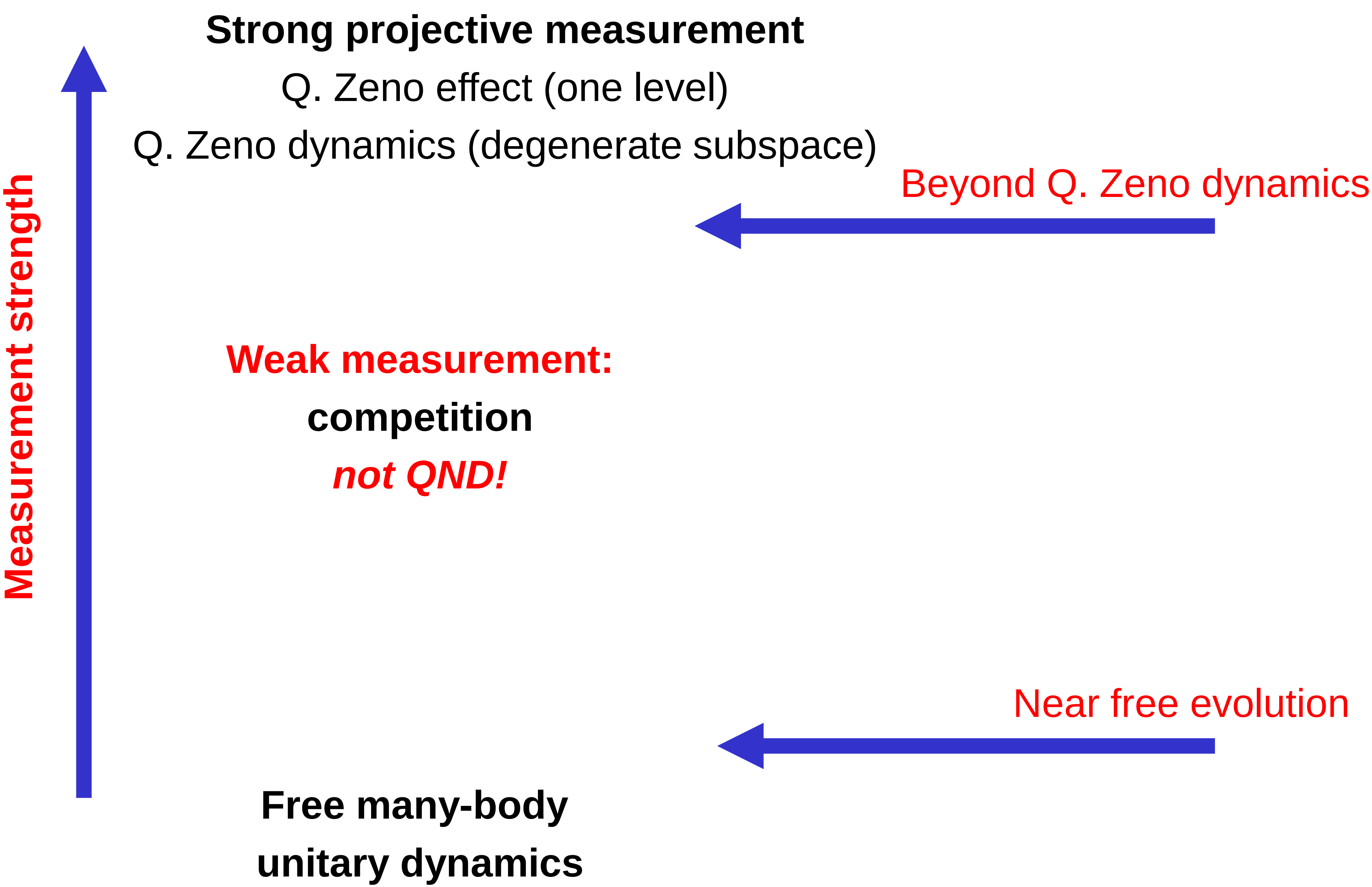
Nature Phys. 3, 319 (2007)

Ensemble average vs Single run measurements

Expectation values: multiple measurements required

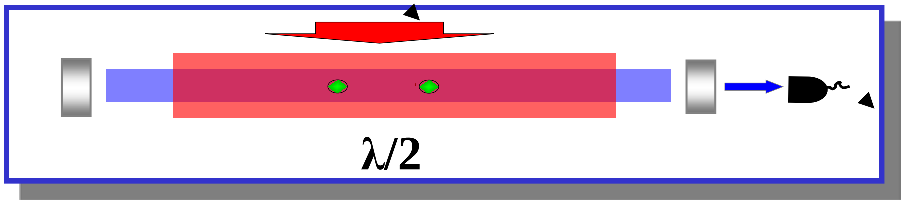
SINGLE RUN → single quantum trajectory → measurement back-action (entanglement)

Weak measurement vs unitary dynamics competition



Light-matter entanglement

Usual 2-slits Young experiment



Young experiment with ONE slit in the superposition of TWO positions

$$|\Psi\rangle_{\text{atom}} \sim |1\rangle_{\text{left}}|0\rangle_{\text{right}} + |0\rangle_{\text{left}}|1\rangle_{\text{right}}$$

SF for 1 atom at 2 sites

$$|\Psi\rangle_{\text{atom-light}} \sim |1\rangle_{\text{left}}|0\rangle_{\text{right}}|\alpha\rangle + |0\rangle_{\text{left}}|1\rangle_{\text{right}}|-\alpha\rangle$$

Light-matter entanglement

Measurement backaction: measuring light one affects the atomic state

Global structured measurement backaction

Due to the light-matter entanglement, measurement of one subsystem (light) affects another subsystem (quantum gas)

0. Initial light-matter state

$$|\Psi(0)\rangle = |\Psi^a(0)\rangle|\alpha_0\rangle = \sum_q c_q^0 |q_1, \ldots, q_M\rangle|\alpha_0\rangle$$

1. No-count process:
non-Hermitian evolution

$$H = H_0 - i\kappa a_1^\dagger a_1$$

$$|\Psi_c(t)\rangle = \frac{1}{F(t)} \sum_q c_q^0 e^{\Phi_q(t)} |q_1, \ldots, q_M\rangle|\alpha_q(t)\rangle$$

(Coherent light states are correlated with atomic Fock states)

2. One-count process:
***global* quantum jump**

$$|\Psi_c(t_+)\rangle \sim a_1|\Psi_c(t_-)\rangle$$

$$a_1 \sim \hat{D} = \sum_{l=1}^{R} \hat{N}_l e^{i2\pi l/R}$$

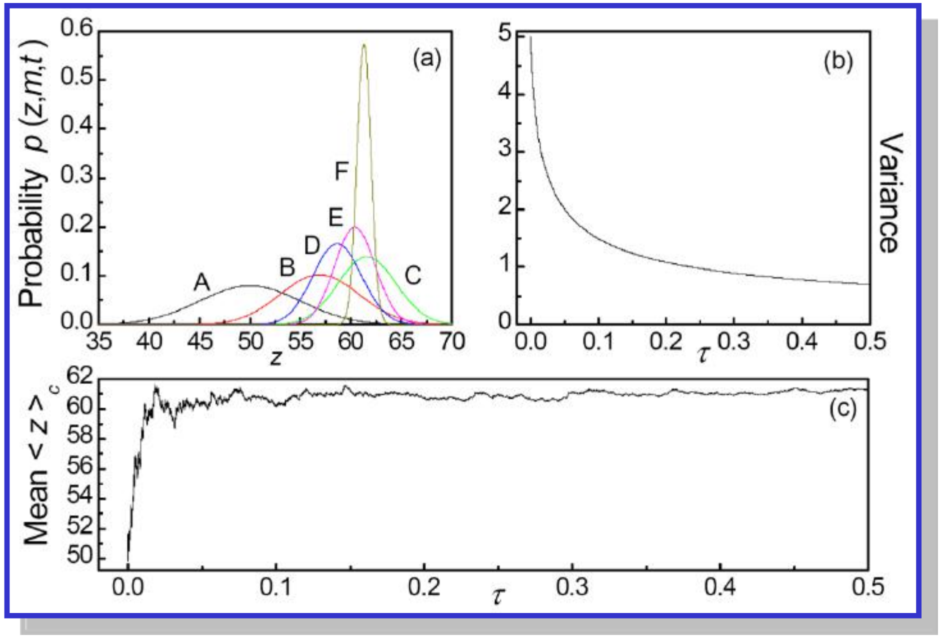
Quantum trajectory after *m* photocounts

$$|\Psi_c(m,t)\rangle = \frac{1}{F(t)} \sum_q \alpha_q^m e^{\Phi_q(t)} c_q^0 |q_1, \ldots, q_M\rangle|\alpha_q\rangle$$

Projection to a region of the initial Hilbert space still contains quantum superpositions. Quantum Zeno dynamics (*and beyond!*).

QND measurements: State preparation

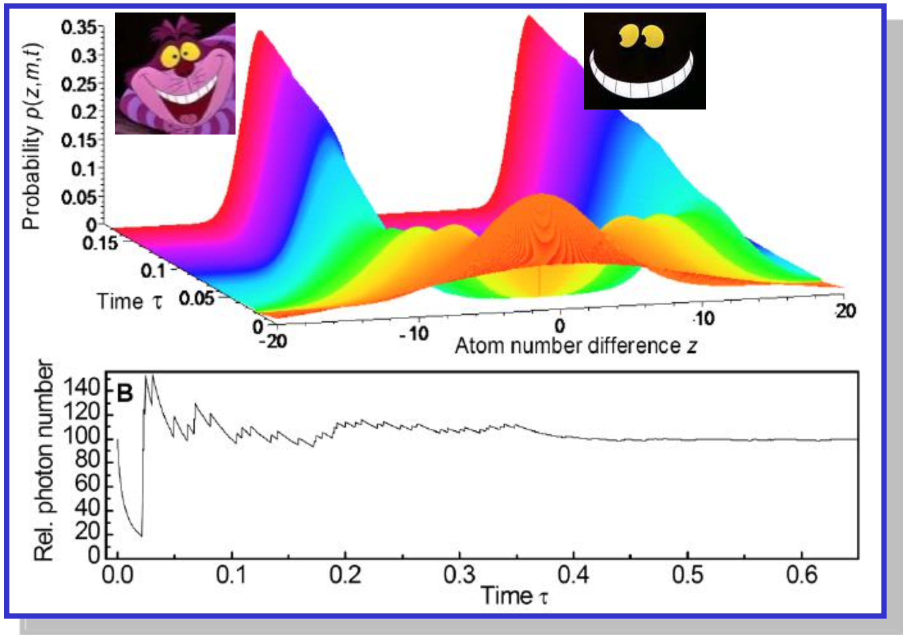
Bragg angle



Many-body Fock state (number squeezing)

$$|\Psi_c\rangle = |z_1, N - z_1\rangle|\alpha_{z_1}\rangle$$

Diffraction minimum



Schrödinger cat state (superposition)

$$|\Psi_c\rangle = (|z_1\rangle|\alpha_{z_1}\rangle + (-1)^m|-z_1\rangle|-\alpha_{z_1}\rangle)/\sqrt{2}$$

Characteristics at a single quantum trajectory (Quantum Monte Carlo)

Quantum state collapse is pre-selected by the optical geometry

→ Introducing and tailoring decoherence in many-body systems

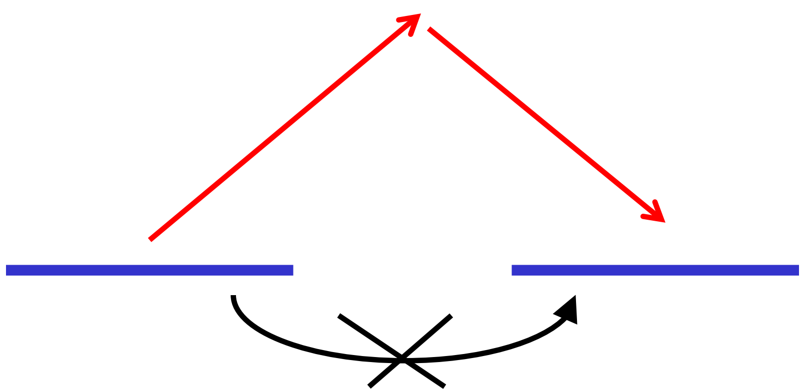
Weaker measurement: beyond Q. Zeno dynamics

PRA 93, 023632 (2016)
PRA 94, 012123 (2016)

$$H = H_0 - i\kappa a_1^\dagger a_1 \quad a_1 \sim \hat{D} = \sum_{l=1}^{R} \hat{N}_l e^{i2\pi l/R}$$

Bose-Hubbard Hamiltonian: $$\mathcal{H}^b = -t_0 \sum_{\langle i,j \rangle} (\hat{b}_i^\dagger \hat{b}_j + h.c) - \mu \sum_i \hat{n}_i + \frac{U}{2} \sum_i \hat{n}_i(\hat{n}_i - 1)$$

Non-Hermitian evolution via Raman-like transitions



Levels in a Q. Zeno subspace

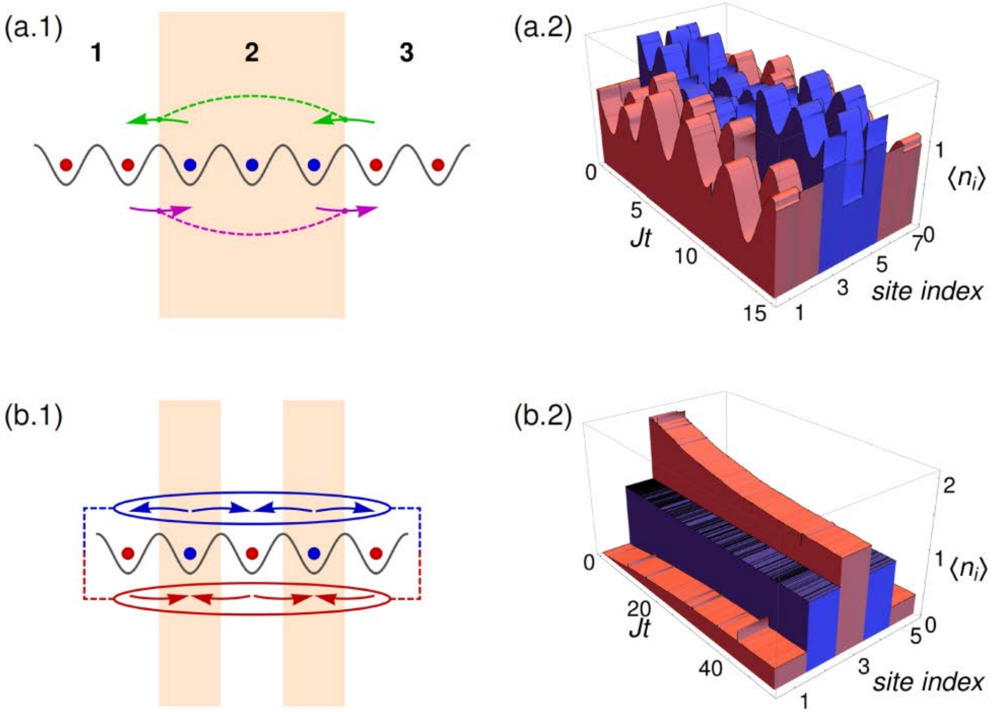
Rate:
tunnelling out / measurement strength

Quantum Zeno and long-range correlated tunneling

Suppression of the standard tunnelling (*quantum Zeno dynamics*).

Long-range correlated tunnelling (*via virtual processes, beyond QZD*).
Engineered extended Hamiltonians

Entangled systems and nonlocal correlated baths



$$\hat{H}_Z = \hat{P}_0 \Big[-J \sum_{\langle i,j\rangle} b_i^\dagger b_j - i\frac{J^2}{A\gamma}\sum_{\varphi} \times \sum_{\substack{\langle i\in\varphi, j\in\varphi'\rangle \\ \langle k\in\varphi', l\in\varphi\rangle}} b_i^\dagger b_j b_k^\dagger b_l \Big]\hat{P}_0,$$

$$\hat{a} = C(\hat{D} + \hat{B}) \qquad \gamma = |C|^2\kappa$$

$$C = \frac{g_{\text{out}} g_{\text{in}} a_0}{\Delta_a\left(\Delta_p + i\kappa\right)}$$

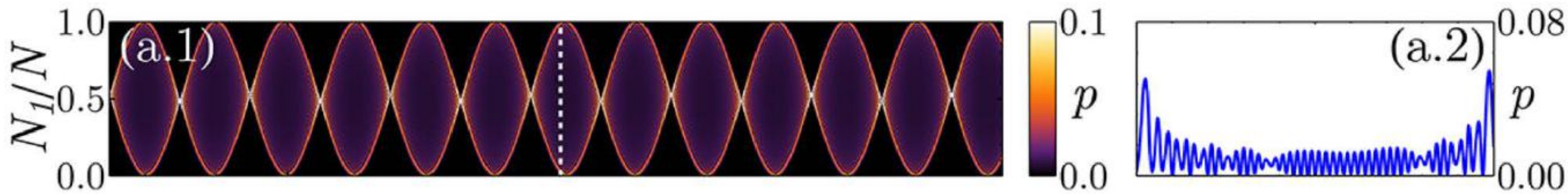
PRA 93, 023632 (2016)
PRA 94, 012123 (2016)

Very weak measurement

2 modes (odd and even sites)

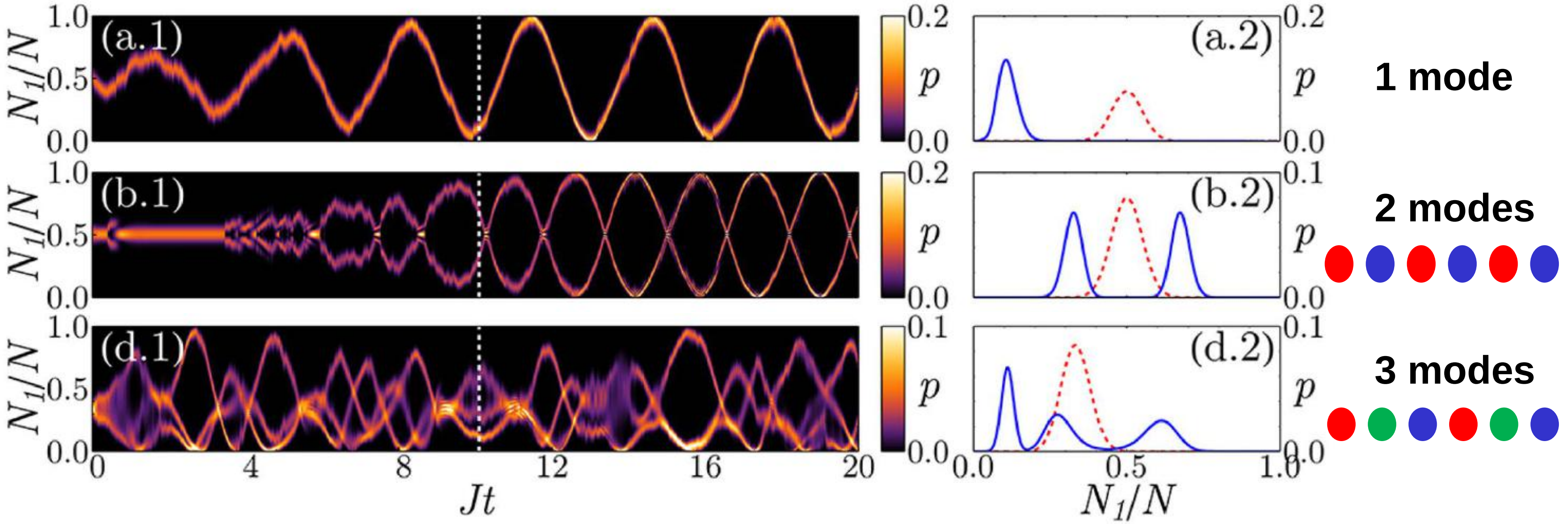
NJP 18, 073017 (2016)

Without measurement: spreading of the distribution



With measurement: competition leads to giant oscillations

multimode **Schrödinger cat (NOON) states**



Fermions: Measurement-induced AFM order

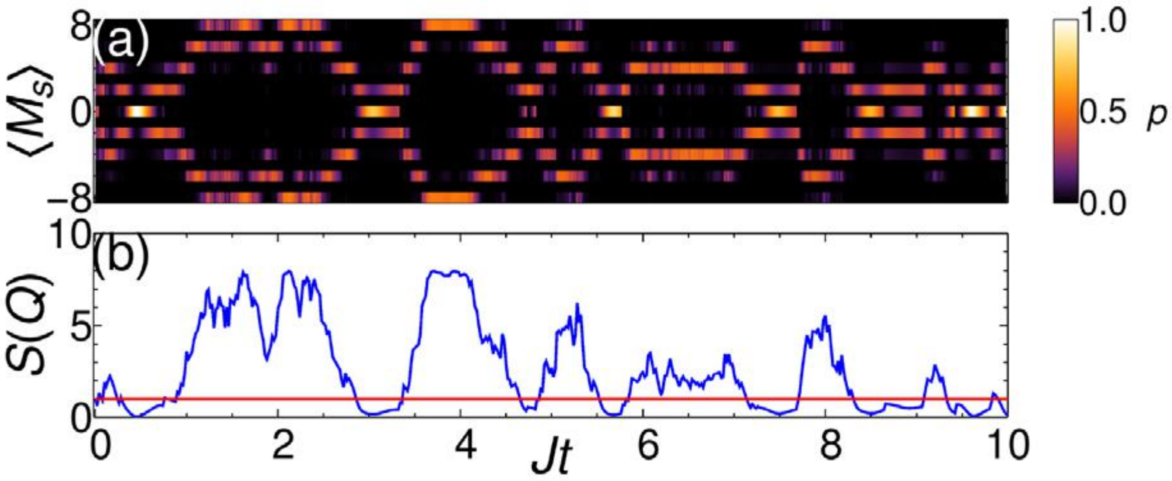
Similar to the measurement of density fluctuations in real space: in diffraction minimum,

$$\langle a^\dagger_{\text{out}} a_{\text{out}} \rangle \sim \langle (\hat{N}_{\text{odd}} - \hat{N}_{\text{even}})^2 \rangle$$

the light scattering can be sensitive to the **spin magnetization** ***M:*** $\langle (\hat{N}_\uparrow - \hat{N}_\downarrow)^2 \rangle$

Staggered magnetization:

$$a_{1y} = C \sum_{i=1} (-1)^i \hat{m}_i = C(\hat{M}_{\text{even}} - \hat{M}_{\text{odd}}) \equiv C\hat{M}_s$$



Competition of ***weak*** measurement backaction with tunnelling leads to **giant oscillations** of staggered magnetization: **antiferromagnetic order**

Feedback control

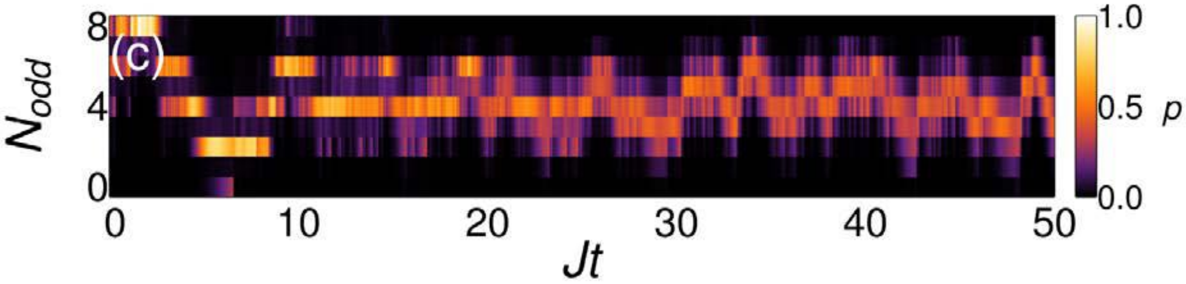
Scientific Rep. 6, 31196 (2016)

Optica (OSA) 3, 1213 (2016)

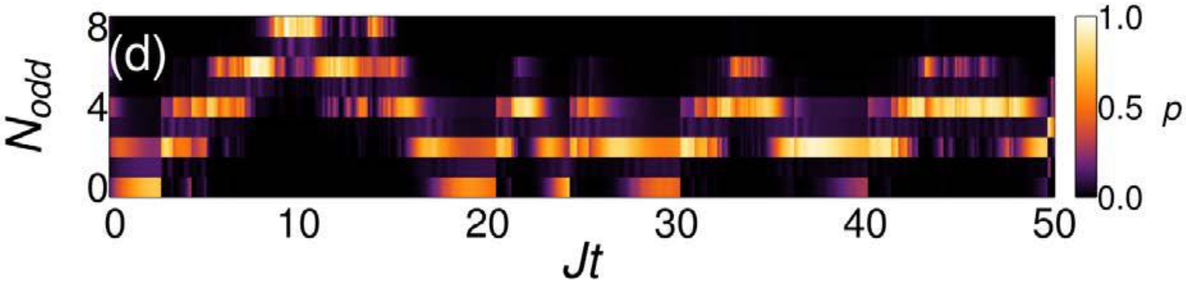
Fermions: Break-up and protection of pairs

PRA 93, 023632 (2016)

Ground state: pairs of **strongly interacting fermions**



Density measurement leads to the **break-up of the pairs**

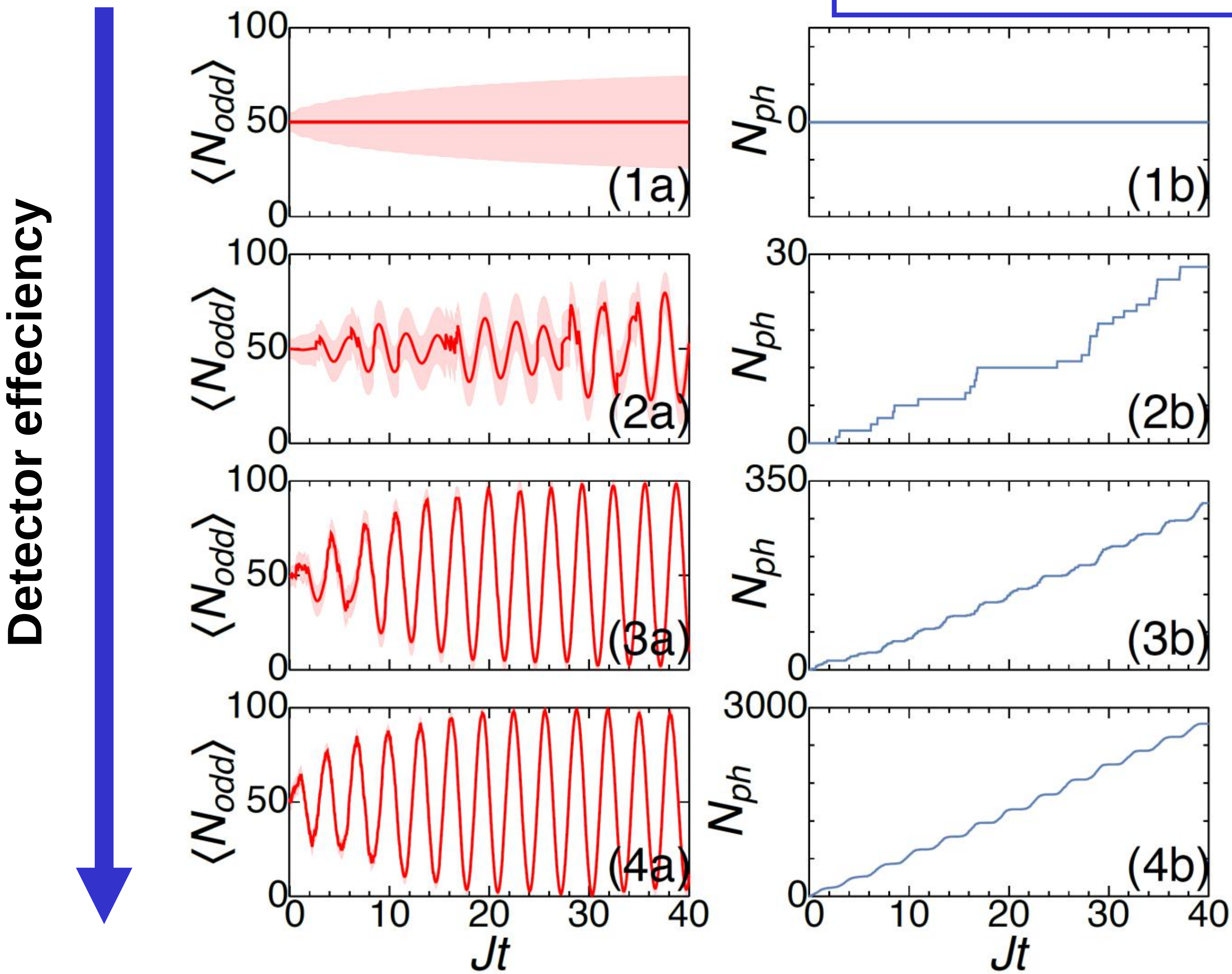


Adding the magnetization measurement, leads to the **protection of the pairs**

Imperfect detection

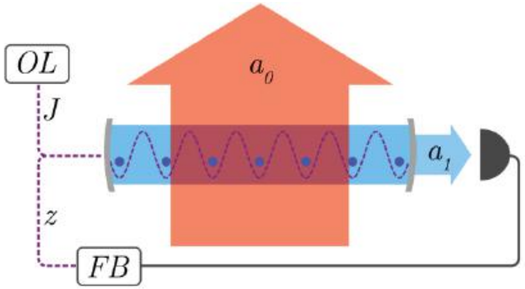
Stochastic master equation:
from dissipation to measurements

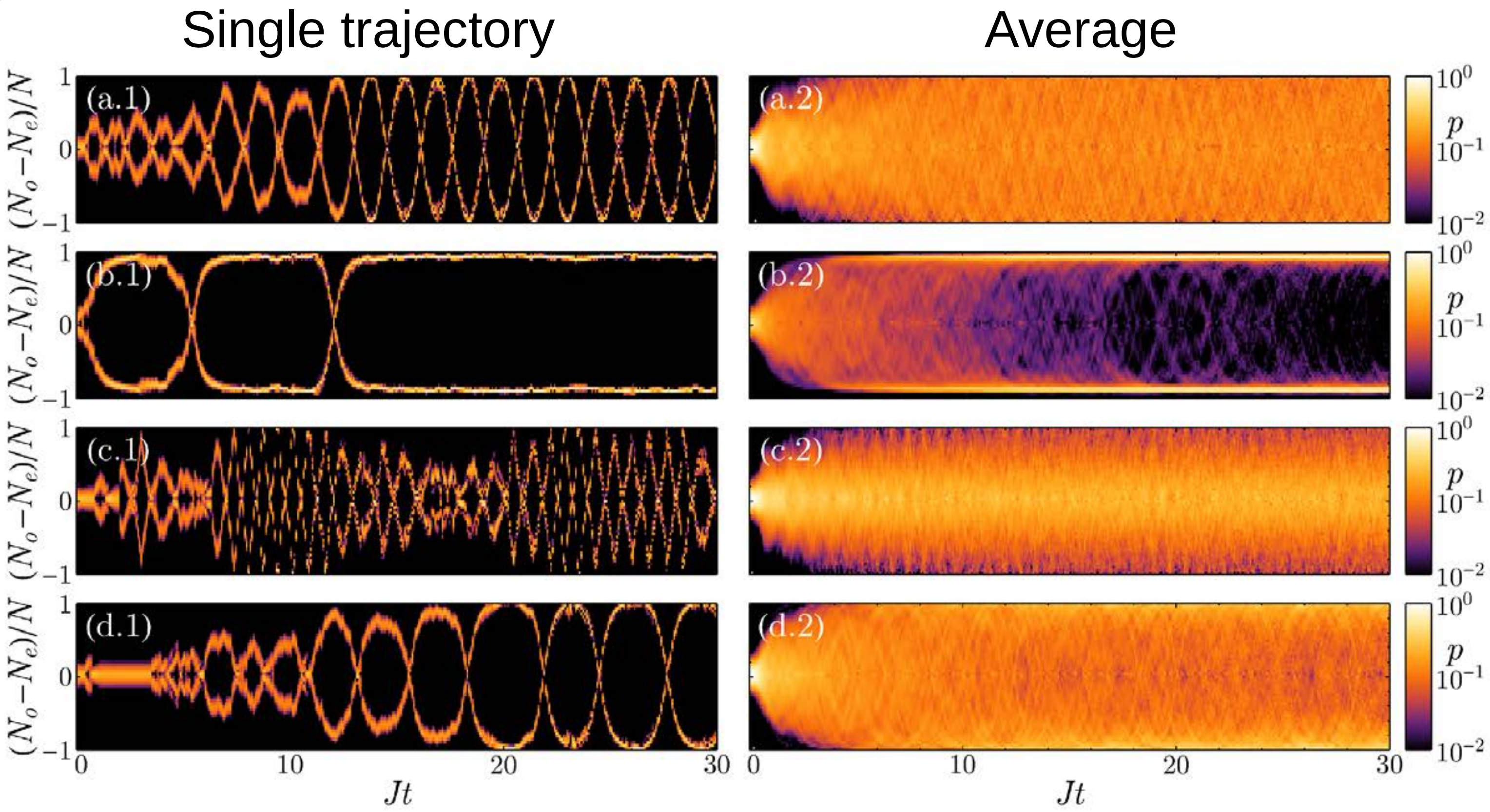
NJP 18, 073017 (2016)
Scientific Rep. 7, 42597 (2017)



Feedback control (beyond dissipative transitions)

Optica (OSA) 3, 1213 (2016)





- **Stabilization**
- **Frequency tuning**

Conclusions

- **New systems at the crossroad of several disciplines**

- **Opportunities for novel phenomena and methods**

- **Quantum engineering and technologies**
  - **Quantum simulations** (broader range of trial Hamiltonians)
  - **Quantum metrology and sensing** (NOON states, inertial sensors)

    Ph. Bouyer and A. Bertoldi (IOGS, Bordeaux)

  - **Quantum computation** (genuinely murtipartite entanglement)
  - **Quantum machine learning** (hidden Markov chains, Bayesian networks, reinforcement learning, feedback)

References (for all lectures)

Books

P. Meystre, Atom optics (2001);
M. Scully and S. Zubairy, Quantum optics (1997);
S. Haroche and J.-M. Raimond, Exploring quantum (2006);

Reviews

I. B. Mekhov and H. Ritsch, Journ. Phys. B. 45, 102001 (2012);
H. Ritsch, P. Domokos, F. Brennecke, and T. Esslinger, Rev. Mod. Phys. 85, 553 (2013);
M. J. Hartmann, J. Opt. 18, 104005 (2016);
C. Noh, S. R. Clark, D. Jaksch, D. G. Angelakis, arXiv:1705.04439 (2017);
M. Greiner, PhD Thesis (2003);

Quantum optical lattices

S. Caballero Benitez & I. B. Mekhov, Phys. Rev. Lett. 115, 243604 (2015);
S. Caballero-Benitez, G. Mazzucchi, & I. B. Mekhov, Phys. Rev. A 93, 063632 (2016);

Quantum measurements and feedback control of many-body systems

I. B. Mekhov & H. Ritsch, Nature Phys. 3, 319 (2007);
T. Elliott, W. Kozlowski, S. Caballero-Benitez, I. B. Mekhov, Phys. Rev. Lett. 114, 113604 (2015);
G. Mazzucchi, S. Caballero-Benitez, D. Ivanov, & I. B. Mekhov, Optica (OSA), 3, 1213 (2016);
W. Kozlowski, S. Caballero-Benitez, & I. B. Mekhov, Scientific Rep. 7, 42597 (2017).