Quantum optics of many-body systems

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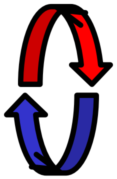
Lecture 2

universite PARIS-SACLAY

Previous lecture 1

Classical optics
light waves
material devices

light

matter

Atom optics
matter waves
light forces

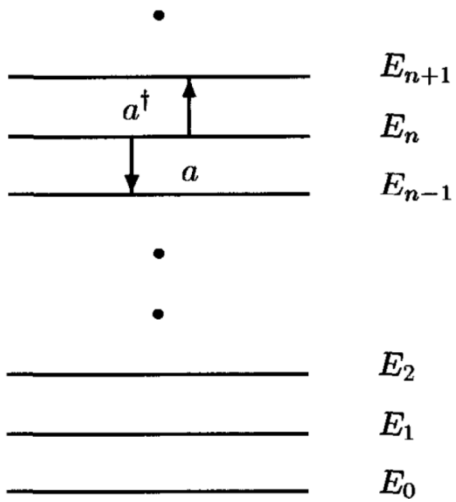
Devices:
beam splitters, mirrors, diffraction gratings, cavities

Quantum optics
classical (hot) atoms ☹
quantum light states ☺

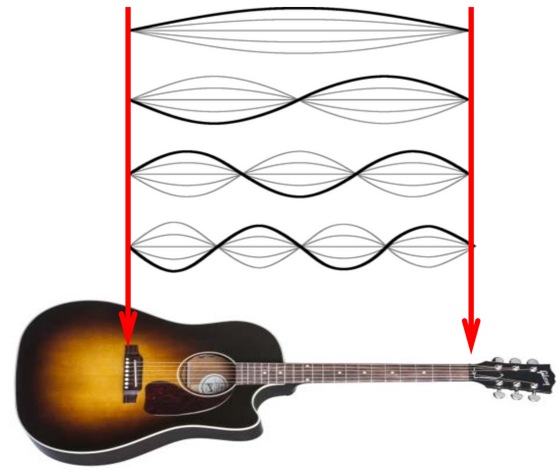
Quantum atom optics
☺ quantum atomic states
☹ classical light

☺ **Quantum optics of quantum gases** ☺

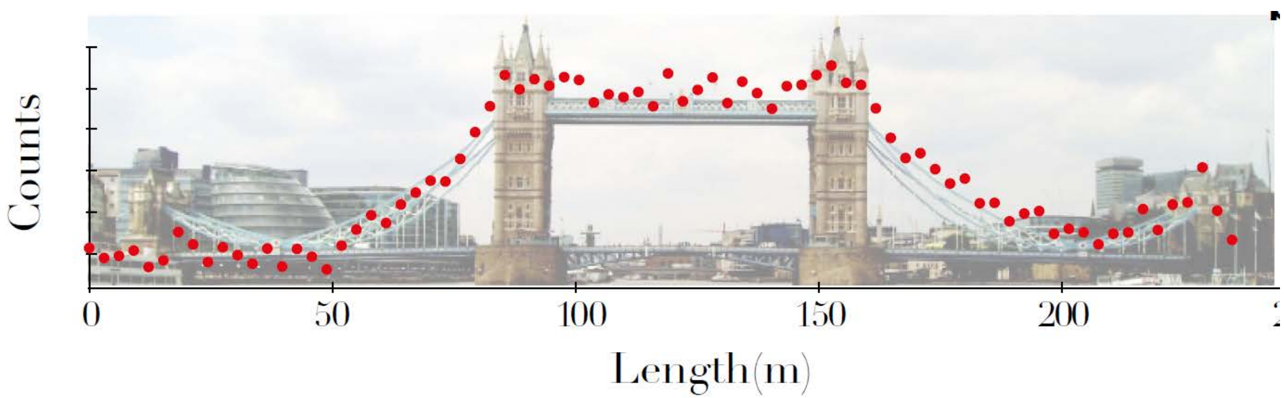
- **Photons: Discreteness of energy in a single mode, NOT discreteness of classical modes**



$$\mathcal{H} = \hbar \sum_j \omega_j \left(a_j^\dagger a_j + \frac{1}{2} \right)$$



A single photon:



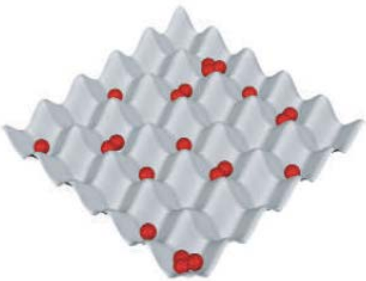
A. Kuhn, Oxford

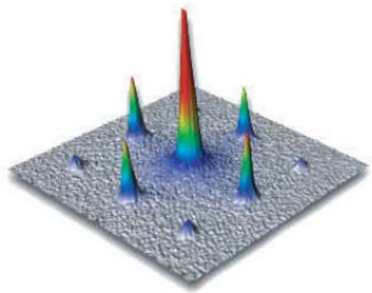
Previous lecture 1

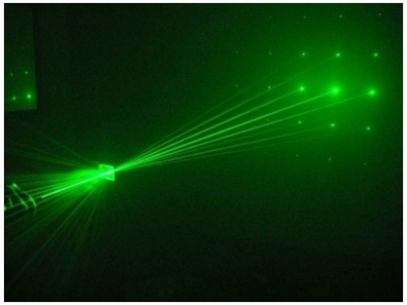
Coherent state

Superfluid state

$$|\Psi_{\mathrm{Coh}}\rangle = \prod_{i=1}^{M} |\mathrm{Coh}\rangle_i$$



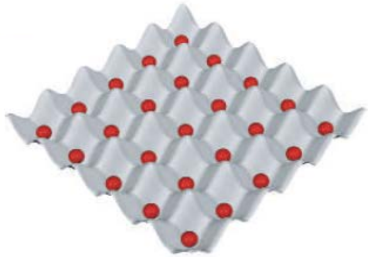


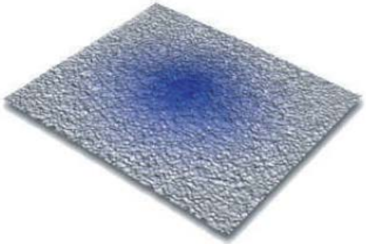


Fock state

Mott insulator

$$|1, 1, 1, ...1, ...\rangle$$





Mott insulator
sub-Poissonian distribution
nonclassical statistics

Further plan

- **Light-matter interaction**

  cavity QED, vacuum Rabi oscillations, Dicke superradiance (long-range interaction)

- **Manipulation of matter-waves by light**

  light forces for cooling and trapping

- **Quantum matter waves**

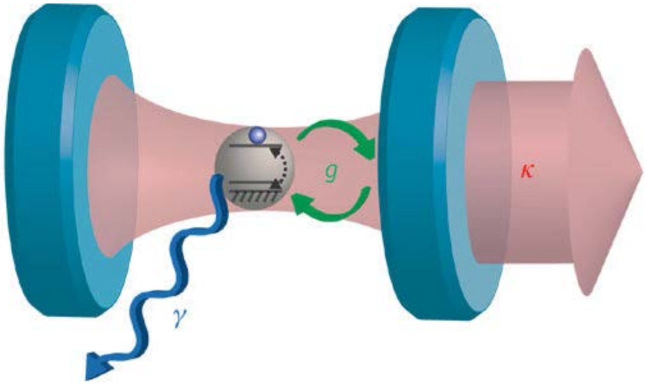
  second quantization of matter fields

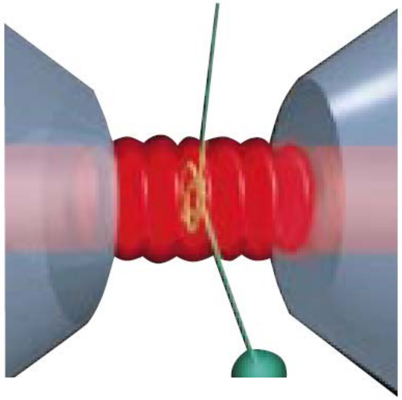
- **Quantum gases in optical lattices**

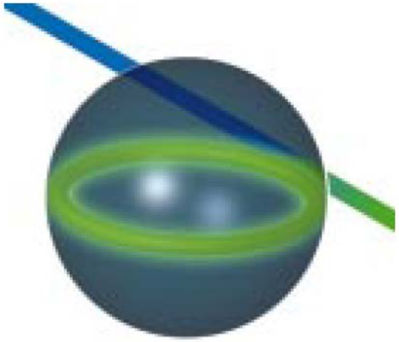
  superfluid, Mott insulator states

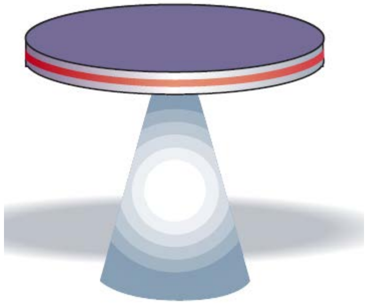
Light-matter interaction

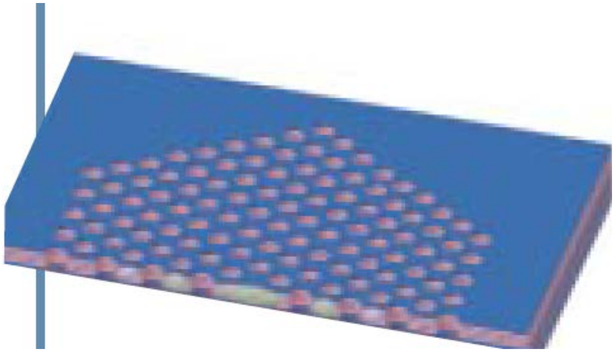
Single-mode cavity, two-level atom(s)

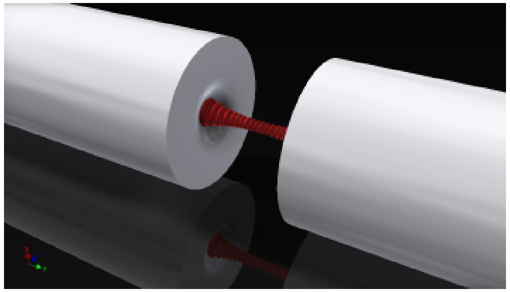




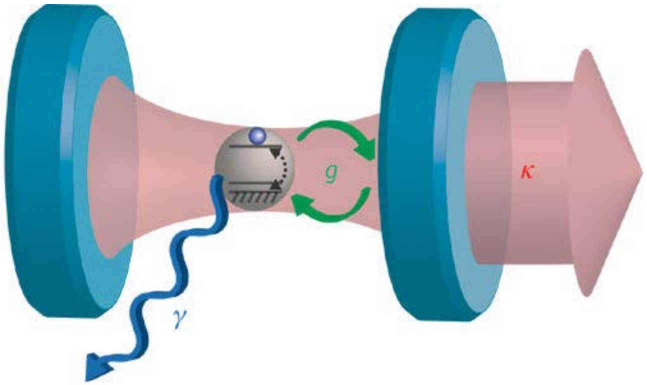






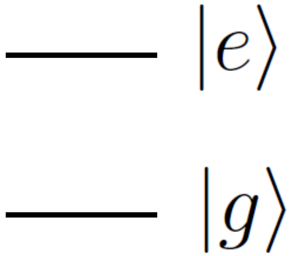


Light-matter interaction



Cavity QED (quantum electrodynamics):

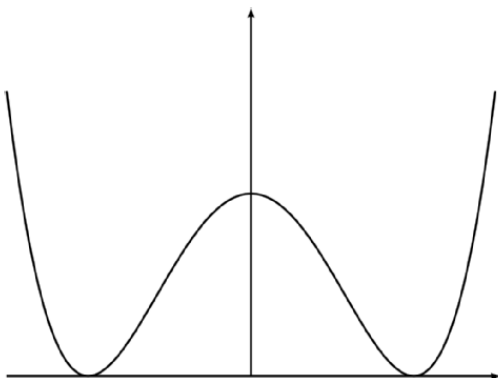
Two-level atom coupled to a single mode cavity



Atomic excited (e) and ground (g) states

Analogy for ultracold atoms:

Double-well potential (left and right wells)



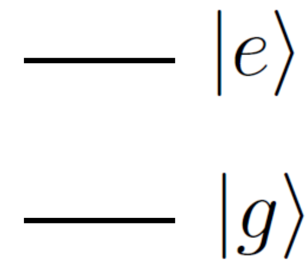
Full Hamiltonian: atom, light, and dipole interaction terms

$$\mathcal{H} = \mathcal{H}_{\text{Atom}} + \mathcal{H}_{\text{light}} + e\mathbf{r}\mathbf{E}$$

$$\mathcal{H}_{\text{light}} = \hbar\omega\left(a^{\dagger}a + \frac{1}{2}\right)$$

atom transition operators:

$$\sigma_{i,j} = |i\rangle\langle j| \qquad i, j = e, g$$



Using identity: $\sum_i |i\rangle\langle i| = I$

$$\mathcal{H}_{\text{Atom}} = \sum_i E_i |i\rangle\langle i| = \sum_i E_i \sigma_{i,i}$$

Interaction part:

$$e\mathbf{r} = \sum_{i,j} e|i\rangle\langle i|\mathbf{r}|j\rangle\langle j| = \sum_{i,j} d_{ij}\sigma_{i,j}$$

Dipole transition matrix element:

$$d_{ij} = e\langle i|\mathbf{r}|j\rangle \qquad d_{eg} = e\langle e|\mathbf{r}|g\rangle$$

Electric field for a single mode and fixed coordinate

$$E = \mathcal{E}(a + a^\dagger) \qquad \mathcal{E}_j = \sqrt{\frac{\hbar\omega_j}{\varepsilon_0 V}}$$

Interaction with a two-level atom

$$\mathcal{H} = \hbar\omega a^\dagger a + \frac{1}{2}\hbar\omega_a\sigma_z + \hbar g(\sigma_+ + \sigma_-)(a + a^\dagger)$$

Standard spin (Pauli) operator

$$\sigma_+ = |e\rangle\langle g| \quad \sigma_- = |g\rangle\langle e|$$

raising and lowering operators

$$\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$$

$$[\sigma_-, \sigma_+] = -\sigma_z \qquad [\sigma_-, \sigma_z] = 2\sigma_-$$

Light-matter coupling constant

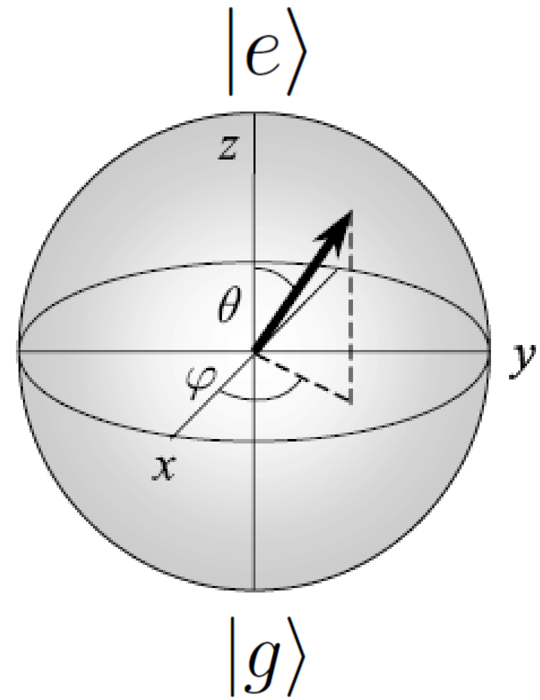
$$g = \sqrt{\frac{d^2\omega_a}{2\hbar\varepsilon_0 V}}$$

$$\mathcal{E}_j = \sqrt{\frac{\hbar\omega_j}{\varepsilon_0 V}}$$

$$\langle n|E^2|n\rangle = 2\mathcal{E}^2\left(n + \frac{1}{2}\right) \neq 0$$

Bloch sphere

$$|\psi\rangle = \cos\frac{\theta}{2}|e\rangle + e^{i\varphi}\sin\frac{\theta}{2}|g\rangle$$



$$\sigma_+ = \sigma_x + i\sigma_y \qquad \sigma_- = \sigma_x - i\sigma_y \qquad \sigma_x^2 = \sigma_y^2 = \sigma_z^2 = I$$

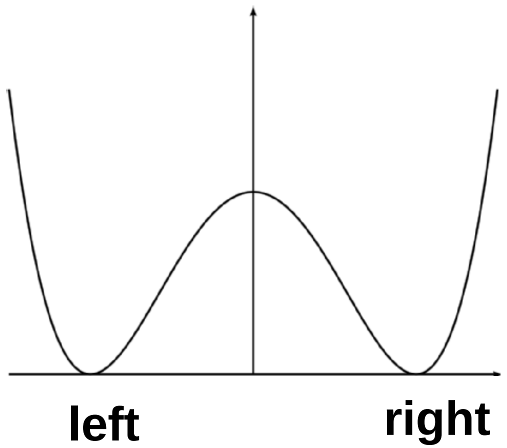
Jaynes-Cummings model, rotating wave approximation

$$\mathcal{H} = \hbar\omega a^\dagger a + \frac{1}{2}\hbar\omega_a\sigma_z + \hbar g(\sigma_+ + \sigma_-)(a + a^\dagger)$$

$$\boxed{\mathcal{H} = \hbar\omega a^\dagger a + \frac{1}{2}\hbar\omega_a\sigma_z + \hbar g(\sigma_+ a + a^\dagger\sigma_-)}$$

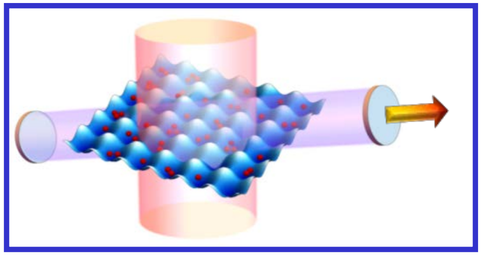
$$\sigma_+ = |e\rangle\langle g|$$

Cold atoms: Josephson oscillations



$$J|\mathrm{left}\rangle\langle\mathrm{right}|$$

Can it be an operator?



Quantum potential?
Quantum optical lattices?

T. Esslinger, A. Hemmerich (2015, 2016)

Vacuum Rabi oscillations

$$i\hbar\frac{\partial}{\partial t}|\psi\rangle = \mathcal{V}|\psi\rangle$$

$$\hbar g(\sigma_+ a + a^\dagger \sigma_-)$$

$$|\psi(t)\rangle = \sum_n (c_{e,n}(t)|e,n\rangle + c_{g,n}(t)|g,n\rangle)$$

Light-matter entanglement

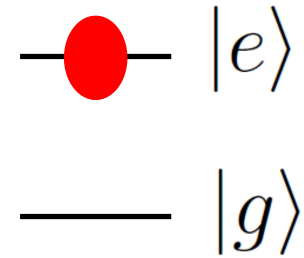
Coupling only between the pairs of levels

$$|e,n\rangle \longleftrightarrow |g,n+1\rangle$$

$$\dot{c}_{e,n} = -ig\sqrt{n+1}c_{g,n+1}$$
$$\dot{c}_{g,n+1} = -ig\sqrt{n+1}c_{e,n}$$

oscillations

Initial conditions: atom in the excited state, no photons



$$t = 0 \qquad c_{e,n}(0) = c_n(0) \qquad c_{g,n+1}(0) = 0$$

$$c_{e,n}(t) = c_n(0)\cos\frac{\Omega_n t}{2} \qquad c_{g,n+1}(t) = -ic_n(0)\sin\frac{\Omega_n t}{2}$$

$$\Omega_n = 2g\sqrt{n+1}$$

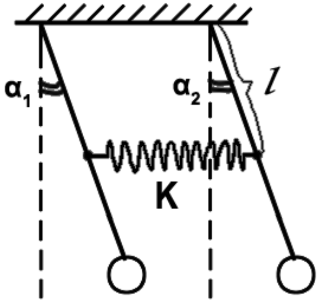
Population inversion

$$W(t) = \sum_n \left(|c_{e,n}(t)|^2 - |c_{g,n}(t)|^2\right) \qquad \rho_{nn}(0) = |c_n(0)|^2$$

$$W(t) = \sum_n \rho_{nn}(0)\cos(\Omega_n t)$$

In general, multi-frequency oscillations

Initial vacuum field



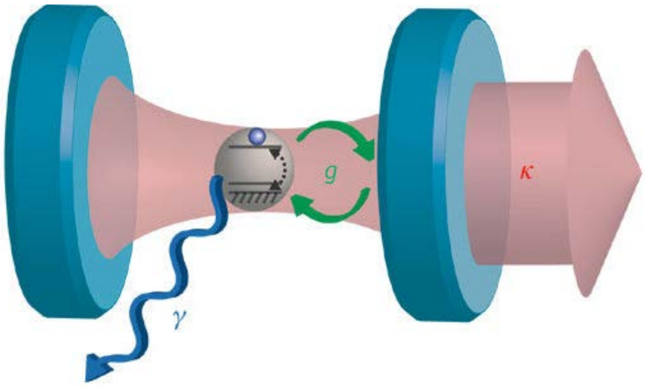
$$\rho_{nn}(0) = |c_n(0)|^2 \qquad \rho_{nn}(0) = \delta_{n0}$$

$$W(t) = \cos(\Omega_0 t) \qquad \Omega_0 = 2g$$ Vacuum Rabi frequency

Vacuum Rabi oscillations

What is quantum? – They start with no photons present: due to the vacuum fluctuations

Reversible spontaneous emission



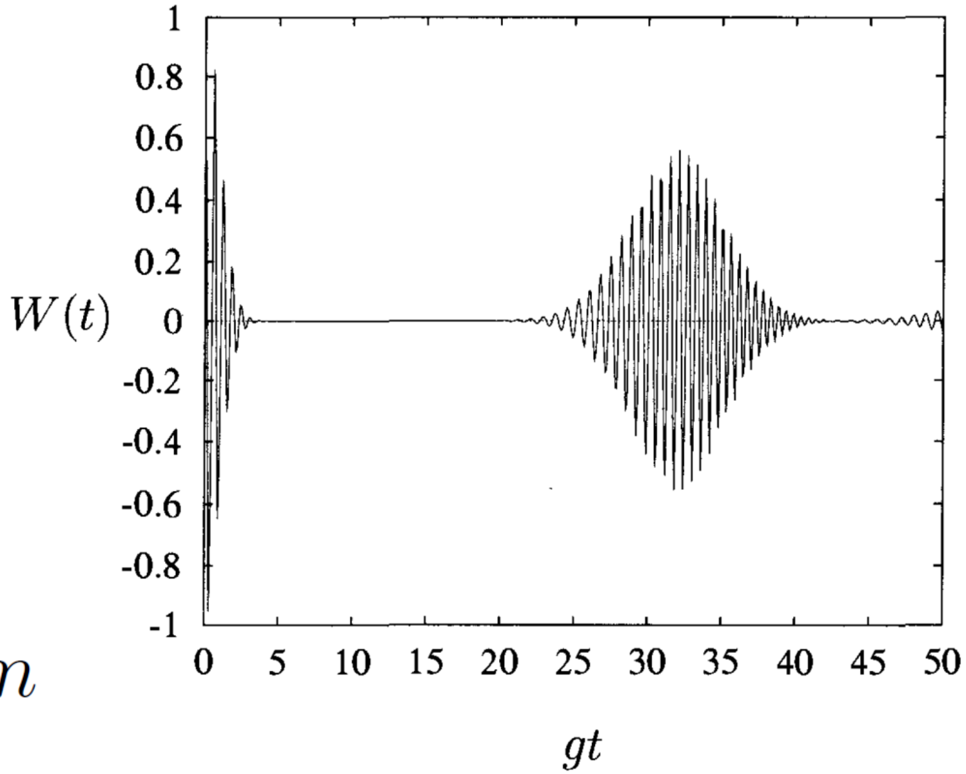
Collapse and revival

Initial coherent field

$$\rho_{nn}(0) = \frac{\langle n \rangle^n e^{-\langle n \rangle}}{n!}$$

$$W(t) = \sum_n \rho_{nn}(0) \cos(\Omega_n t)$$

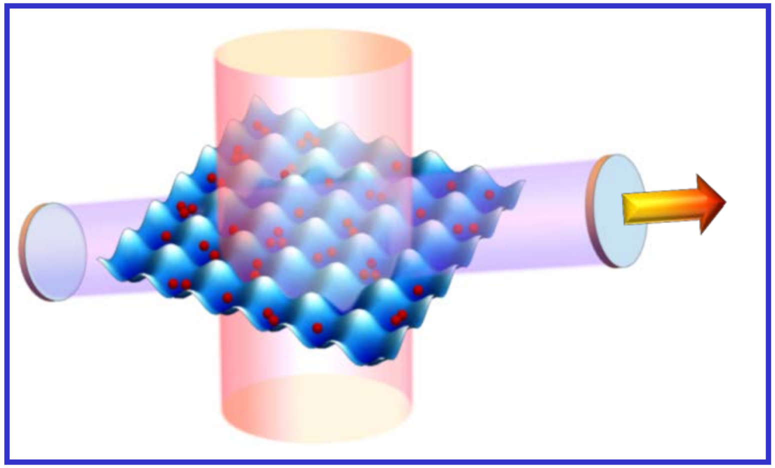
$$(\Omega_{\langle n \rangle} - \Omega_{\langle n \rangle - 1}) t_r = 2\pi m$$

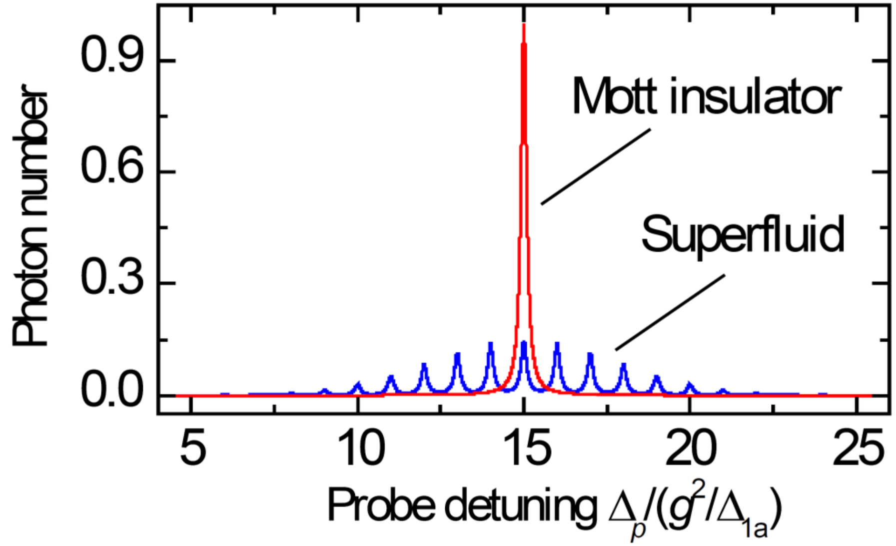


$\langle n \rangle \gg 1$ $\quad t_r \approx 2\pi m \sqrt{\langle n \rangle}/g$ Due to the discreetness of photons

Analogous effects in quantum atom optics:
due to the discreteness of quantum mater waves

Seeing the matter wave discreteness in a nondestructive way

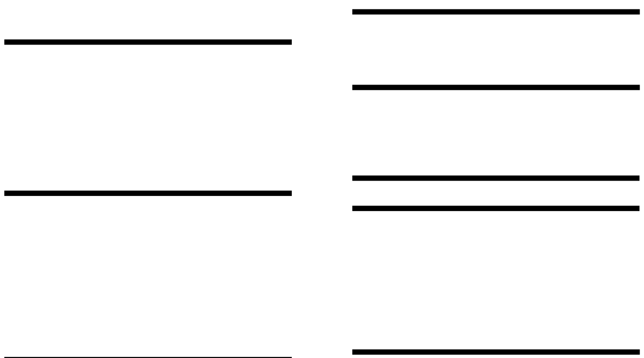




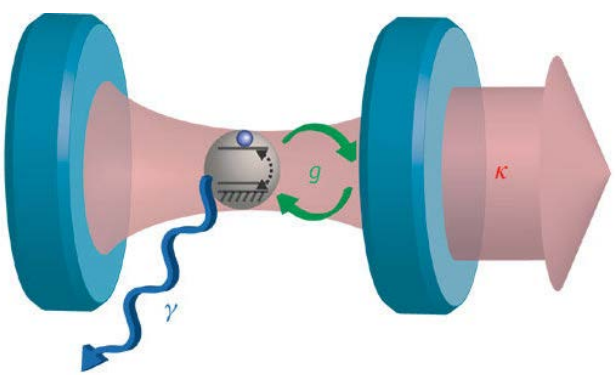
Dressed states

Strong coupling

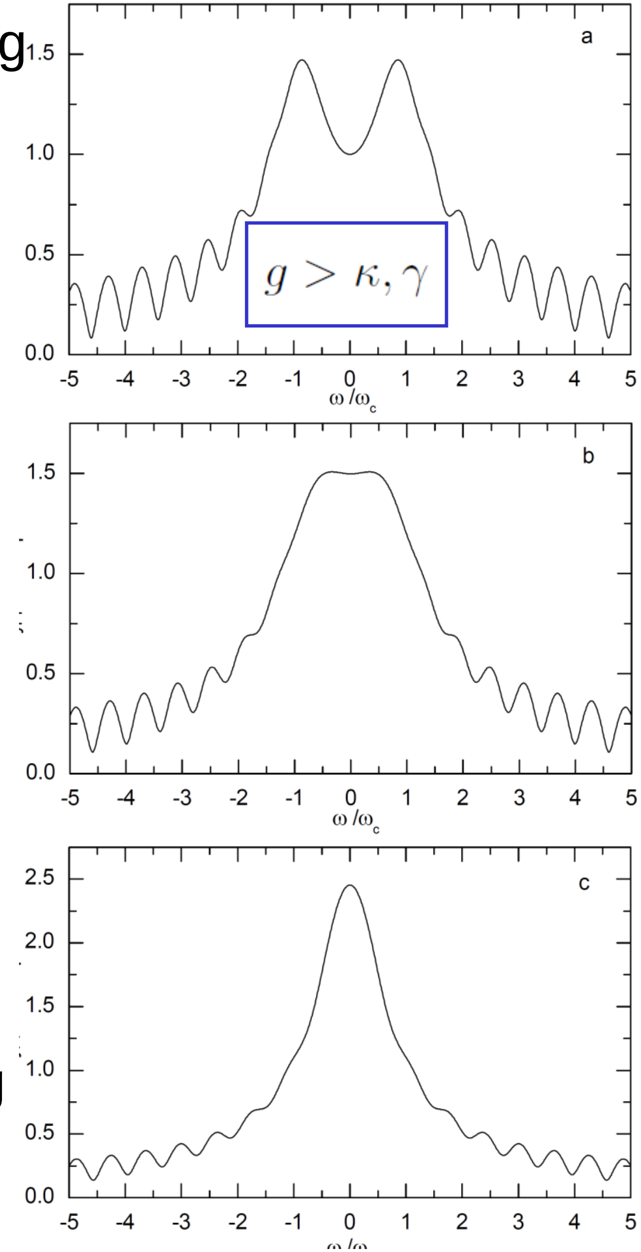
$$\mathcal{H}\Psi_n^{\pm} = \hbar\left(\omega(n - \frac{1}{2}) \pm \frac{\Omega_n}{2}\right)\Psi_n^{\pm}$$



$$\Omega_n = 2g\sqrt{n+1}$$



Weak coupling



Many-atom effects, superradiance

$$\mathcal{H} = \hbar\omega a^\dagger a + \frac{1}{2}\hbar\omega_a \sum_n \sigma_z^n + \hbar g(a \sum_n \sigma_+^n + a^\dagger \sum_n \sigma_-^n)$$

$$J_\pm = \sum_n \sigma_\pm^n \qquad J_z = \frac{1}{2}\sum_n \sigma_z^n$$

Angular momentum: $\mathbf{J} = (J_x, J_y, J_z) \qquad J_\pm = J_x \pm iJ_y$

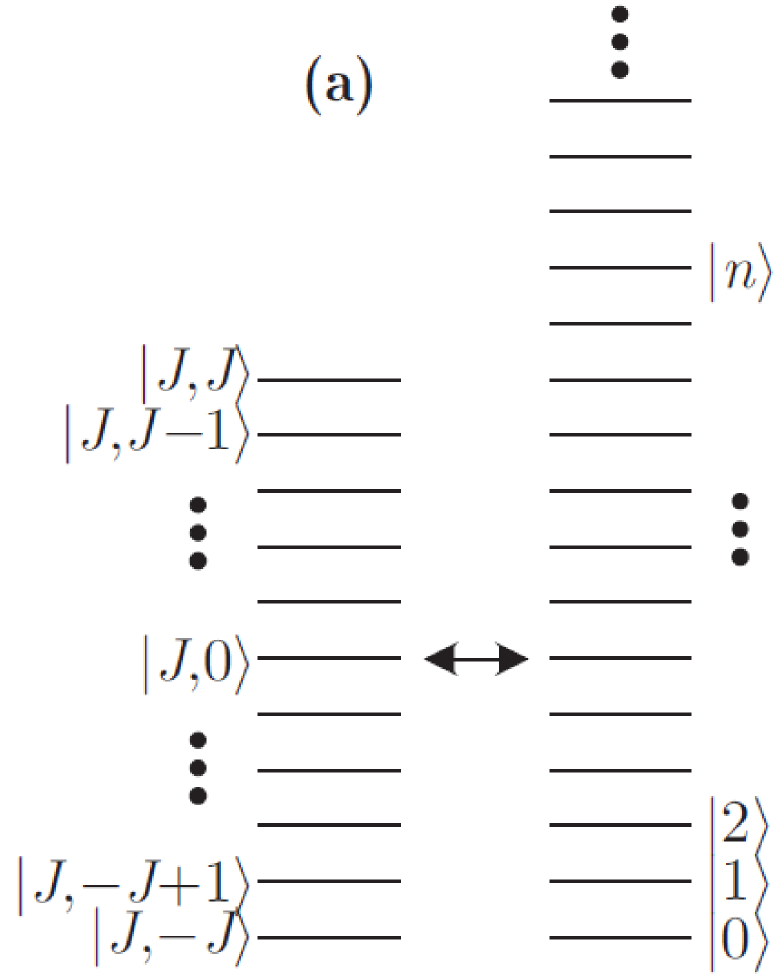
$$\mathcal{H} = \hbar\omega a^\dagger a + \hbar\omega_a J_z + \hbar g(aJ_+ + a^\dagger J_-)$$

$J = N/2$ - maximal

Dicke states $|J, M\rangle$

$$J_Z\,|J, M\rangle = M\,|J, M\rangle$$

$$|e, e, \ldots, e\rangle$$



$$|g, g, \ldots, g\rangle$$

$$J_{\pm}\,|J, M\rangle = \sqrt{(J \pm M + 1)(J \mp M)}\,|J, M \pm 1\rangle$$

Long-range correlation: do not depend on the position, maximal for *M* = 0

$$\langle J, M|\, \sigma_{+,i}\sigma_{-,j}\, |J, M\rangle = \frac{J^2 - M^2}{\mathcal{N}(\mathcal{N}-1)}$$

N/2 excitations are symmetrically shared between all atoms

$\mathcal{N} = 2$ $$(|e,g\rangle + |g,e\rangle)/\sqrt{2}$$ *Entanglement*

Long-range interaction (global)

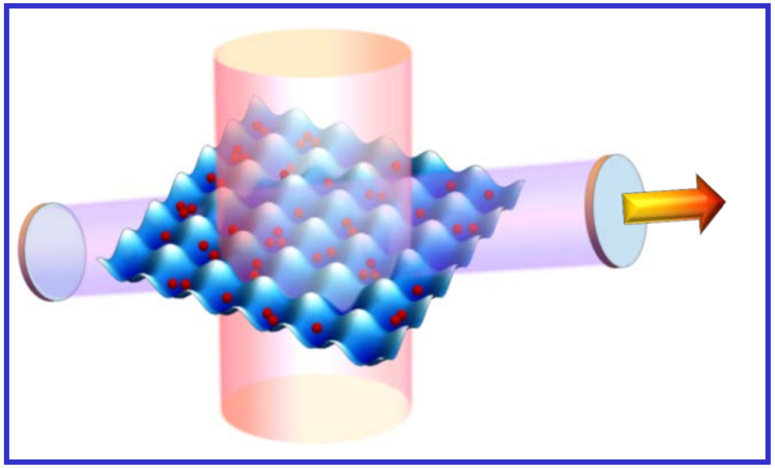
$$\mathcal{H} = \hbar\omega a^\dagger a + \frac{1}{2}\hbar\omega_a \sum_n \sigma_z^n + \hbar g(a \sum_n \sigma_+^n + a^\dagger \sum_n \sigma_-^n)$$

$$\hbar g \sum_{n,k} \sigma_+^n \sigma_-^k$$

$$\hbar g \sum_{n,k} A(r_n - r_k)\sigma_+^n \sigma_-^k$$

How a global interaction can compete with short-range one?

$$\hbar g \sum_{n,k} A(r_n - r_k)\sigma_+^n \sigma_-^k$$



Alternating coupling constants

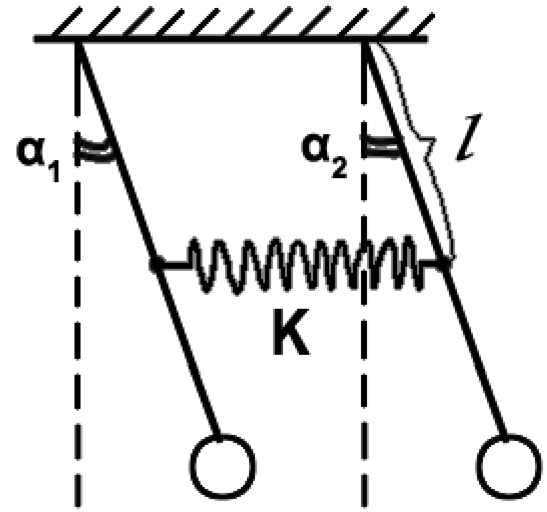
Many modes

Classical view: coupled oscillators

$$\dot{a} = -igN\sigma_-$$

$$\dot{\sigma}_- = ig\sigma_z a$$

$$\dot{\sigma}_z = 2ig(a^\dagger\sigma_- - \sigma_+ a)$$

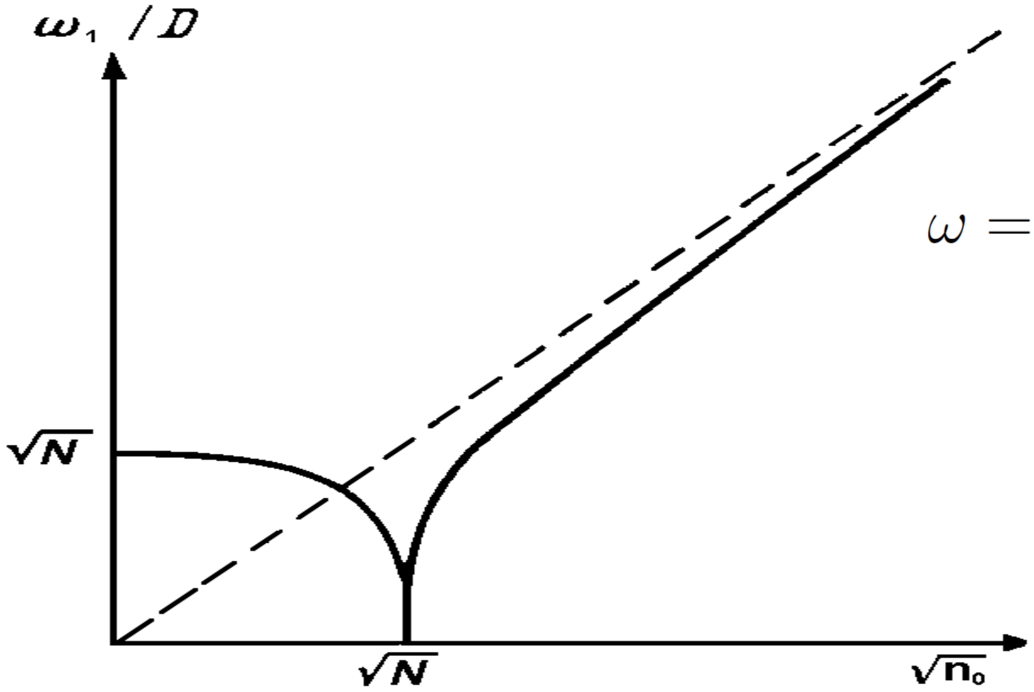


Linear problem

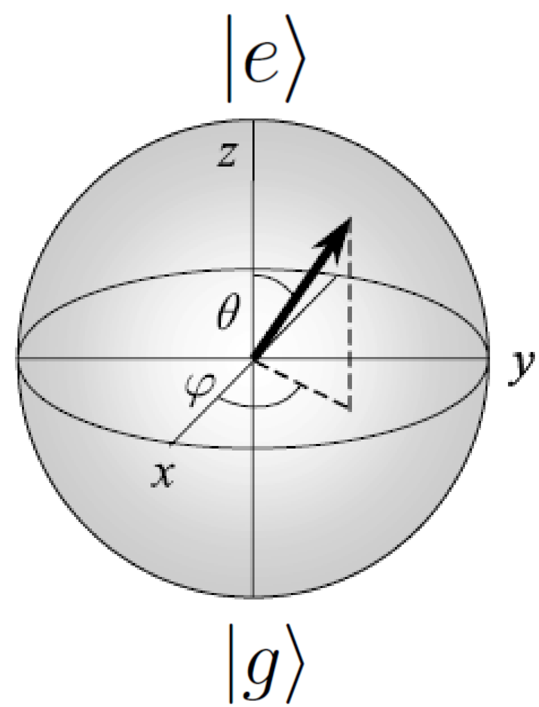
$$\ddot{a} + \omega_0^2 a + 0 \qquad \sigma_z = -1 \qquad \omega_0 = g\sqrt{N}$$

Nonlinear pendulum

$$\ddot{\theta} + \omega_0 \sin\theta = 0$$



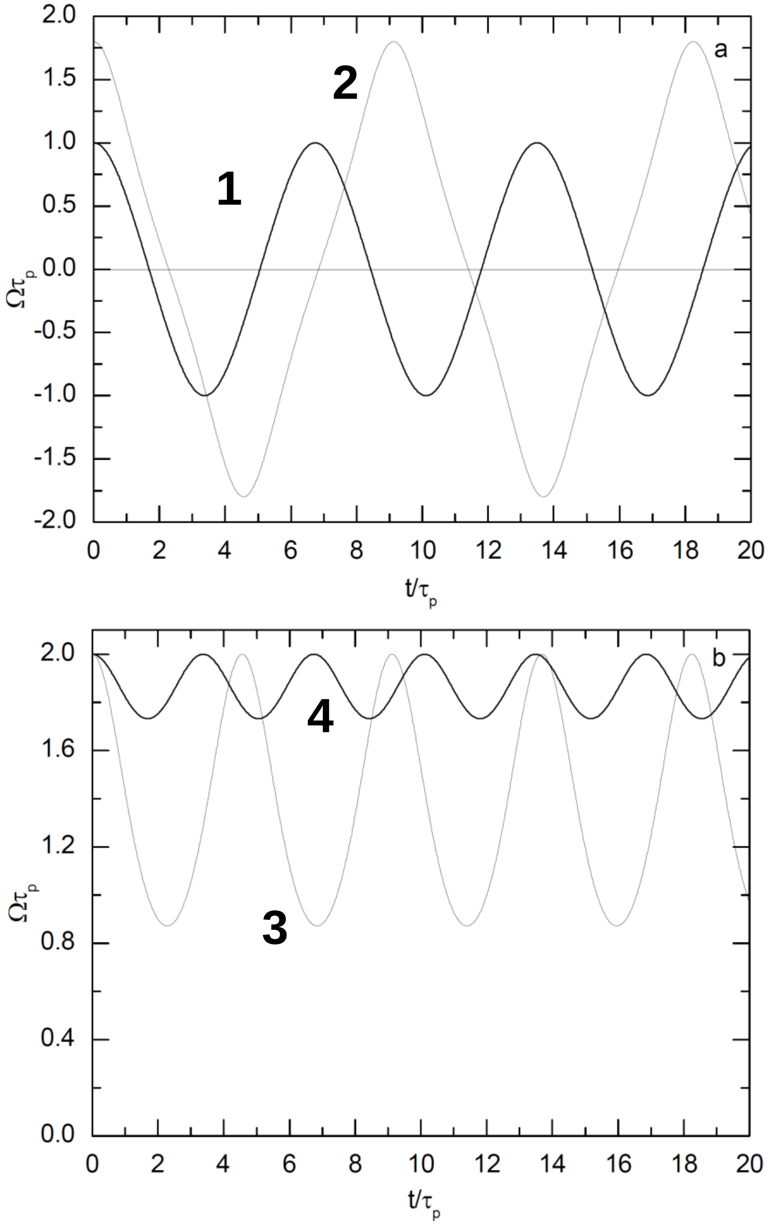
$$\omega = g a_0 = \frac{dE}{\hbar} = \Omega_{\mathrm{Rabi}}$$

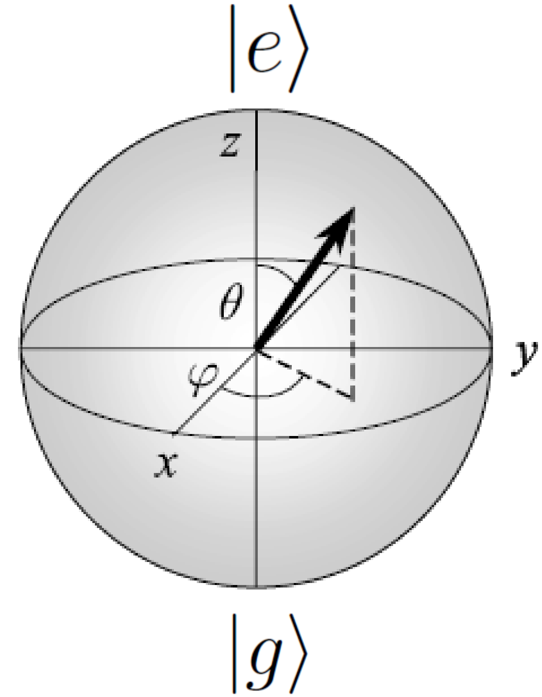


Weak field: “vacuum” Rabi oscillations

Strong field: Rabi oscillations

Strong field degrades collective effects


a
b
1
2
3
4
Ωτp
t/τp

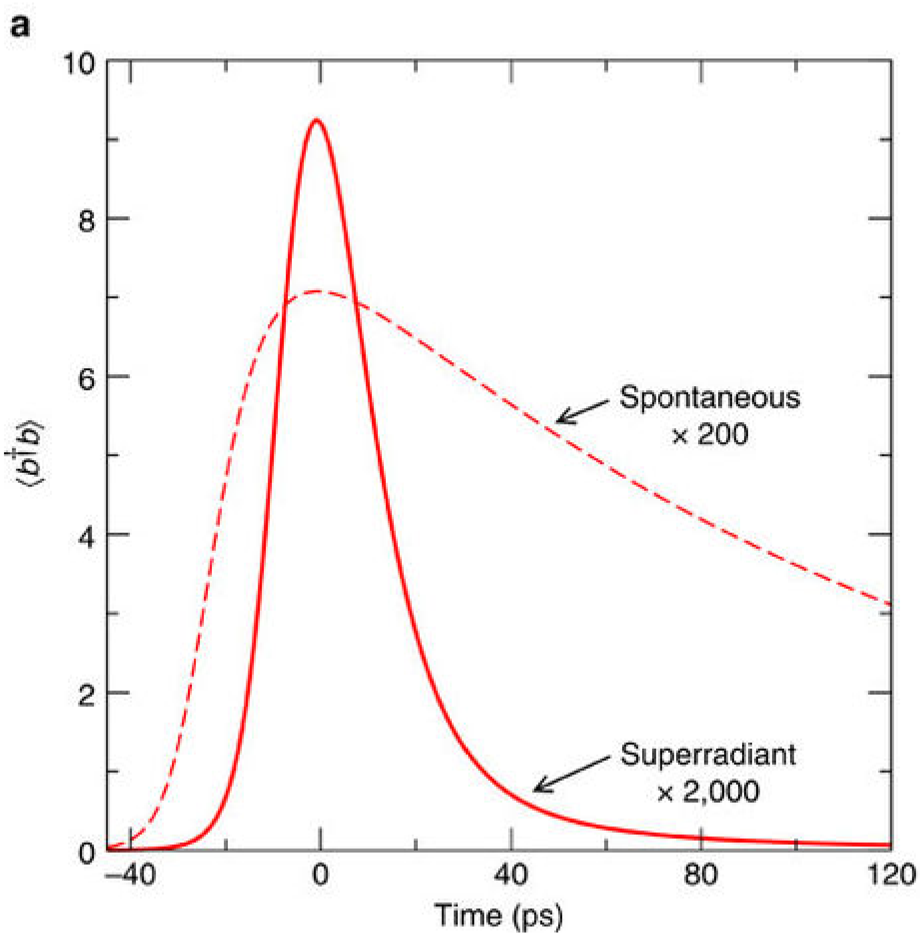

$|e\rangle$
$|g\rangle$
z
x
y
θ
φ

Overdamped regimes: Purcell effect and superradiance

$$\dot{a} = -igN\sigma_- - \kappa a$$

$$\dot{\sigma}_- = ig\sigma_z a - \gamma\sigma_-$$

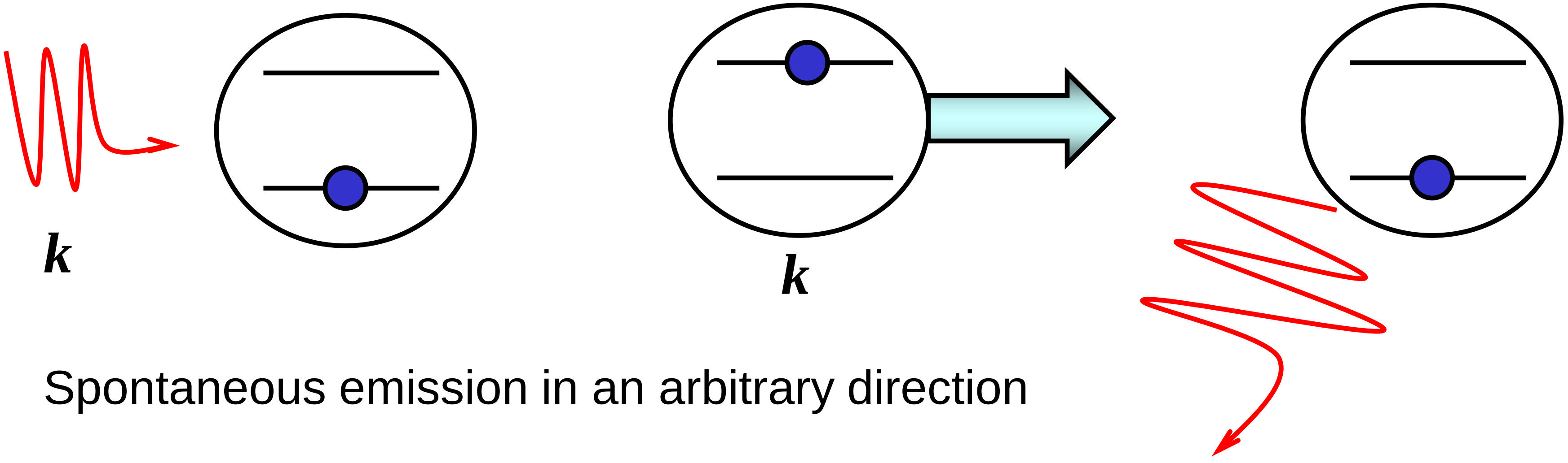
$$\Gamma_{\text{total}} = 2\gamma(1 + 2C) \qquad 2C = \frac{g^2 N}{\kappa\gamma}$$



Collective enhancement of spontaneous emission

Matter wave superradiance (W. Ketterle)

Light forces acting on atoms



Spontaneous emission in an arbitrary direction

In average: recoil momentum $\delta p = \hbar k$

Force (dissipative, absorptive): $F_{\text{diss}} = r\hbar k$

Emission rate: $r = \Gamma \rho_{ee}$

Excited state population $\rho_{ee} = \dfrac{\Omega_R^2}{4\Delta^2 + \Gamma^2 + 2\Omega_R^2}$

$$\Delta = \omega_a - \omega_l$$

Doppler shift for a moving atom $\omega = \omega_l \pm kv$

$$F_{\text{diss}} = \hbar k \Gamma \frac{\Omega_R^2}{4(\Delta \mp kv)^2 + \Gamma^2 + 2\Omega_R^2}$$

Weak light field (no saturation of the atomic transition), slow atomic motion:

$$F_{\text{diss}} = F_0 \pm \beta m v$$

$$F_0 = \hbar k \Gamma \frac{\Omega_R^2}{4\Delta^2 + \Gamma^2} \qquad \beta = 8\hbar k^2 \Gamma \frac{\Omega_R^2 \Delta}{m(4\Delta^2 + \Gamma^2)^2}$$

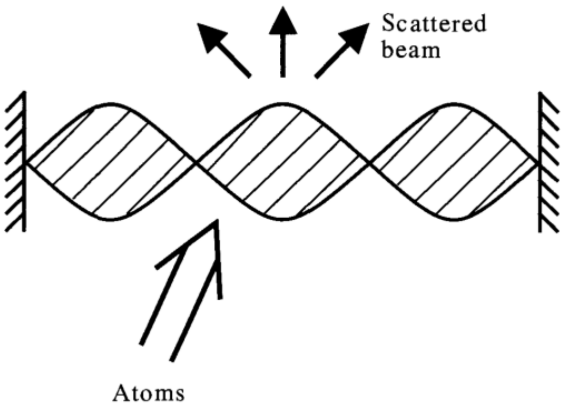
$$\boxed{F_{\text{standing wave}} = -2\beta m v}$$

Doppler cooling

$k_B T \approx \hbar \Gamma$

limit

Diffraction/ beam splitting by light



$$\mathcal{H} = -d\mathcal{E}_0 \sin kz$$

Initial wave: $\psi(z,0) = e^{ik_0 z}$

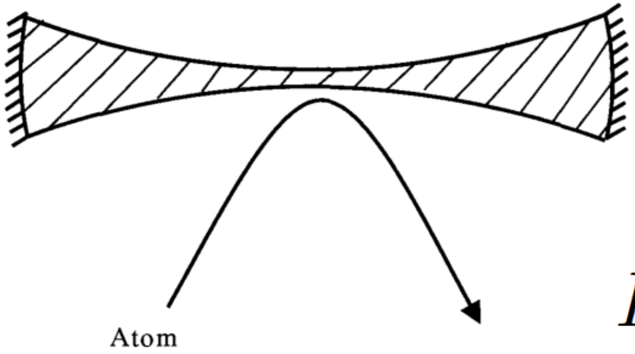
Interaction: $\psi(z,t) = \exp\left(-\frac{i}{\hbar}\mathcal{H}t\right)\psi(z,0) = e^{-i\Omega_R \sin kz} e^{-ik_0 z}$

$$\psi(z,t) = \sum_{n=-\infty}^{\infty} J_n(\Omega_R t) e^{i(k_0+nk)z}$$

Interchange with integer number of photons leads to the diffraction of the atomic wave

Two peaks: beam splitter

Trapping, collimating, reflecting



$$\mathcal{H} = -dE(\mathbf{r}, t)$$

$$E(\mathbf{r}, t) = \frac{1}{2}\mathcal{E}_0(x, y)e^{-i(\omega t - kz)} + \text{c.c.}$$

Induced dipole moment:

$$\langle d \rangle = d_{eg}\rho_{eg}e^{i(\omega t - kz)} + \text{c.c.} \qquad \rho_{eg} = \frac{-2\Omega_R(\Delta + i\Gamma/2)}{4\Delta^2 + \Gamma^2 + 2\Omega_R^2}$$

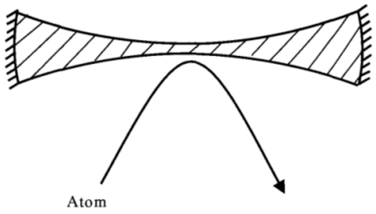
Interaction energy:

$$W = -\frac{\hbar\Omega_R}{2}(\rho_{ge} + \rho_{eg}) = \frac{2\hbar\Delta\Omega_R^2(x, y)}{4\Delta^2 + \Gamma^2 + 2\Omega_R^2}$$

Dipole force:

$$F_{\text{dipole}} = -\nabla W = -\frac{2\hbar\Delta}{4\Delta^2 + \Gamma^2}\nabla\Omega_R^2(x, y)$$

$$|\nabla \Omega_R^2(x,y)| \approx \frac{\Omega_R^2}{a}$$



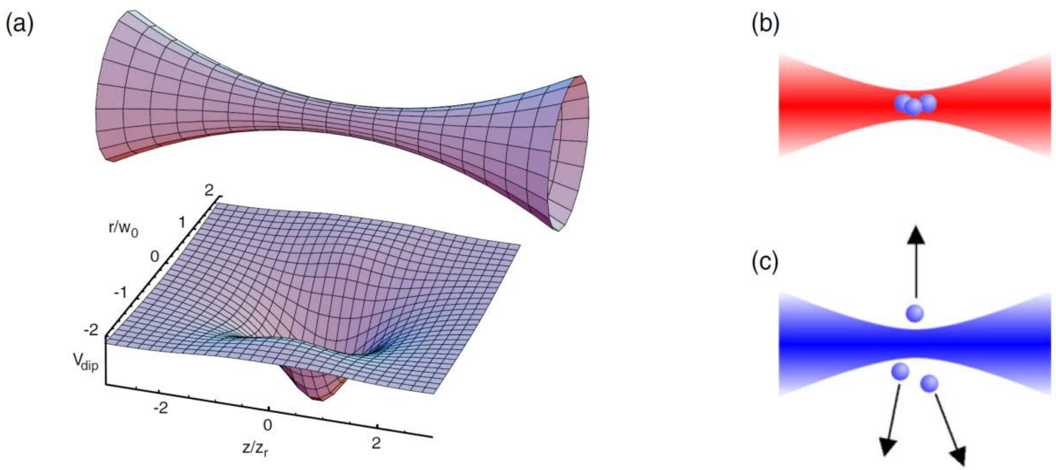
Dipole force:

Depends on the detuning

$$F_{\text{dipole}} = \frac{2\hbar\Delta\Omega_R^2}{a(4\Delta^2+\Gamma^2)}$$

Beam

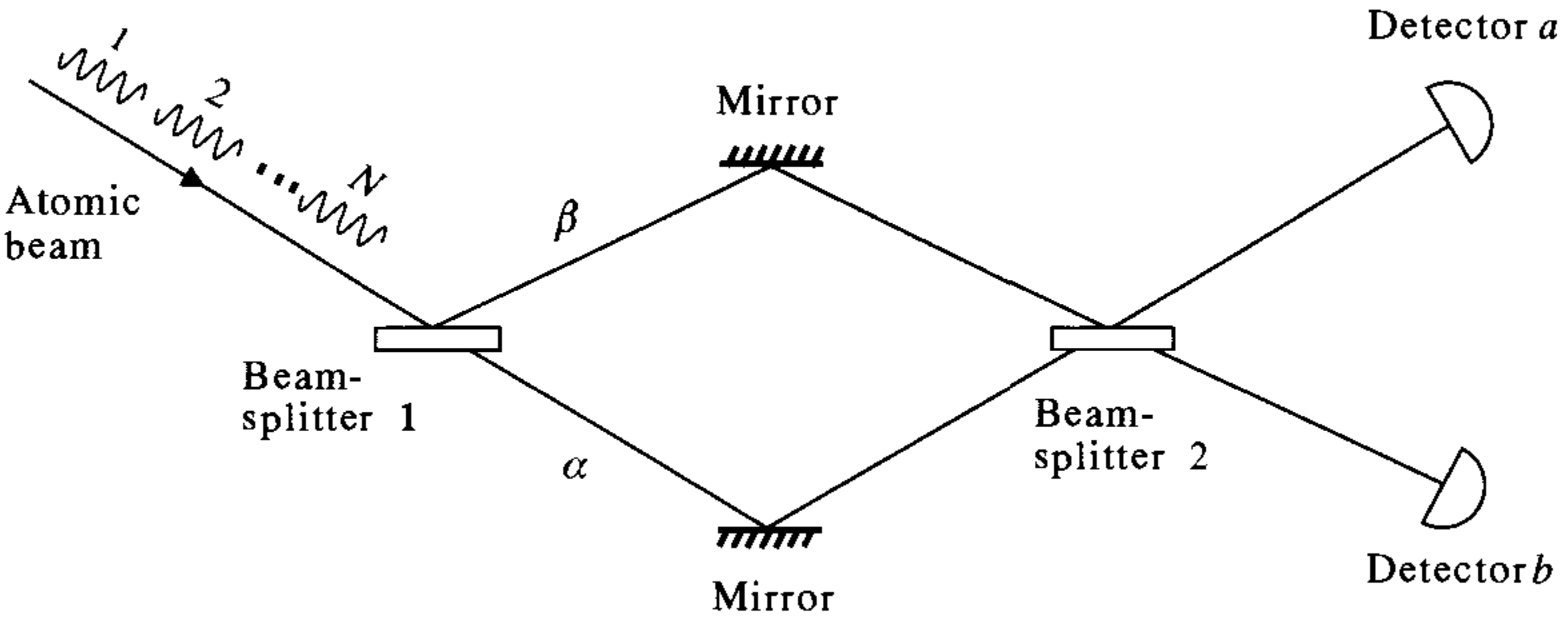
Potential



Trap, lens

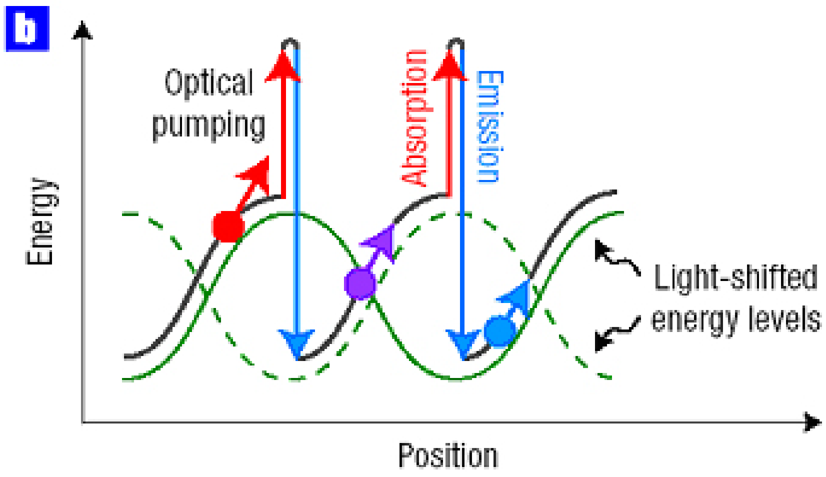
Mirror

Atomic interferometer

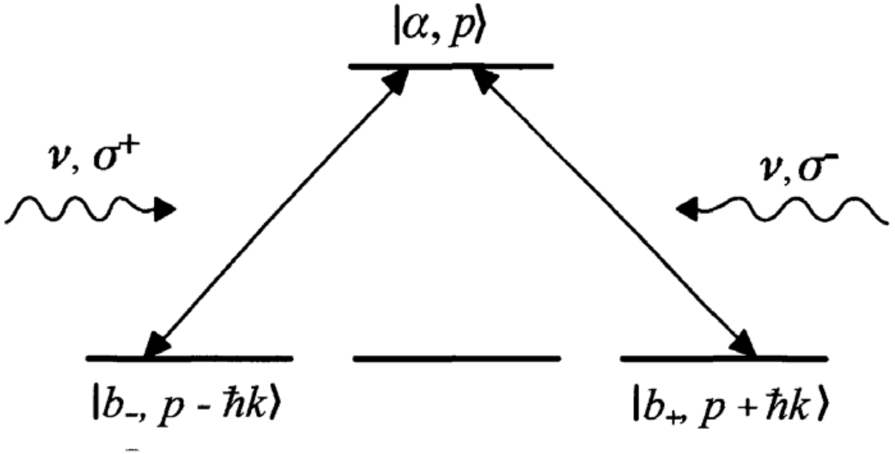


More advanced cooling schemes

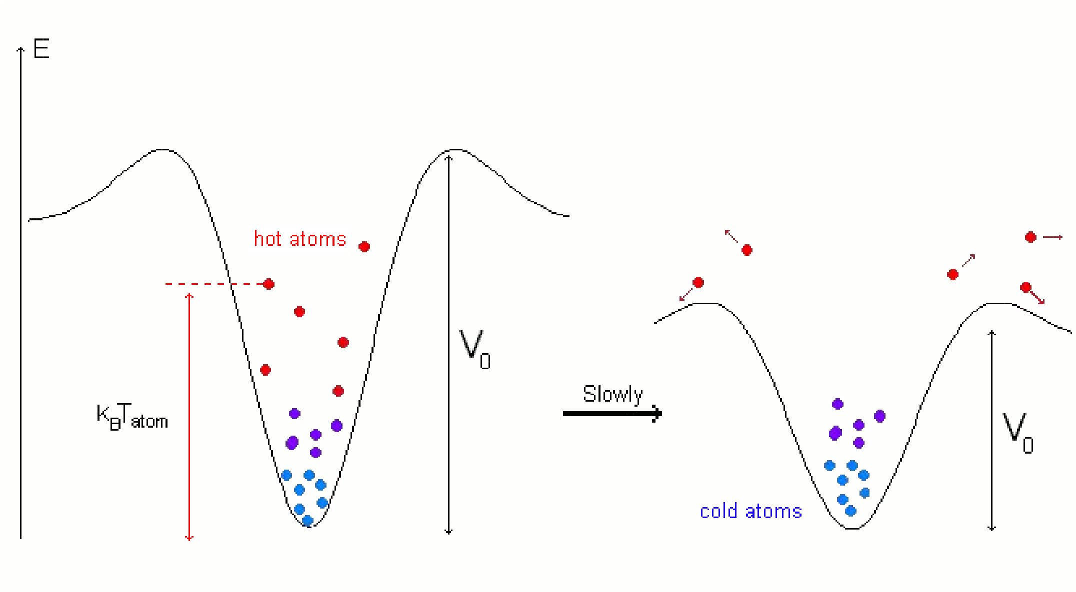
Sisyphus cooling

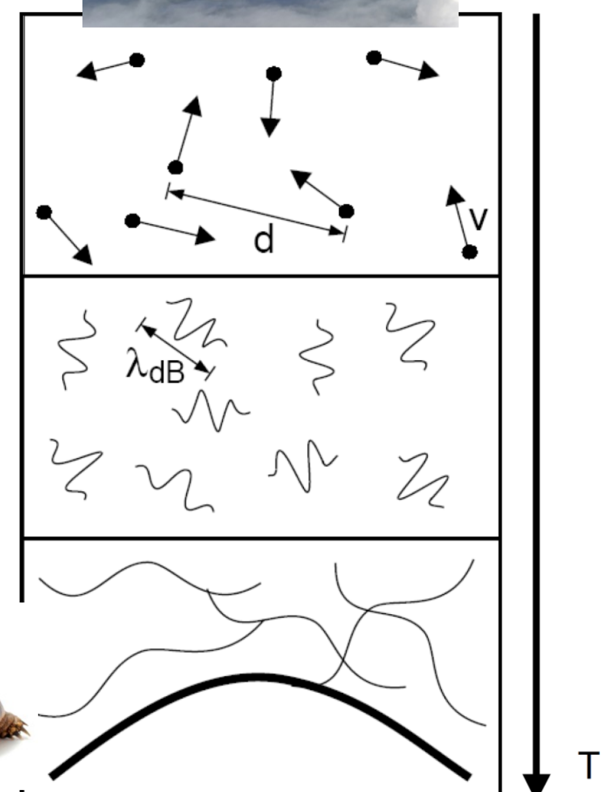


Velocity selective coherent population trapping (VSCPT): dark states

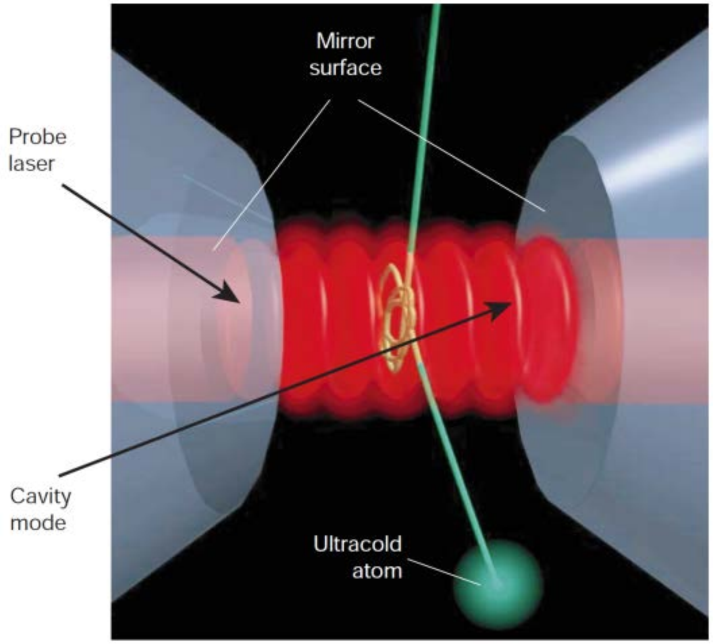


Evaporative cooling





Cavity cooling



Relaxation provided by the cavity (instead of spontaneous emission)

Off-resonant

Does not depend on the level structure

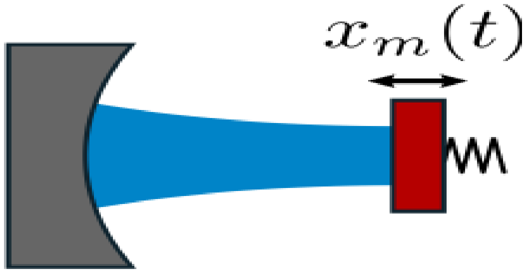
Promising for molecules

Macroscopic particles

Trapping a single atom by a field of a single photon (G. Rempe)

Optomechanics

Cooling a massive object down to its ground state
(Fock state of motion)



Squeezing of motion
Entanglement

Quantum “optics” of phonons

a single mechanical mode of a macroscopic object