Quantum optics of many-body systems

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Introduction

- **General context:** links between several fields

Classical optics

Atom optics

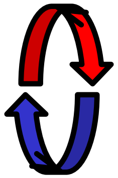
Quantum optics

Quantum atom optics

Quantum optics of many-body systems

- **Systems beyond atomic physics (solid state implementations)**

- **Introduction to quantum optics of light waves**



- **Introduction to quantum “optics” of matter waves**

- **Interfacing quantum light and matter fields**

What are the new physical phenomena and opportunities?

- **Inclusion of the quantum nature of measurement process**

"Optics": How to see and manipulate big quantum systems?
(without demolishing them)

- **Future perspectives for quantum engineering and technologies**

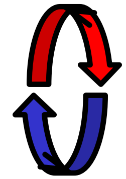
General context

19th century electromagnetism

Classical optics
light waves
material devices

Maxwell's equations

light



matter

1980s laser cooling

Atom optics
matter waves
light forces

Schrödinger equation

Devices:
beam splitters, mirrors, diffraction gratings, cavities

20th century photons, lasers

Quantum optics
classical (hot) atoms ☹
quantum light states ☺

Quantum fluctuations, entanglement

1995/2002 BEC/ Mott insulator

Quantum atom optics
☺ quantum atomic states
☹ classical light

Quantum correlations, entanglement

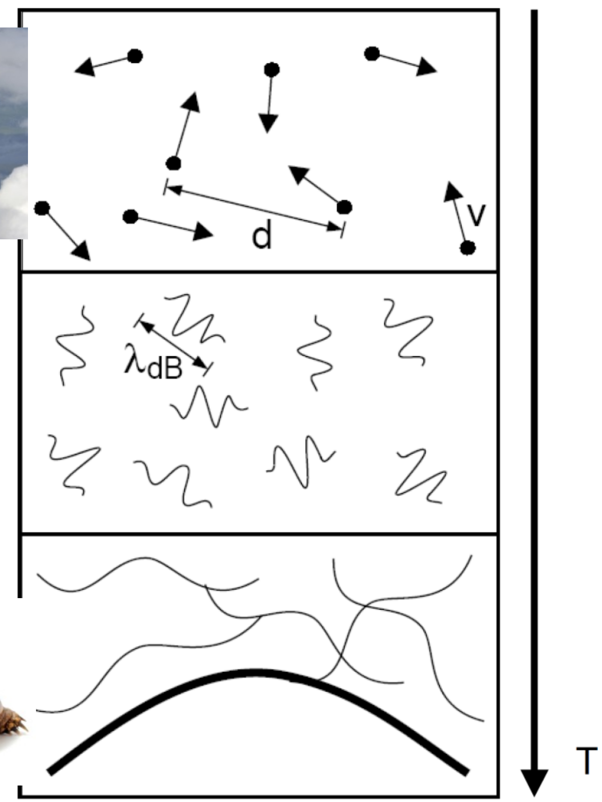
☺ **Quantum optics of quantum gases** ☺

Optics vs Atom optics

Classical optics

Light waves: Maxwell's equations for real (or c-number) functions

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{E}(\mathbf{r},t) - \nabla^2\mathbf{E}(\mathbf{r},t) = 0$$



Atom optics

Low *T* leads to the increase of de Broglie wavelength

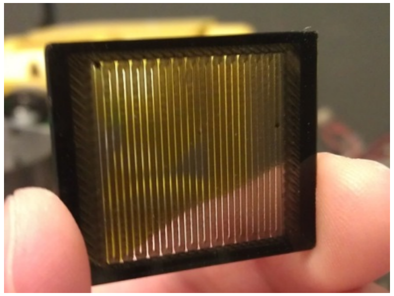
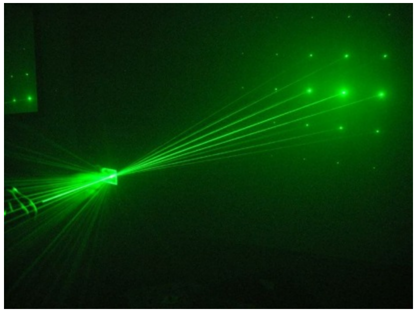
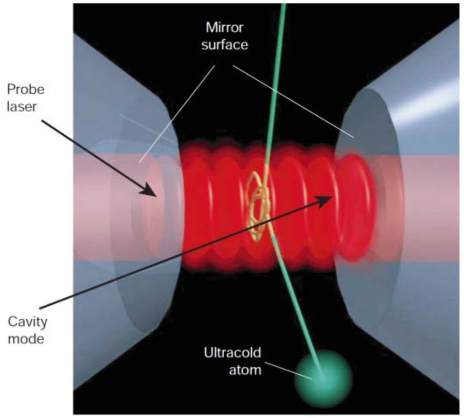
$$\lambda_{\mathrm{dB}} = \frac{h}{mv} \qquad E_K = \frac{1}{2}mv^2 = \frac{3}{2}k_B T$$

Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t)$$

Optics vs Atom optics

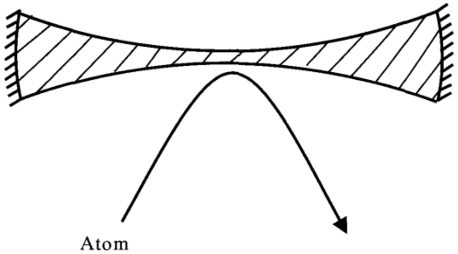
Classical optics: Material devices

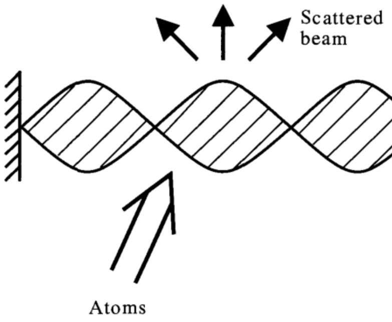





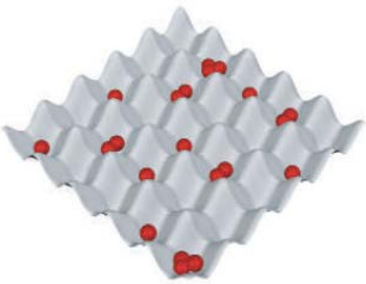
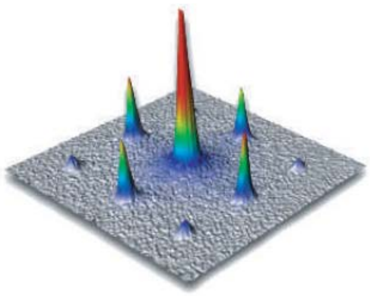
Mirrors **Diffraction gratings,** **lasers – classical light**

Cavities (optical resonators)

Atom optics: light forces and potentials





Mirrors **Diffraction gratings** **Cavities: traps and optical lattices**

Quantum optics vs Quantum atom optics

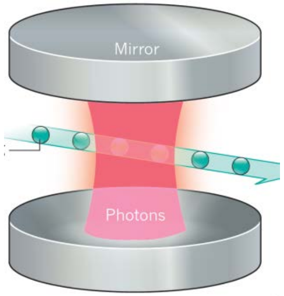
Quantum optics of light

Beyond c-number function: operators $\hat{\mathrm{E}}(\mathbf{r}, t)$

Laser light – the most "classical" one (coherent)

Nonclassical fluctuations, single photons (Fock states)

Cavity quantum electrodynamics
(cavity QED)



Entanglement

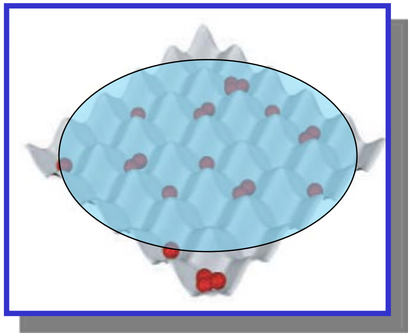
Quantum optics of matter waves

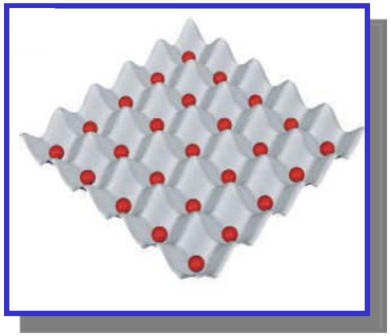
Many-body wave functions are conveniently expressed via the state operators $\hat{\Psi}(\mathbf{r}, t)$

Beyond mean-field approximations

Fluctuations, coherent and Fock states

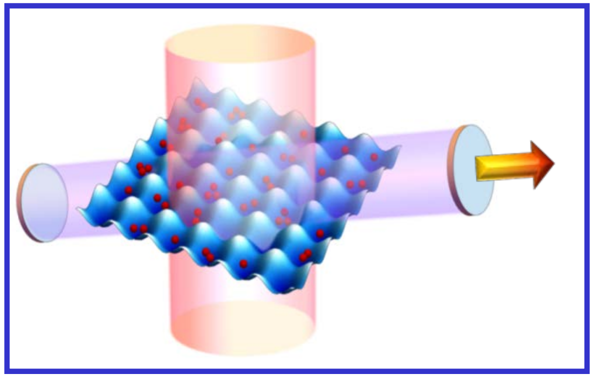
Fermions, strong interactions
("backaction" on light optics)



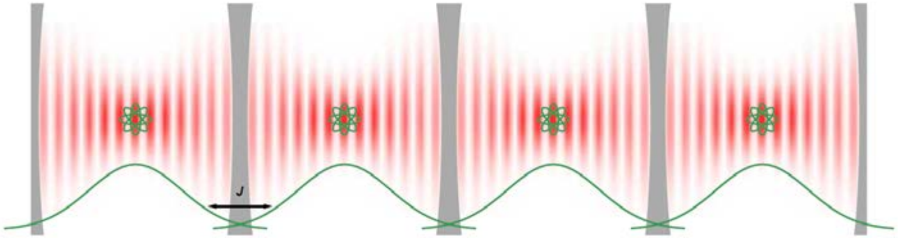


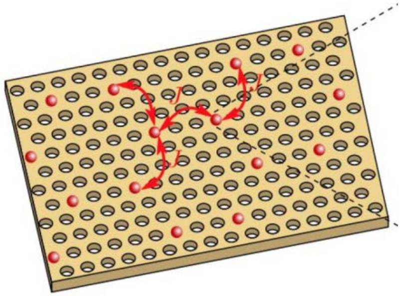
Introducing quantum light in many-body physics

Ultracold atoms inside a cavity

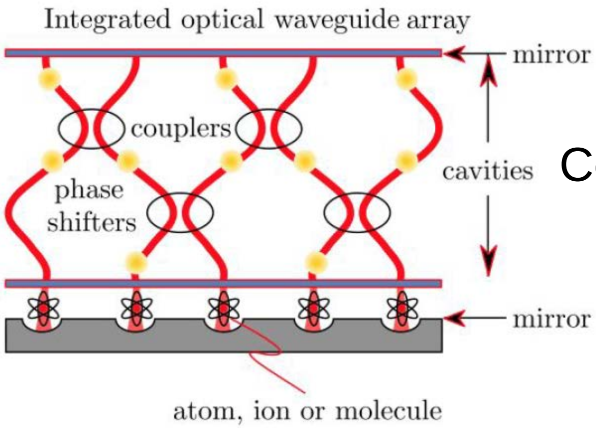


Coupled cavity arrays



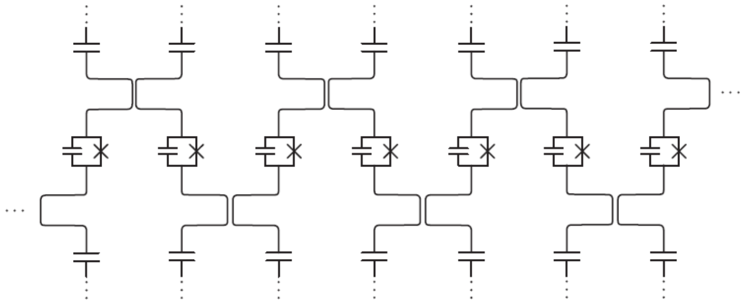


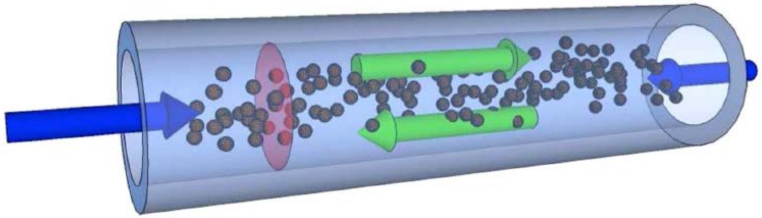
Photonic crystals with quantum dots



Coupled fibres

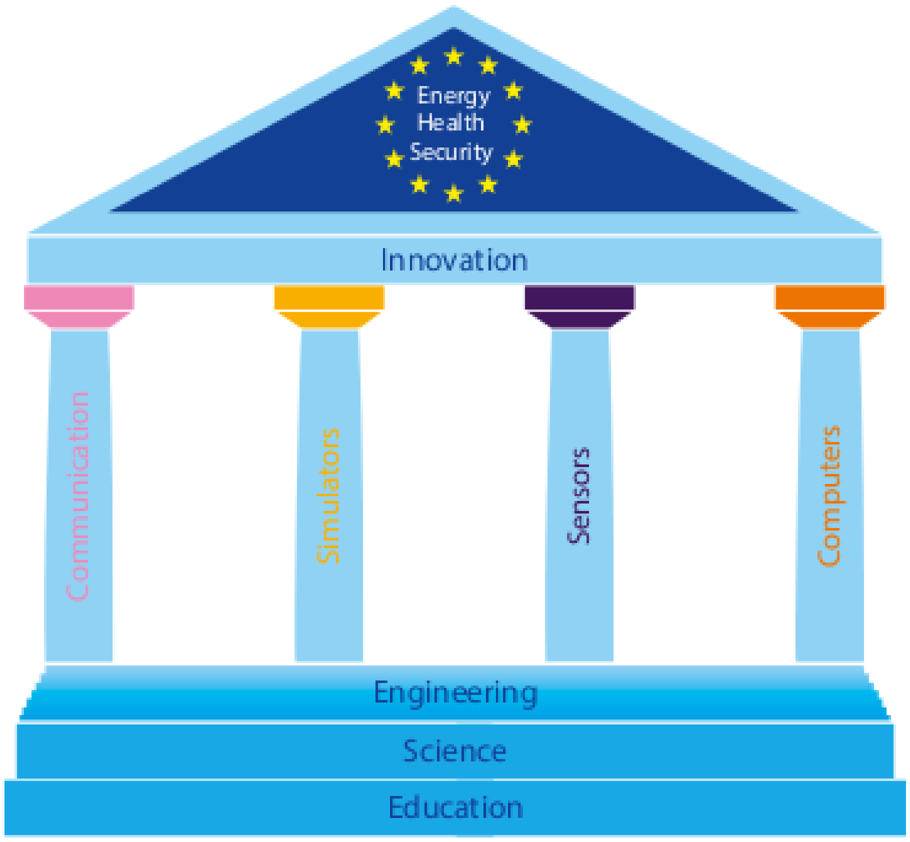
Superconducting resonators





Strongly interacting photons

Towards quantum technologies



Elements of a European programme in quantum technologies.

Quantum computation
Quantum simulations
Quantum communication
Quantum sensors/ metrology

EU Quantum technology Flagship

Multiple disciplines:

Atomic physics, condensed matter, high energy physics (gauge fields), astrophysics (black holes and neutron stars), gravity, biology

Computer science

Industry

What is quantum?

Introduction to quantum optics

- **Quantization of the electromagnetic field**

  Quantum states of light: Fock states, coherent states, squeezed states

  What is a single photon?

- **Light–matter interaction**

  Jaynes–Cummings model, light–atom entanglement,

  cavity QED (quantum electrodynamics)

  Collective effects (many particles): superradiance, Dicke phase transition

Quantization of the electromagnetic field

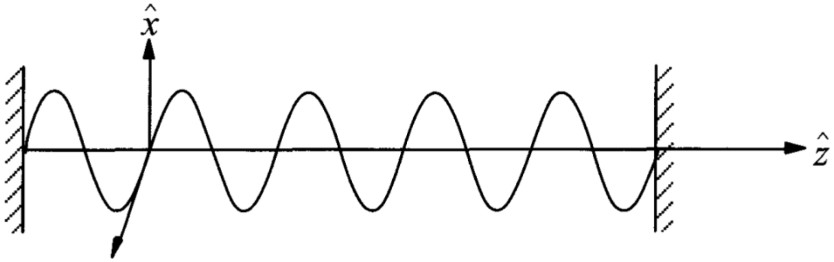
- **Classical Maxwell's equations**

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}, \quad \nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \cdot \mathbf{B} = 0, \quad \nabla \cdot \mathbf{D} = 0$$

$$\mathbf{B} = \mu_0 \mathbf{H}, \quad \mathbf{D} = \varepsilon_0 \mathbf{E}, \quad \mu_0 \varepsilon_0 = 1/c^2$$

Wave equation:

$$\nabla^2 \mathbf{E}(\mathbf{r}, t) - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \mathbf{E}(\mathbf{r}, t) = 0$$



- **Mode expansion**

$$E_x(z, t) = \sum_j A_j \underline{q_j(t)} \sin(k_j z), \quad k_j = j\pi/L \quad (j = 1, 2, 3, \dots)$$ wave vector

$$A_j = \sqrt{\frac{2\omega_j^2 m_j}{V \varepsilon_0}}, \quad \omega_j = j\pi c/L$$ frequency

- **Classical Hamiltonian**

$$\mathcal{H} = \frac{1}{2}\int_V dv(\varepsilon_0 E_x^2 + \mu_0 H_y^2)$$

$$\mathcal{H} = \sum_j \left(\frac{p_j^2}{2m_j} + \frac{1}{2}m_j\omega_j^2 q_j^2\right), \quad p_j = m_j\dot{q}_j$$

a sum of harmonic oscillators !!!
(modes)

- **Quantization:** ***From now - operators***

$$[q_j, p_{j'}] = i\hbar\delta_{jj'},$$

$$[q_j, q_{j'}] = [p_j, p_{j'}] = 0$$

$$a_j e^{-i\omega_j t} = \frac{1}{\sqrt{2m_j \hbar\omega_j}}(m_j\omega_j q_j + ip_j) \qquad a_j^\dagger e^{i\omega_j t} = \frac{1}{\sqrt{2m_j \hbar\omega_j}}(m_j\omega_j q_j - ip_j)$$

- **Hamiltonian of the electromagnetic field**

$$\mathcal{H} = \hbar \sum_j \omega_j \left(a_j^\dagger a_j + \frac{1}{2} \right)$$

Energies of modes

- **Bosonic commutation relations:** ***creation and annihilation operators***

$$[a_j, a_{j'}^\dagger] = \delta_{jj'}$$

$$[a_j, a_{j'}] = [a_j^\dagger, a_{j'}^\dagger] = 0$$

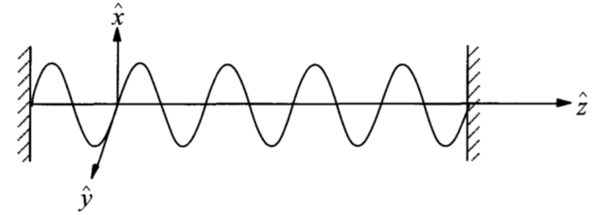
- **Operator of the electric field**

$$E_x(z,t) = \sum_j \mathcal{E}_j (a_j e^{-i\omega_j t} + a_j^\dagger e^{i\omega_j t}) \sin k_j z \qquad \mathcal{E}_j = \sqrt{\frac{\hbar\omega_j}{\varepsilon_0 V}}$$

Done!

Fock states, single photons

- **Single mode.** $[a, a^\dagger] = 1$



Eigenstate $|\psi_n\rangle = |n\rangle$ **Energy eigenvalue** E_n

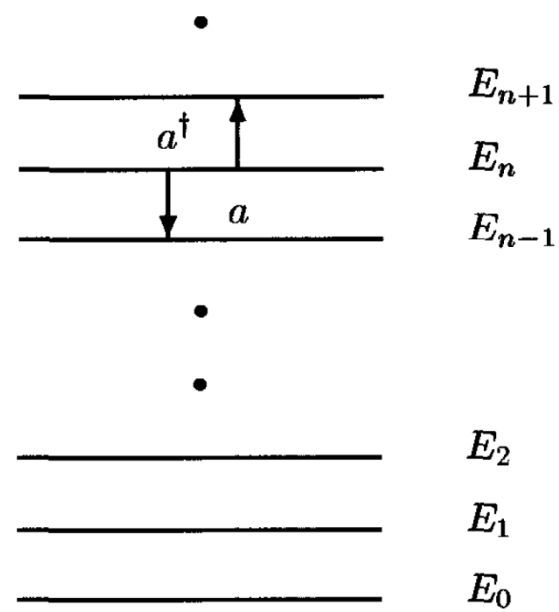
$$\mathcal{H}|n\rangle = \hbar\omega\left(a^\dagger a + \frac{1}{2}\right)|n\rangle = E_n|n\rangle$$

$$\mathcal{H}\underline{\left(a|n\rangle\right)} = \left(E_n - \hbar\omega\right)\underline{\left(a|n\rangle\right)}$$

This is an eigenstate as well:

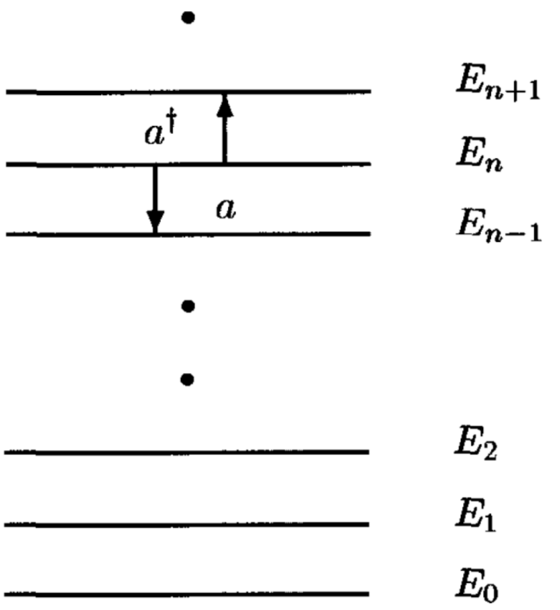
$$|\psi_{n-1}\rangle = |n-1\rangle \sim a|n\rangle$$

$$E_{n-1} = E_n - \hbar\omega$$



Vacuum state: $a|0\rangle = 0 \quad E_0 = \frac{1}{2}\hbar\omega$

Fock (number state): $|n\rangle \quad E_n = \left(n + \frac{1}{2}\right)\hbar\omega$



- **Annihilation and creation operators**

$$a|n\rangle = \sqrt{n}|n-1\rangle \qquad a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

- **Number operator** $a^\dagger a|n\rangle = n|n\rangle \qquad \hat{n} = a^\dagger a$

- **Photon: One excitation in a given mode**

(If in a medium – *polaritons*. Similar procedure for other quasiparticles)

A general state: superposition of Fock states $|\psi\rangle = \sum_n c_n|n\rangle$

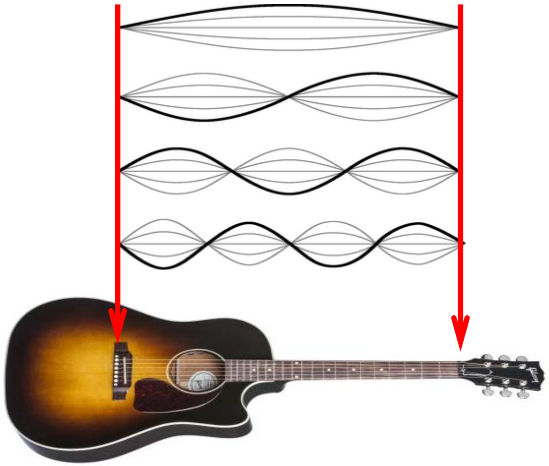
$$E_x(z,t) = \sum_j \mathcal{E}_j (a_j e^{-i\omega_j t} + a_j^\dagger e^{i\omega_j t}) \sin k_j z$$

Mean electric field $\langle n|E|n\rangle = 0$ $\qquad [a, a^\dagger] = 1$

Mean intensity $\langle n|E^2|n\rangle = 2\mathcal{E}^2\left(n + \frac{1}{2}\right) \neq 0$ $\qquad \mathcal{E}_j = \sqrt{\frac{\hbar\omega_j}{\varepsilon_0 V}}$

Very strong fluctuations of the field around zero value, **even in the vacuum**

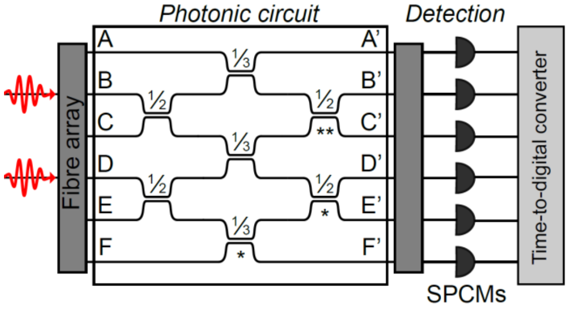
- **Light quantization, photons: Discreteness of energy in a single mode**
- **NOT due to the discrete classical modes in a cavity !!!**



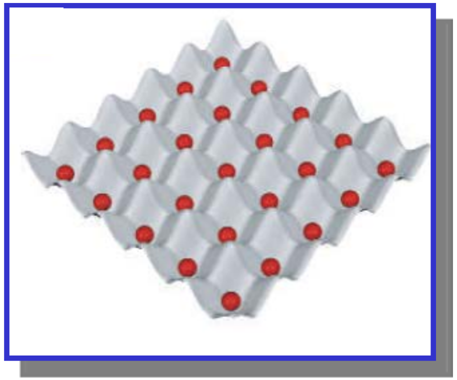
- **<u>Several modes</u>**

$$|\{n_j\}\rangle = |n_1\rangle|n_2\rangle...|n_j\rangle... = |n_1, n_2, n_3, ...n_j, ...\rangle$$

- **Multimode Fock state** $|1, 1, 1, ...1, ...\rangle$ **very difficult in quantum optics**



Photonic circuits (for Q. computations)



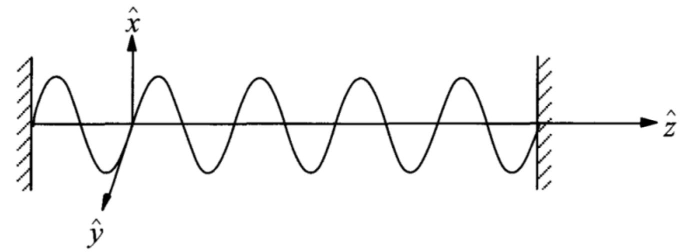
- **Atom optics: achieved routinely !!! Mott insulator state**

$$|1, 1, 1, ...1, ...\rangle$$

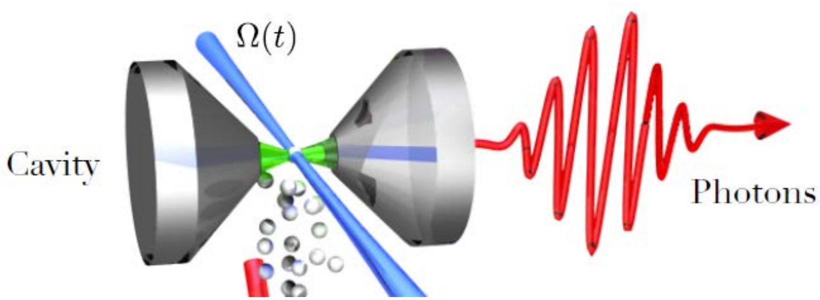
How does a single photon look like?

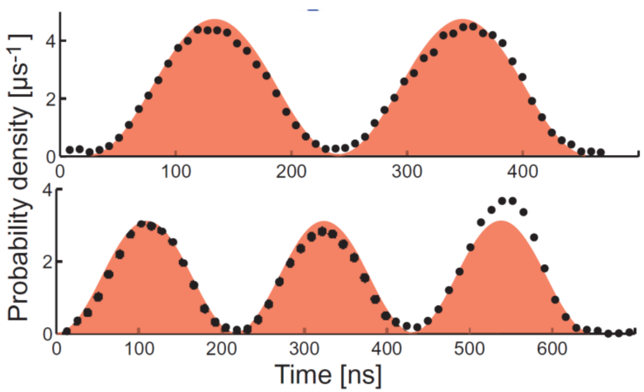
- **Various modes are possible**

$$E_x(z,t) = \sum_j \mathcal{E}_j (a_j e^{-i\omega_j t} + a_j^\dagger e^{i\omega_j t}) \sin k_j z$$

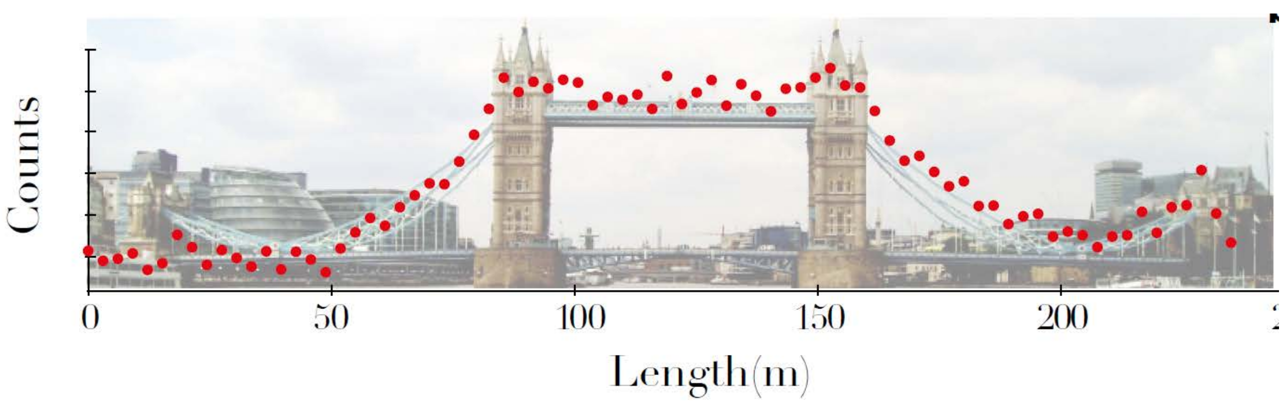


- **Photons leaking from the cavity**





Qubits, qutrits, ququads



A. Kuhn, Oxford

Coherent states

A general photon state (single mode) $$|\psi\rangle = \sum_n c_n |n\rangle$$

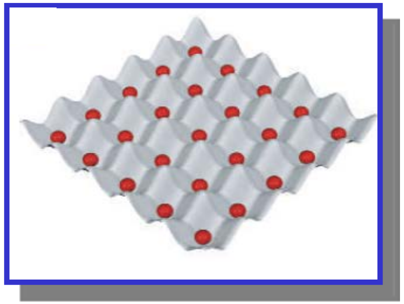
Eigenstate of the annihilation operator $$a|\alpha\rangle = \alpha|\alpha\rangle$$

Superposition of Fock states $$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_0^\infty \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

Mean photon number $$\langle\alpha|a^\dagger a|\alpha\rangle = |\alpha|^2$$

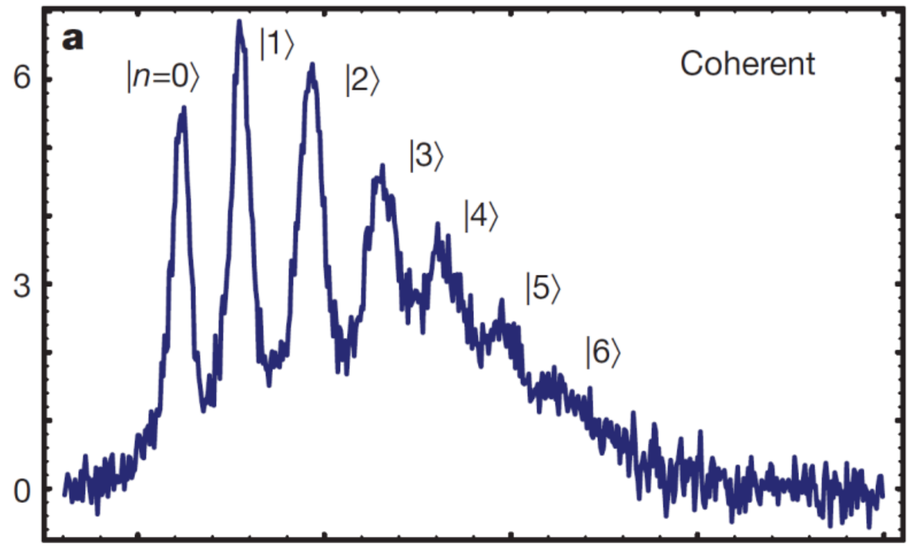
The most "classical" state (but, still this is a quantum state, superposition)

(in quantum atom optics, the light really classical)



Photon number distribution: Poissonian

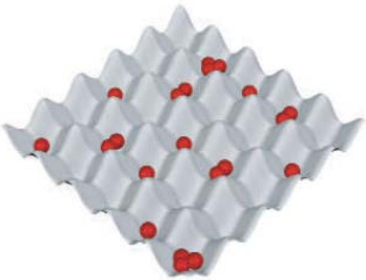
$$p(n) = \langle n|\alpha\rangle\langle\alpha|n\rangle = \frac{\langle n\rangle^n e^{-\langle n\rangle}}{n!}$$

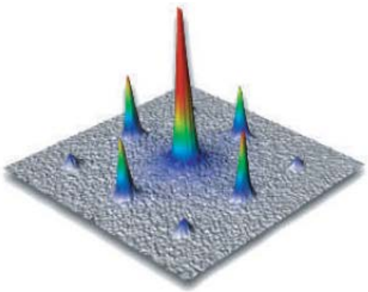


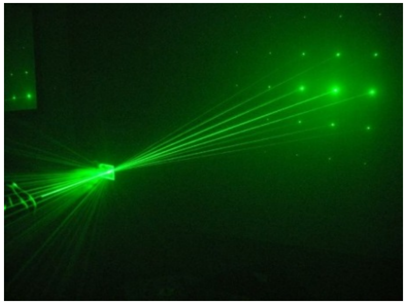
cavity QED and circuit QED measurements

In quantum atom optics: superfluid state (multimode, many-body)

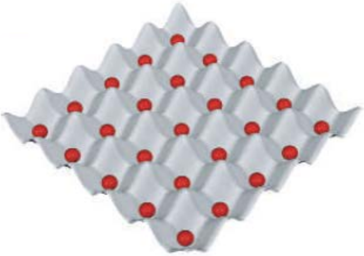
$$|\Psi_{\mathrm{Coh}}\rangle = \prod_{i=1}^{M} |\mathrm{Coh}\rangle_i$$

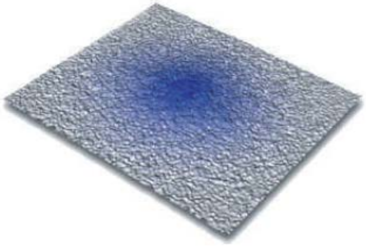






$$|1, 1, 1, \ldots 1, \ldots\rangle$$

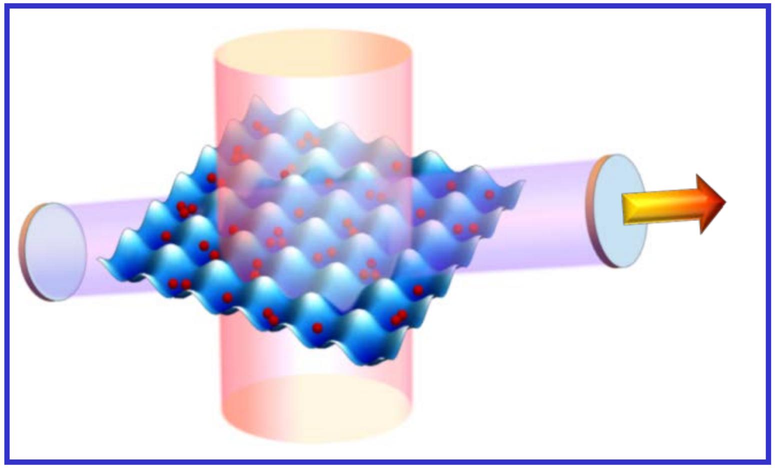


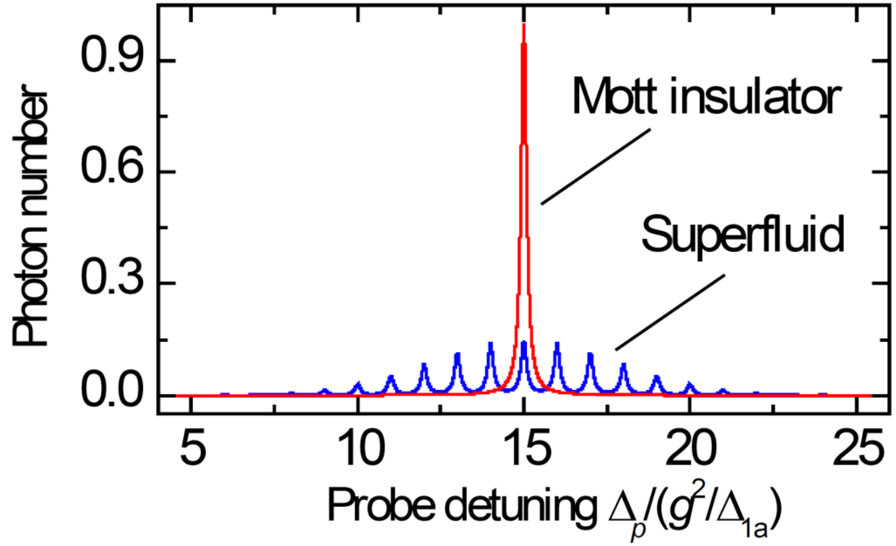


Mott insulator
sub-Poissonian distribution
nonclassical statistics

Non-destructive measurement of the atom number distribution:

Mapping atom statistics on the properties of light





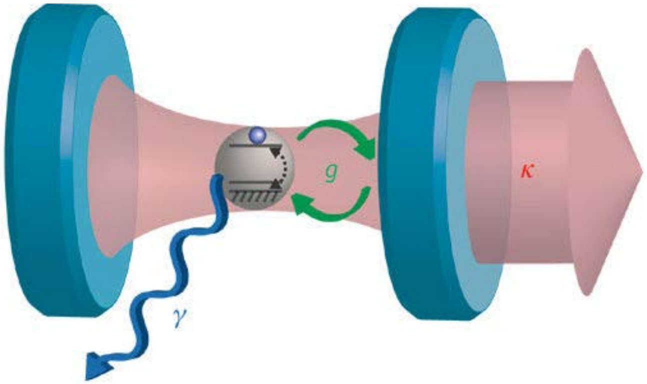
Superfluid to Mott insulator phase transition:

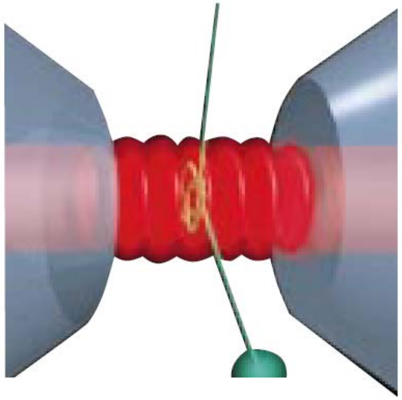
from Poissonian to sub-Poissonian statistics

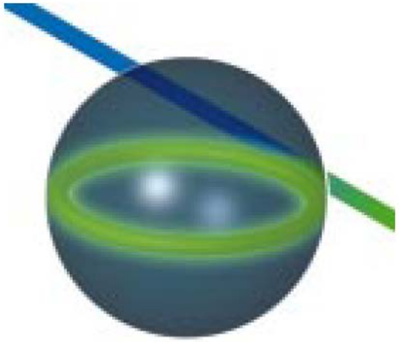
from "classical" to nonclassical statistics: atom number squeezing

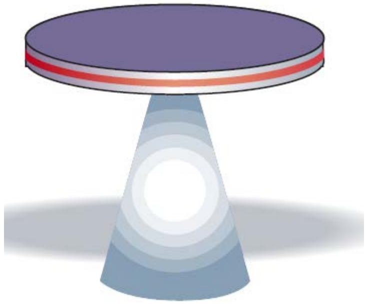
Light-matter interaction

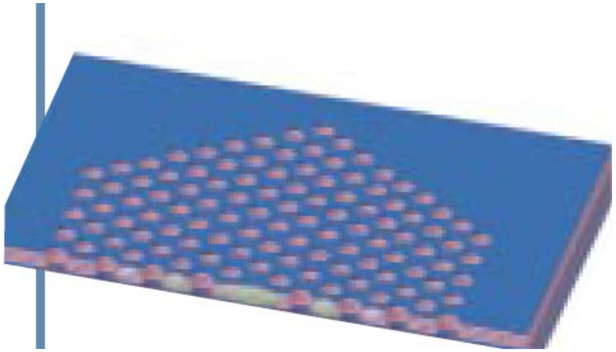
Single-mode cavity, two-level atom(s)

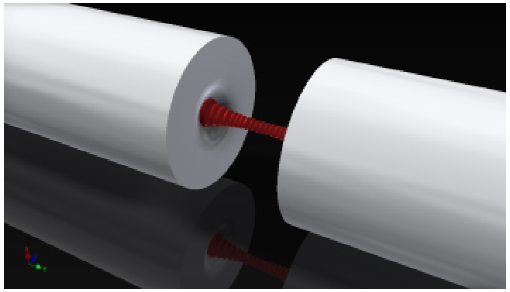






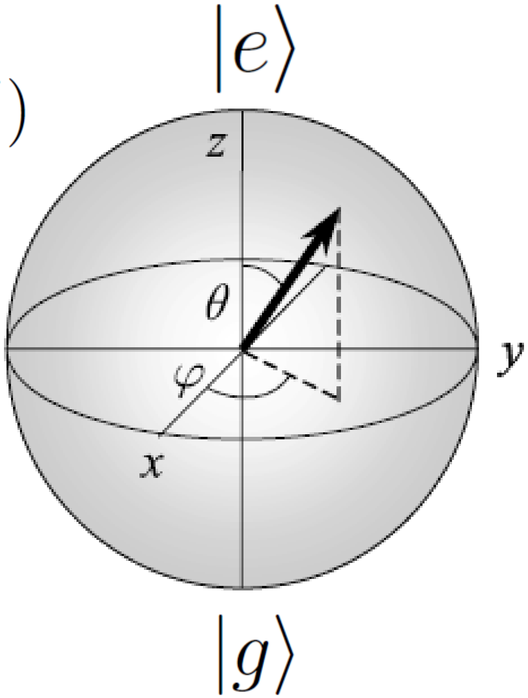






Interaction with a two-level atom

$$\mathcal{H} = \hbar\omega a^\dagger a + \frac{1}{2}\hbar\omega_a\sigma_z + \hbar g(\sigma_+ + \sigma_-)(a + a^\dagger)$$



Standard spin (Pauli) operator

$$\sigma_+ = |e\rangle\langle g| \qquad \sigma_- = |g\rangle\langle e|$$

$$\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$$

Light-matter coupling constant

$$g = \sqrt{\frac{d^2\omega_a}{2\hbar\varepsilon_0 V}}$$

$$\mathcal{E}_j = \sqrt{\frac{\hbar\omega_j}{\varepsilon_0 V}}$$

$$\langle n|E^2|n\rangle = 2\mathcal{E}^2\left(n + \frac{1}{2}\right) \neq 0$$

Jaynes-Cummings model, rotating wave approximation

$$\mathcal{H} = \hbar\omega a^\dagger a + \frac{1}{2}\hbar\omega_a\sigma_z + \hbar g(\sigma_+ a + a^\dagger\sigma_-)$$

Very useful model in cavity QED:
vacuum Rabi oscillations (reversible spontaneous emission), collapse and revival of oscillations, light-matter entanglement, etc.

Strong coupling regime of light-matter interaction $g > \kappa, \gamma$

But limited …

Superradiant Dicke phase transition

no RWA, but approximation of linear dipoles (small excitation number)
Holstein-Primakoff approximation, bosonization

$$\sigma_- \approx c \qquad \sigma_+ \approx c^\dagger \quad \sigma_z = 2c^\dagger c$$

Polaritons (linear superposition of light and matter excitations)

$$P_{1,2} = \alpha_{1,2} a + \beta_{1,2} a^\dagger + \gamma_{1,2} c + \delta_{1,2} c^\dagger$$

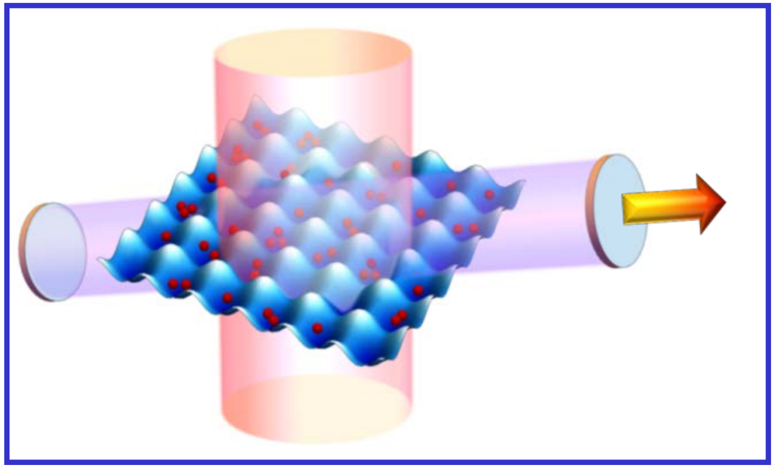
Diagonalized Hamiltonian

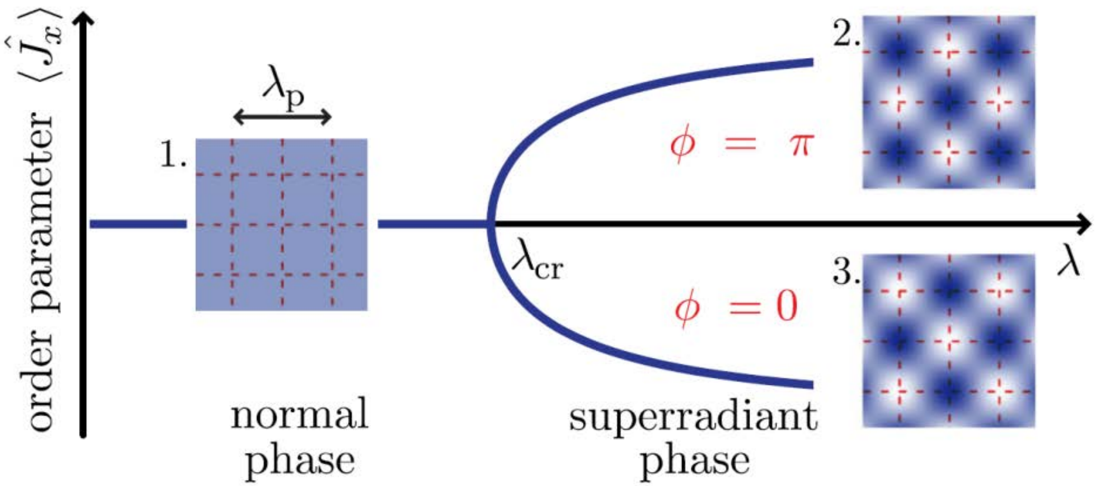
$$\mathcal{H} = \lambda_1 P_1^\dagger P_1 + \lambda_2 P_2^\dagger P_2 \qquad \lambda_{1,2} = \omega_a \sqrt{1 \pm \frac{g}{\omega_a}}$$

Superstrong coupling regime: instability $\quad g > \omega_a$

Unrealistic to obtain with two-level atoms, even including the collective enhancement $\quad g\sqrt{N}$

Realization with a BEC in a cavity





T. Esslinger: without a lattice (2010), with a lattice (2015). **Supersolid-like state (?)**