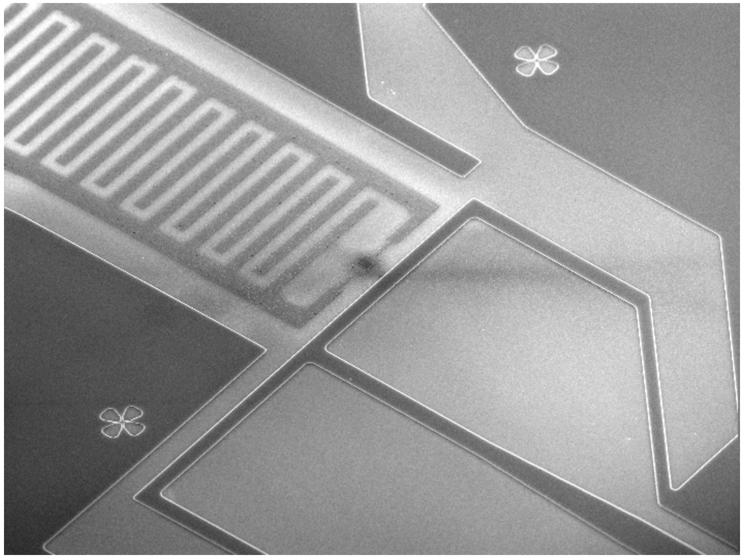
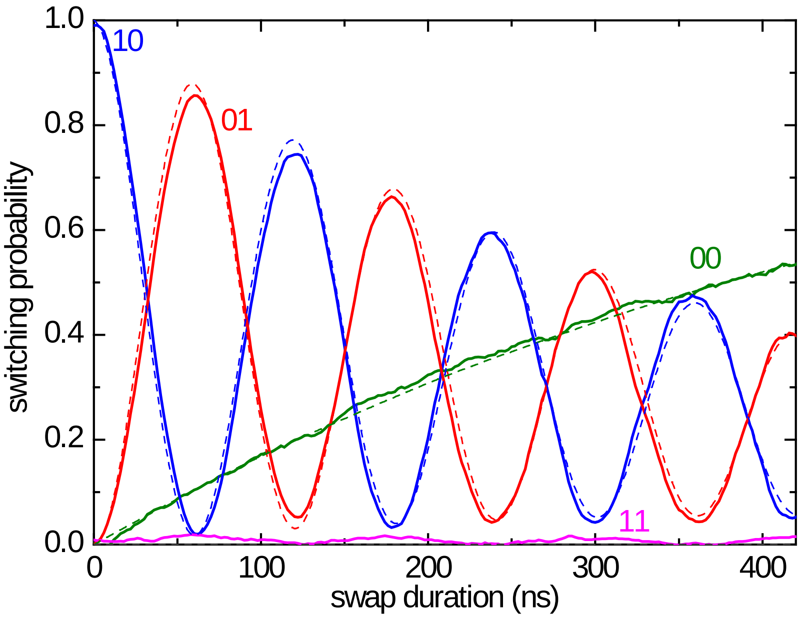




Electrical quantum engineering with superconducting circuits

P. Bertet & R. Heeres

SPEC, CEA Saclay (France),
Quantronics group

Outline

Lecture 1: Basics of superconducting qubits

1) Introduction: Hamiltonian of an electrical circuit
2) The Cooper-pair box
3) Decoherence of superconducting qubits

Lecture 2: Qubit readout and circuit quantum electrodynamics

1) Readout using a resonator
2) Amplification & Feedback
3) Quantum state engineering

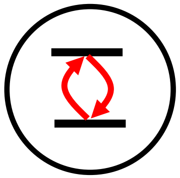
Lecture 3: Multi-qubit gates and algorithms

1) Coupling schemes
2) Two-qubit gates and Grover algorithm
3) Elementary quantum error correction

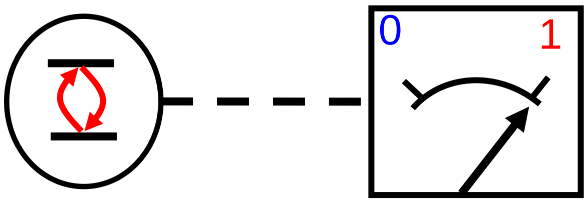
Lecture 4: Introduction to Hybrid Quantum Devices

Requirements for QC

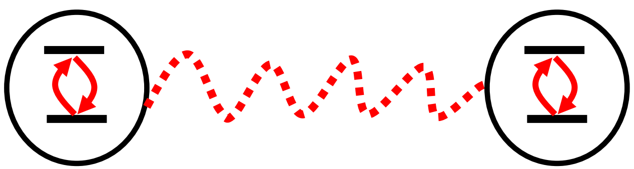
**High-Fidelity
Single Qubit Operations**



**High-Fidelity Readout
of Individual Qubits**

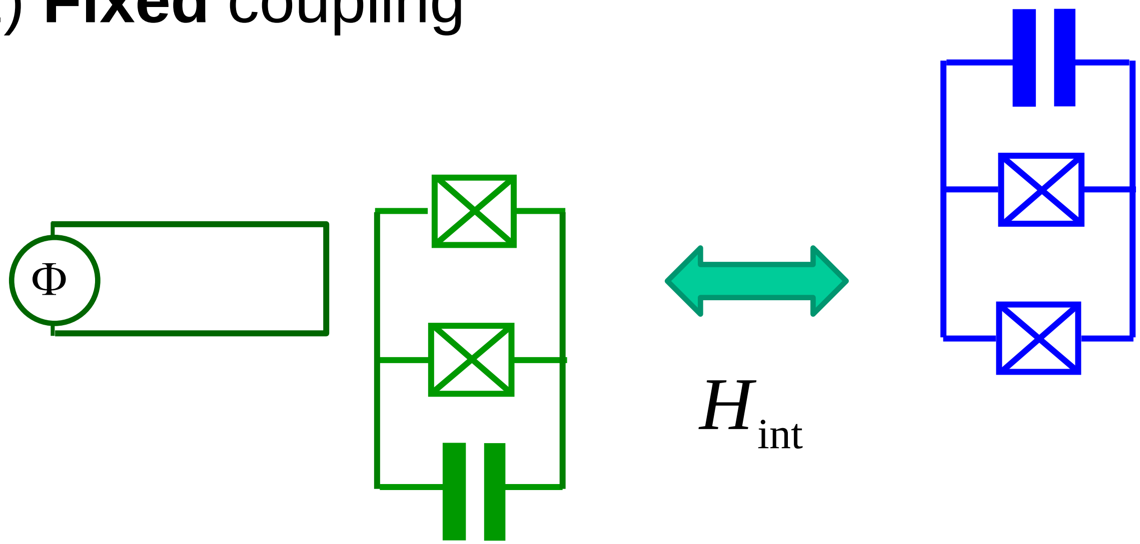


**Deterministic, On-Demand
Entanglement between Qubits**

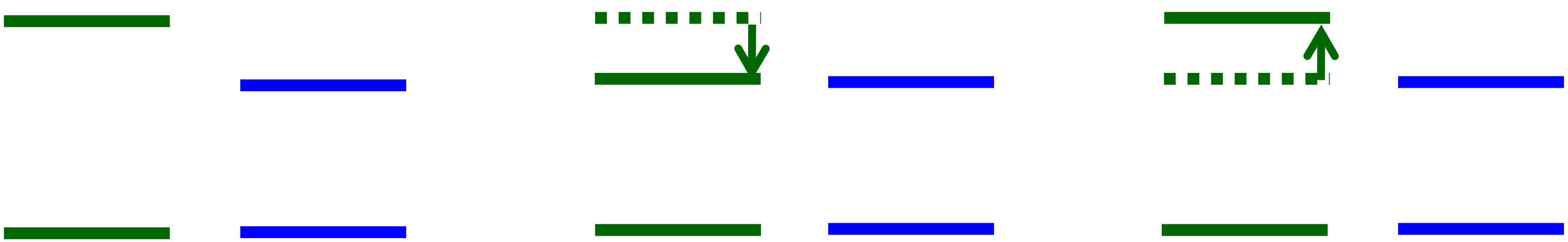


Coupling strategies

1) **Fixed** coupling

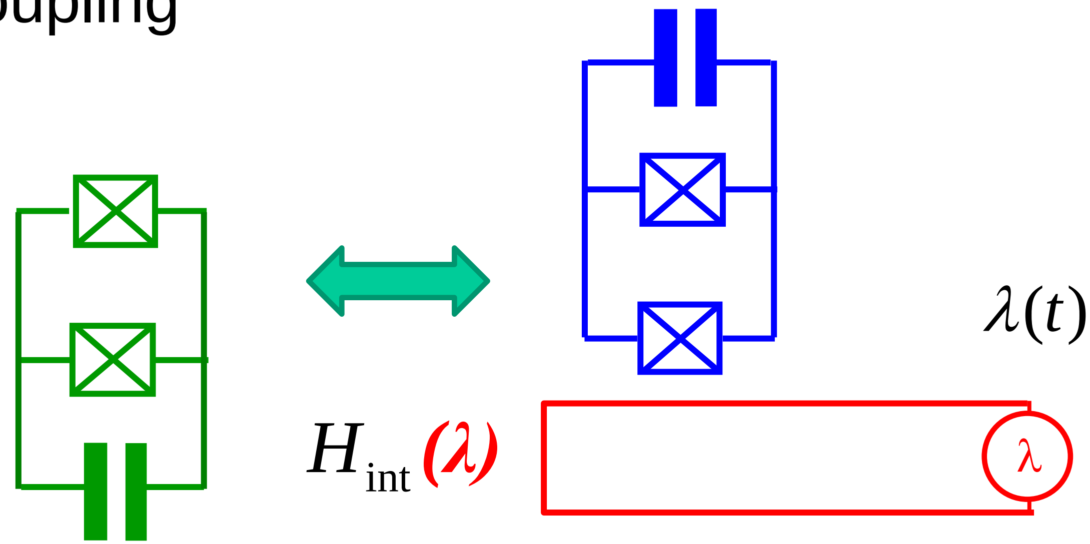


Entanglement on-demand ???
« Tune-and-go » strategy



Coupling strategies

2) **Tunable** coupling

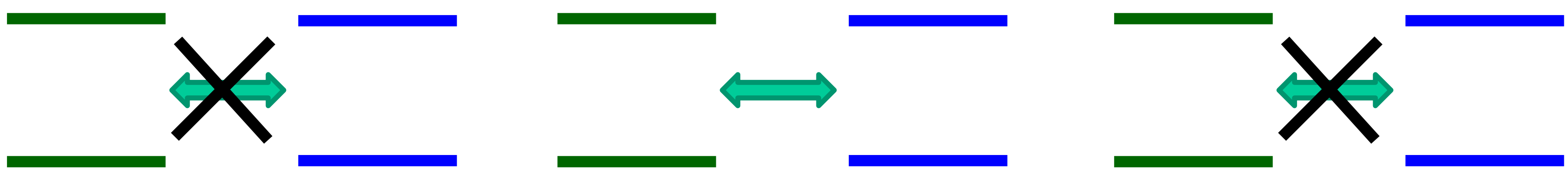


$\lambda(t) = \lambda_{OFF}$

$H_{\text{int}}(\lambda)$

Entanglement on-demand ???

A) Tune ON/OFF the coupling with qubits on resonance



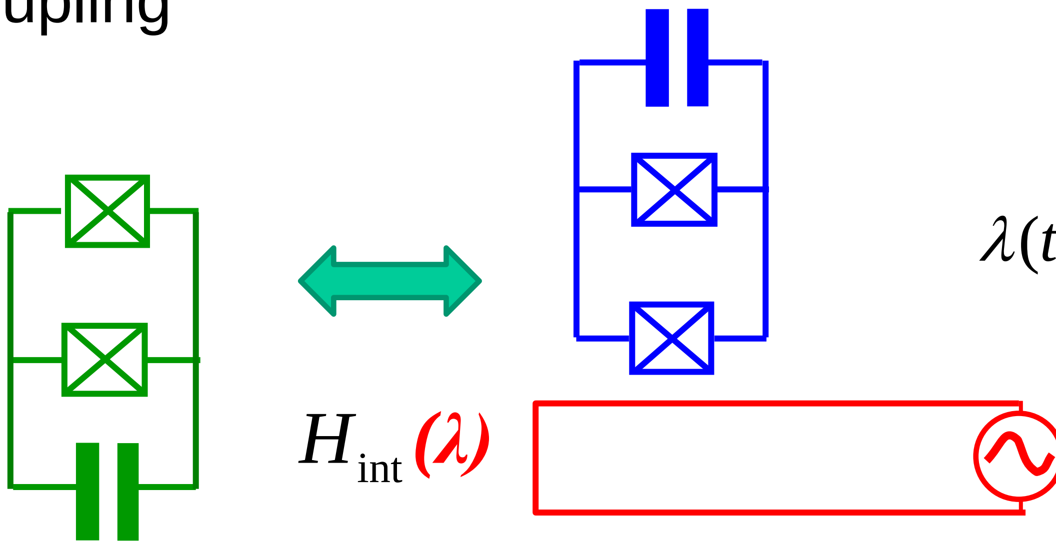
Coupling OFF (λ_{OFF})

Coupling activated for τ by λ_{ON}

Entangled qubits Interaction OFF (λ_{OFF})

Coupling strategies

2) **Tunable** coupling



$$\lambda(t) = \lambda_{OFF}$$

Entanglement on-demand ???
B) Modulate coupling

IN THIS LECTURE : ONLY FIXED COUPLING

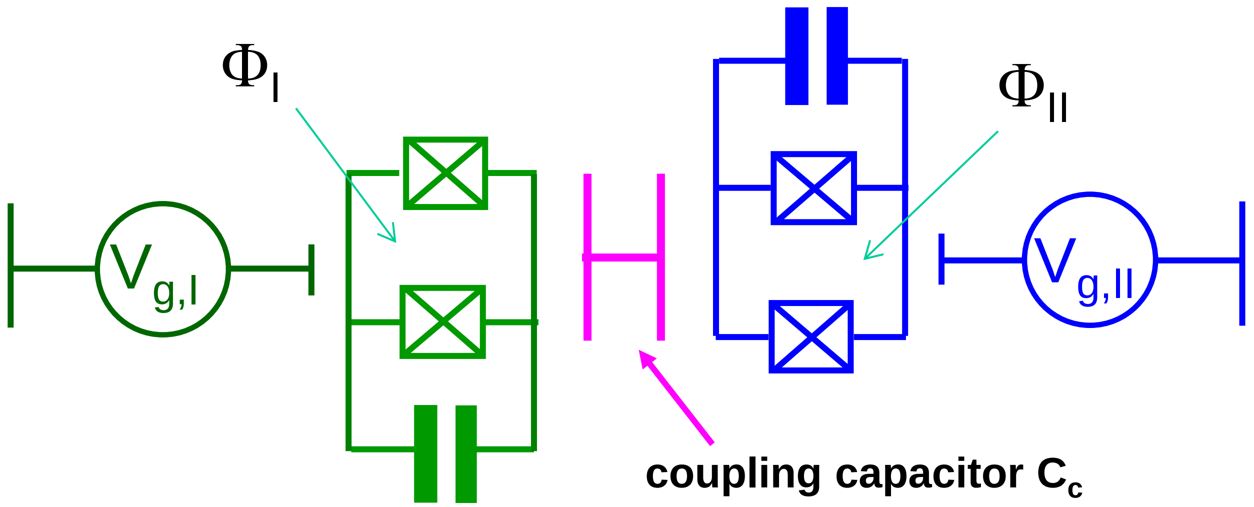
Coupling OFF (λ_{OFF})

Coupling ON by modulating λ

Coupling OFF (λ_{OFF})

How to couple transmon qubits ?

1) Direct capacitive coupling



$$H = E_{c,I}(\hat{N}_I - N_{g,I})^2 - E_{J,I}(\Phi_I)\cos\hat{\theta}_I$$
$$+E_{c,II}(\hat{N}_{II} - N_{g,II})^2 - E_{J,II}(\Phi_{II})\cos\hat{\theta}_{II}$$
$$+2\frac{E_{c,I}E_{c,II}}{E_{cc}}(\hat{N}_I - N_{g,I})(\hat{N}_{II} - N_{g,II})$$

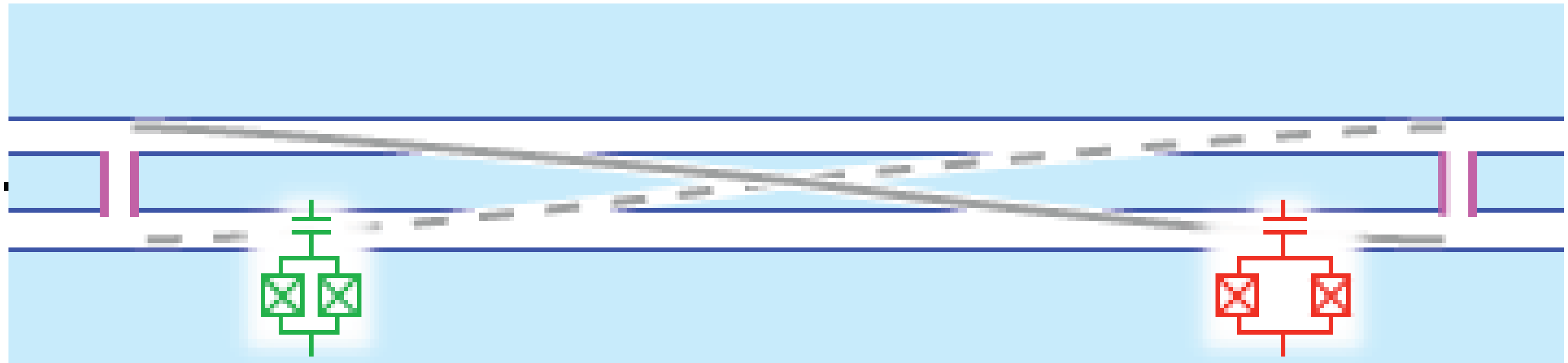
$$\Rightarrow \begin{cases} H_{q,I} = -\dfrac{\hbar\omega_{01}^{I}(\Phi_I)}{2}\sigma_{z,I} \\ H_{q,II} = -\dfrac{\hbar\omega_{01}^{II}(\Phi_{II})}{2}\sigma_{z,II} \\ H_c = \hbar g \sigma_{x,I}\sigma_{x,II} \end{cases}$$

$$\approx \hbar g(\sigma_I^{+}\sigma_{II}^{-} + \sigma_I^{-}\sigma_{II}^{+})$$

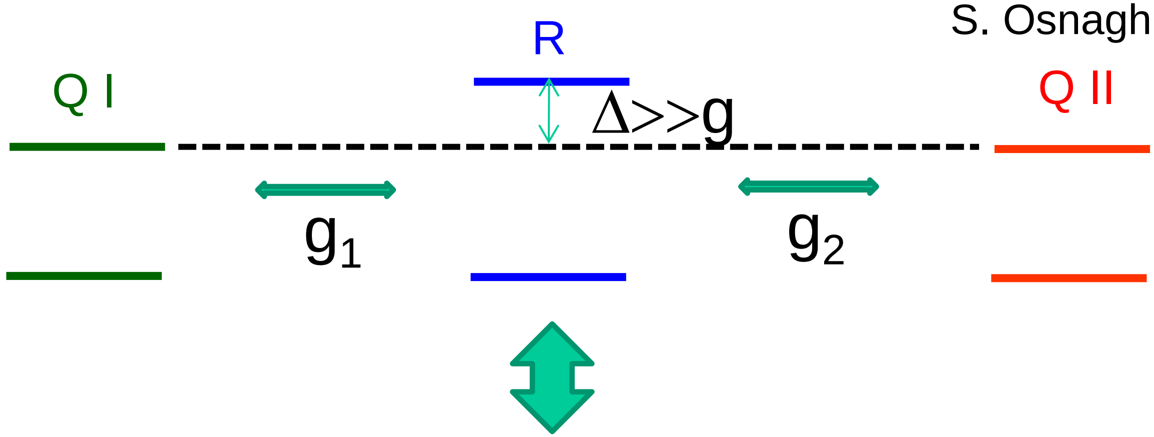
$$\hbar g = (2e)^2 \frac{C_c}{C_I C_{II}} \left| \langle 0_I | \hat{N}_I | 1_I \rangle \langle 0_{II} | \hat{N}_{II} | 1_{II} \rangle \right|$$

How to couple transmon qubits ?

2) Cavity mediated qubit-qubit coupling



J. Majer et al., Nature **449**, 443 (2007)
S. Osnaghi et al., PRL (2001)



Q I

$$g_{eff}=g_1 g_2/\Delta$$

Q II

$$H_{eff}=\hbar g_{eff}\left(\sigma_I^+\sigma_{II}^-+\sigma_I^-\sigma_{II}^+\right)$$

iSWAP Gate

$$H/\hbar = -\frac{\omega_{01}^{I}}{2}\sigma_z^{I} - \frac{\omega_{01}^{II}}{2}\sigma_z^{II} + \underbrace{g\left(\sigma_+^{I}\sigma_-^{II} + \sigma_-^{I}\sigma_+^{II}\right)}_{H_{\text{int}}}$$

→ « Natural » universal gate : $\sqrt{iSWAP}$

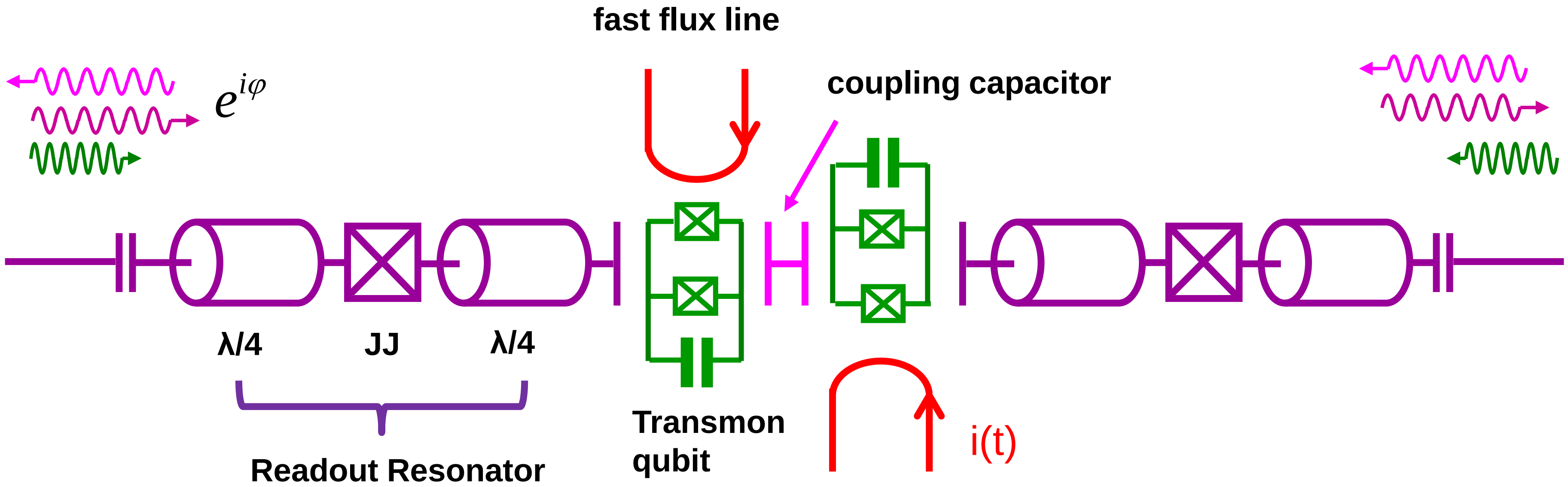
On resonance, ($\omega_{01}^{I} = \omega_{01}^{II}$)

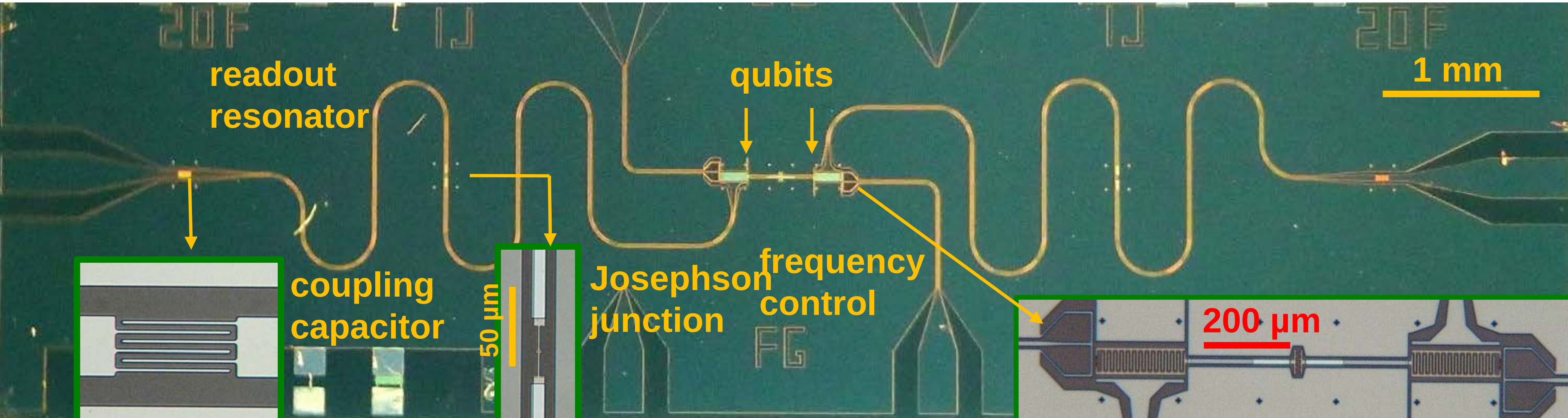
$$\begin{matrix} 00 & 10 & 01 & 11 \end{matrix}$$

$$U_{\text{int}}(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(gt) & -i\sin(gt) & 0 \\ 0 & -i\sin(gt) & \cos(gt) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad U_{\text{int}}\left(\frac{\pi}{2g}\right) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & -i/\sqrt{2} & 0 \\ 0 & -i/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \sqrt{iSWAP}$$

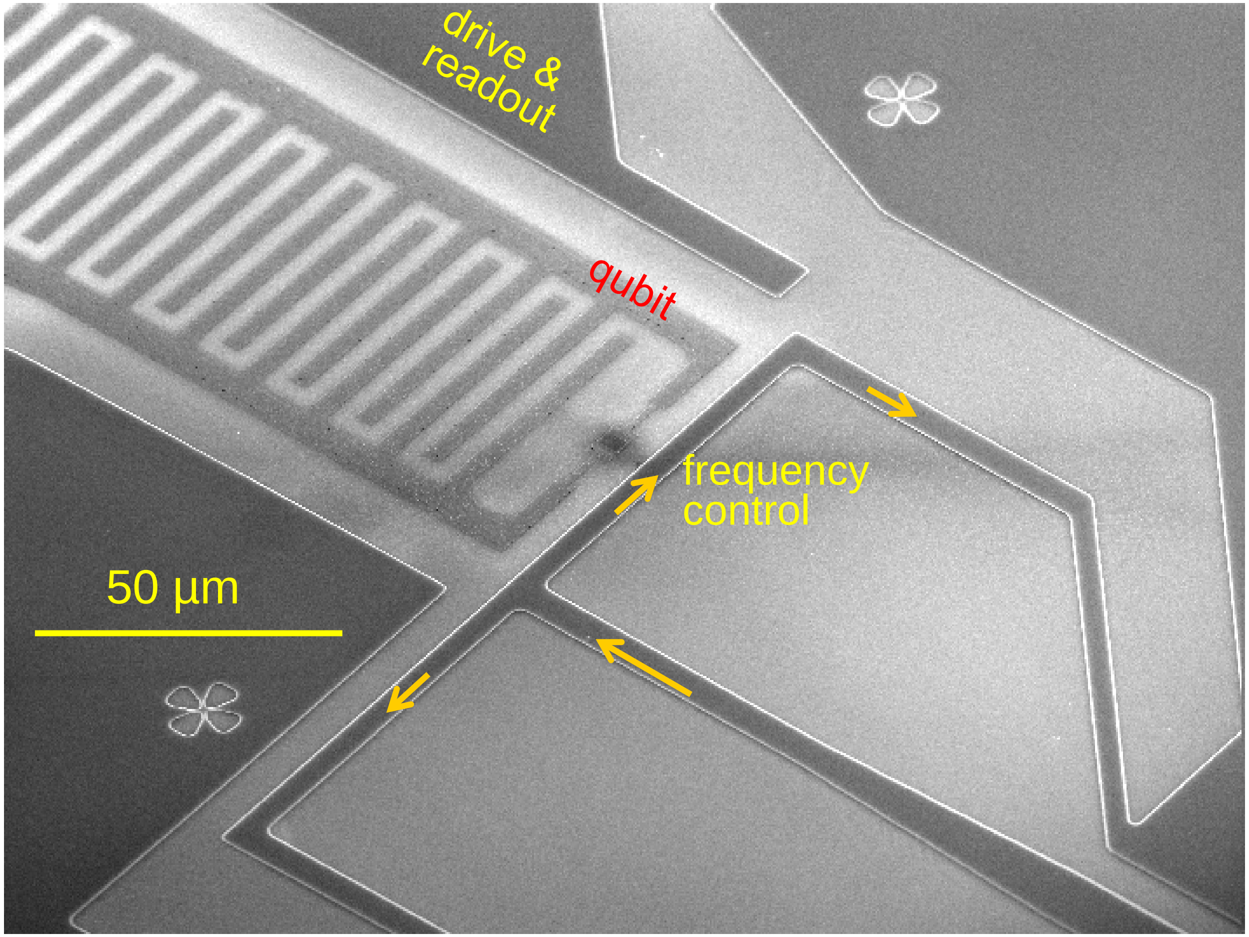
Example : capacitively coupled transmons with individual readout

(Saclay, 2011)

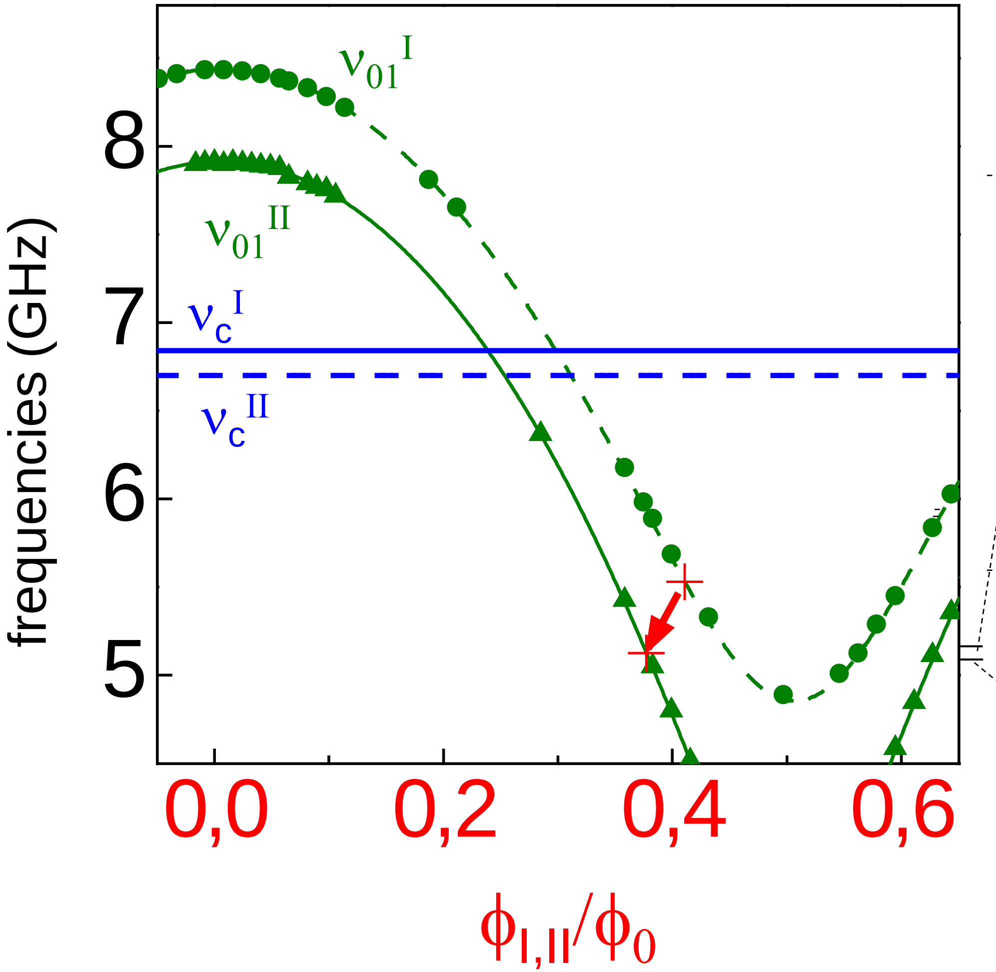


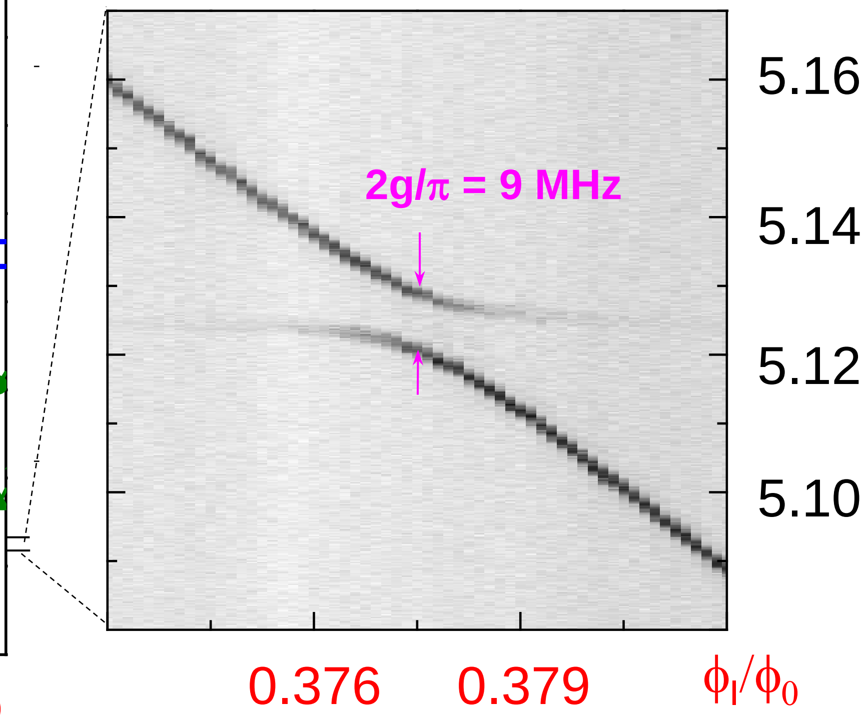


Example : capacitively coupled transmons with individual readout



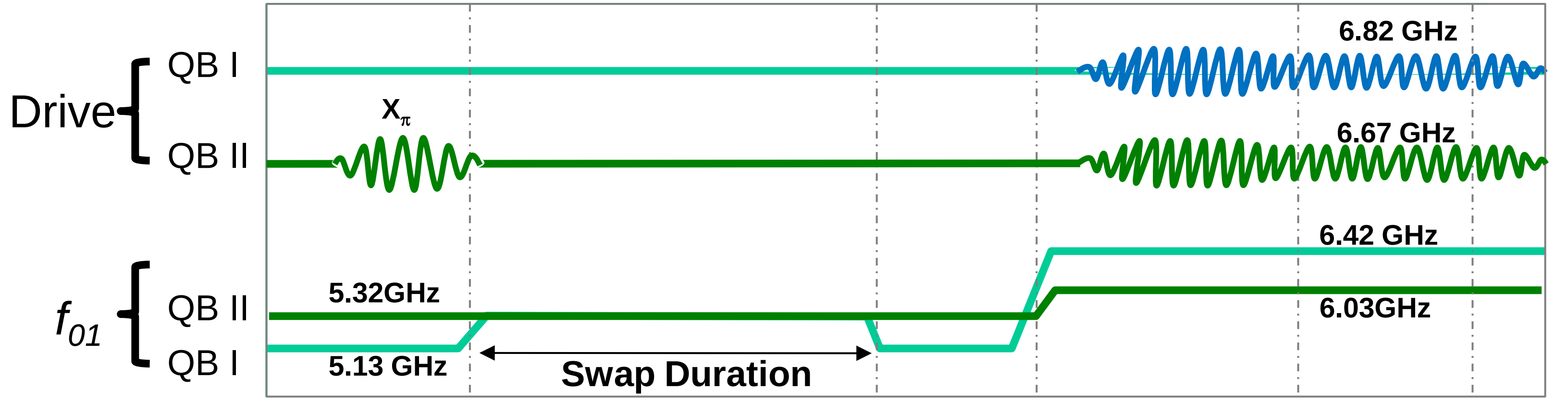
Spectroscopy

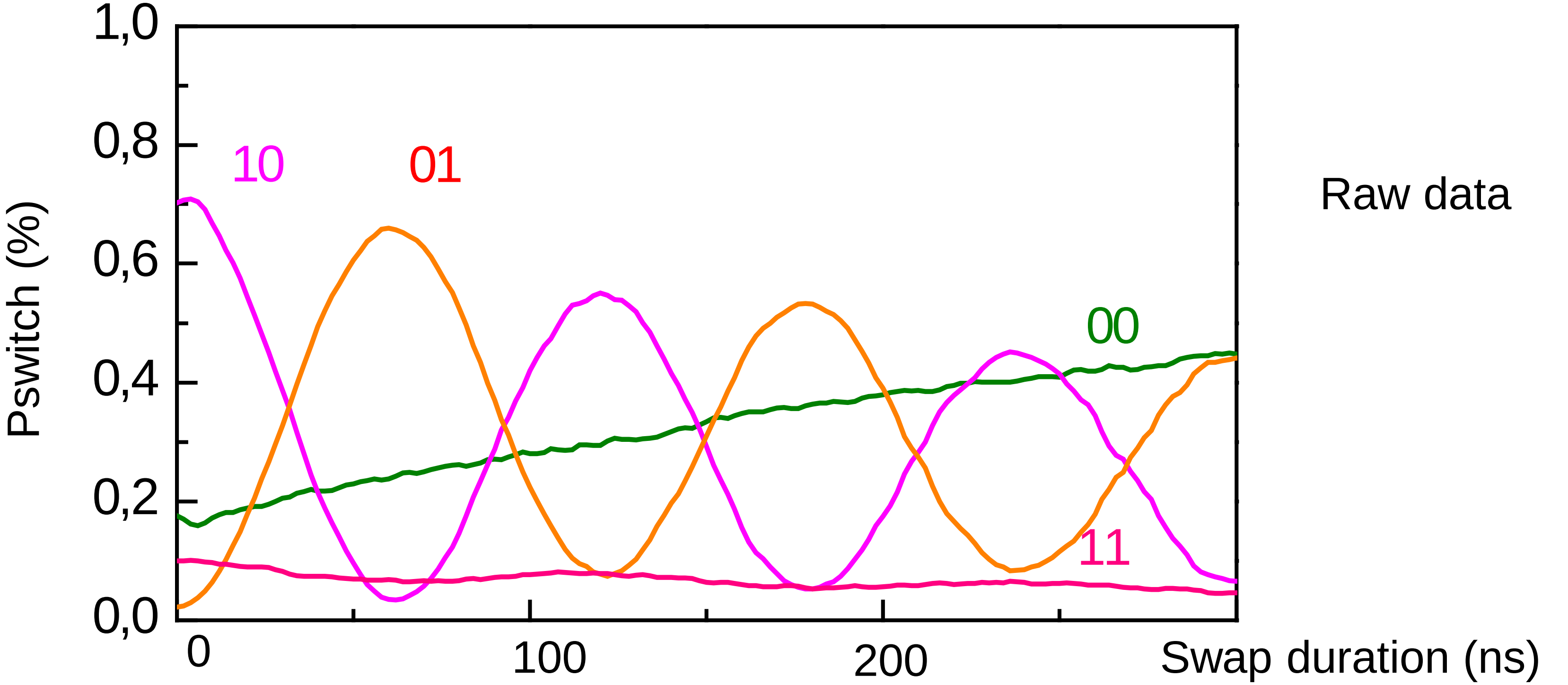




A. Dewes et al., PRL (2011)

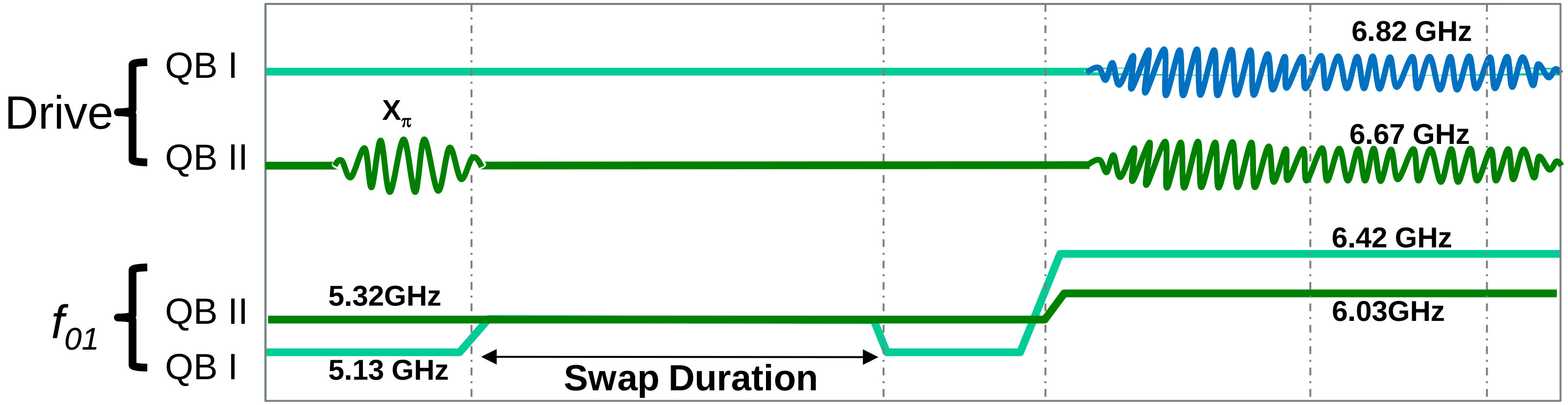
SWAP between two transmon qubits

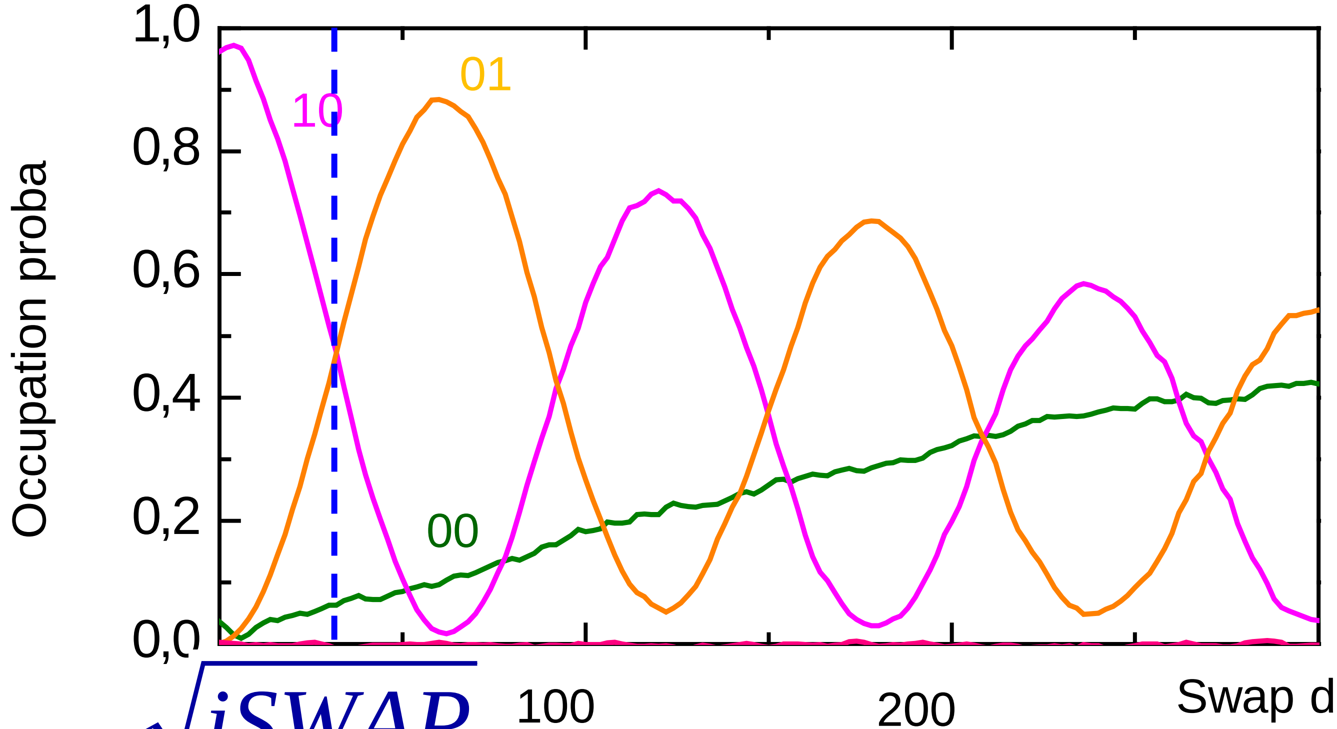




Raw data

SWAP between two transmon qubits



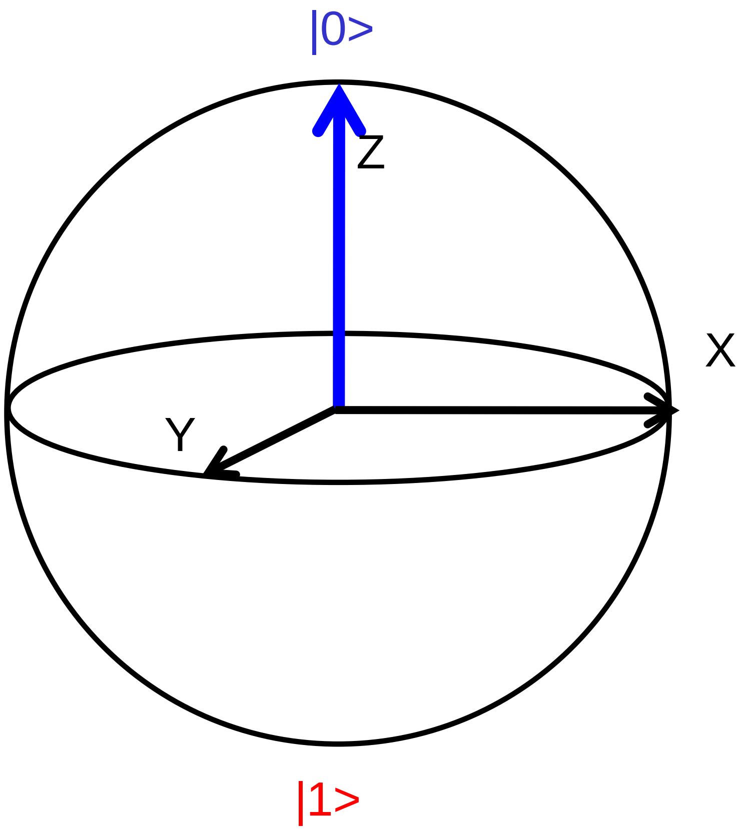


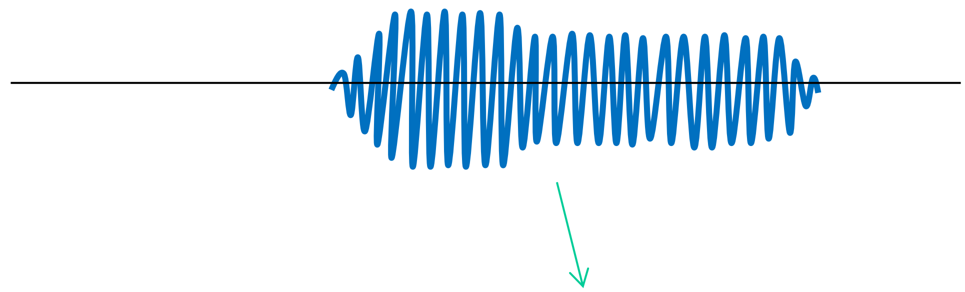
$\sqrt{iSWAP}$

Data corrected from readout errors

How to quantify entanglement ??

Need to measure ρ_{exp} ⟹ Quantum state tomography

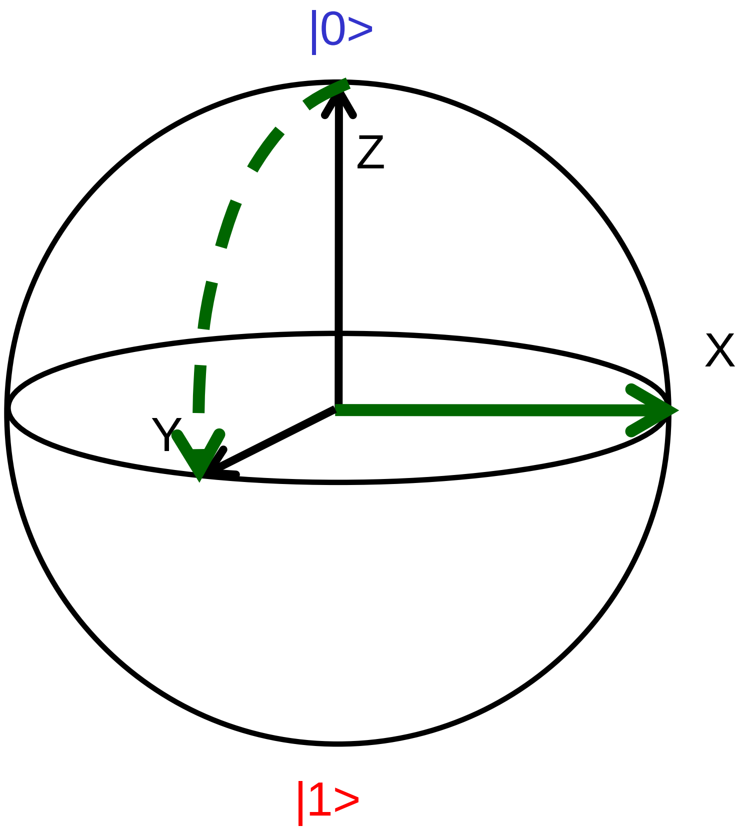


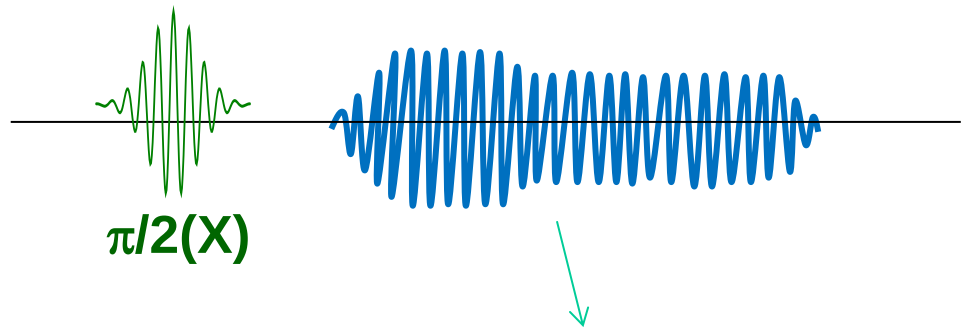


$$P_{switch} \,\square\, \left(1+\langle\sigma_z\rangle\right)/2$$

How to quantify entanglement ??

Need to measure ρ_{exp} ⟹ Quantum state tomography

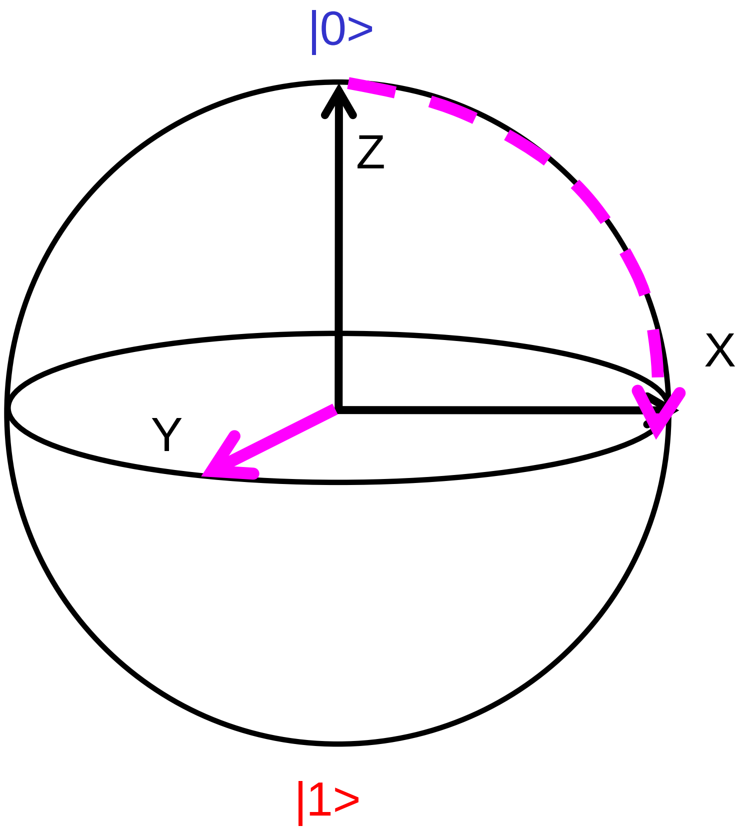


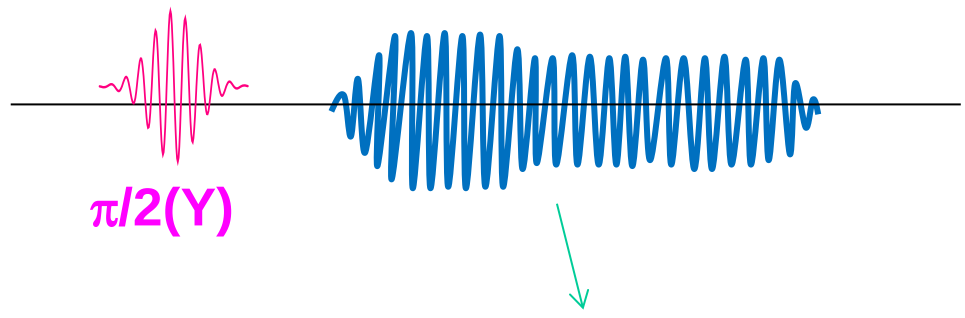


$$P_{switch} \approx \left(1 + \langle \sigma_y \rangle\right)/2$$

How to quantify entanglement ??

Need to measure ρ_{exp} ⟹ Quantum state tomography

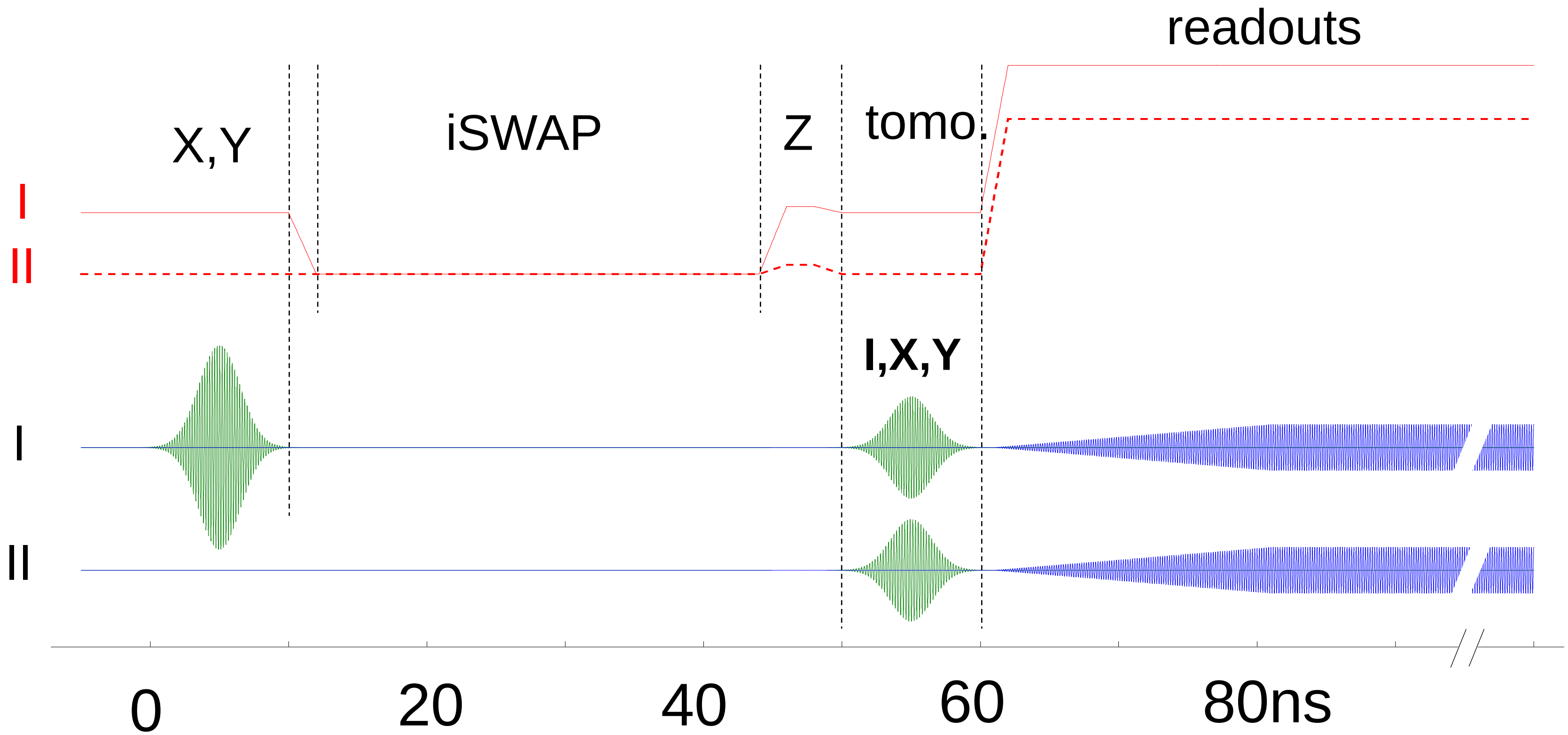




$$P_{switch} \,\square\, \left(1+\langle\sigma_x\rangle\right)/2$$

M. Steffen et al., Phys. Rev. Lett. **97**, 050502 (2006)

How to quantify entanglement ??

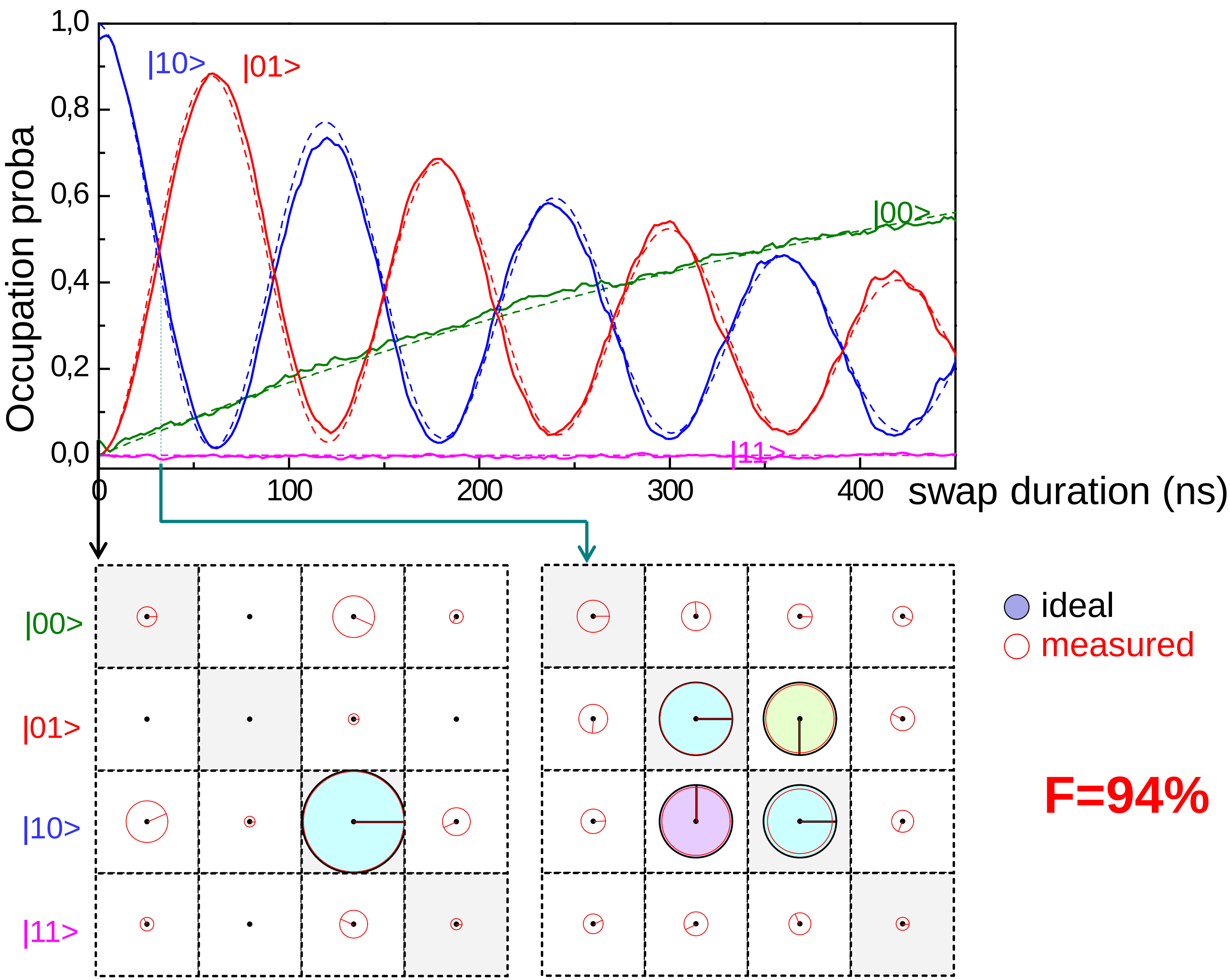


3*3 rotations*3 independent probabilities (P_{00},P_{01},P_{10}) = 27 measured numbers

➡ Fit experimental density matrix ρ_{exp}

➡ Compute fidelity $F = Tr\left(\rho_{th}{}^{1/2}\rho_{exp}\rho_{th}{}^{1/2}\right)$

How to quantify entanglement ??



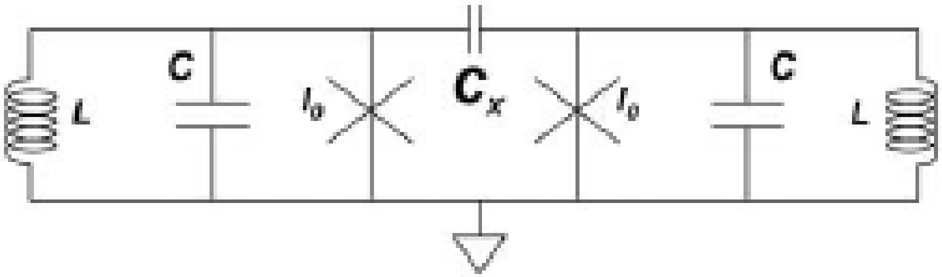
F=98%

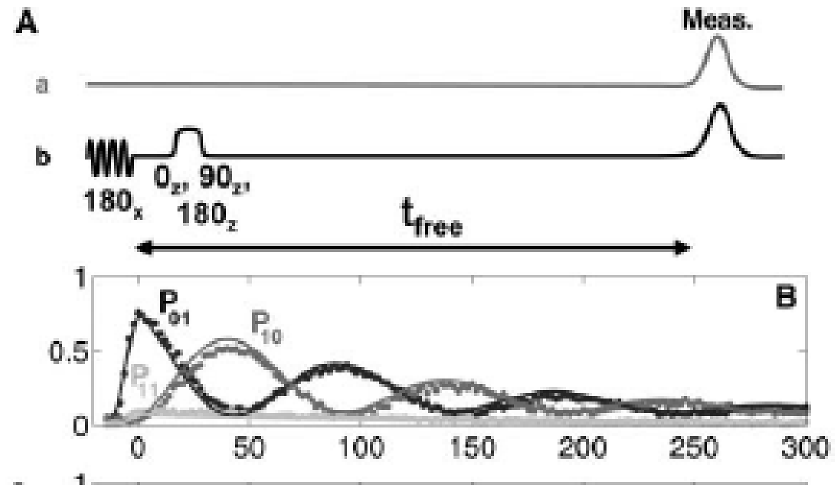
F=94%

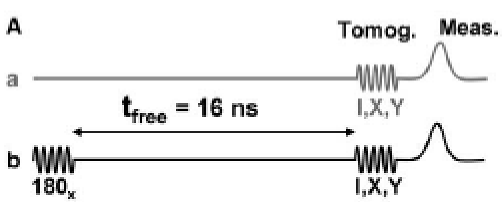
A. Dewes et al., PRL (2011)

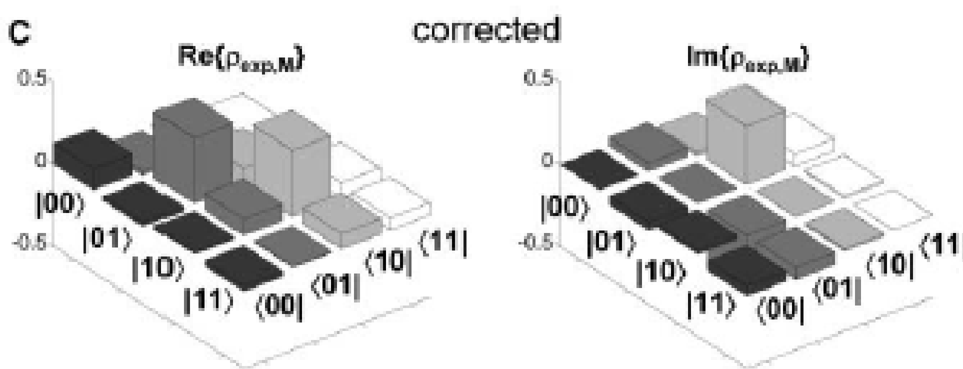
SWAP gate of capacitively coupled phase qubits

M. Steffen et al., Science 313, 1423 (2006)





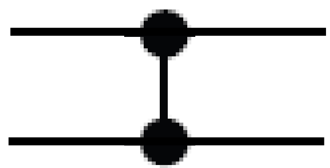




F=0.87

Other universal two-qubit gates

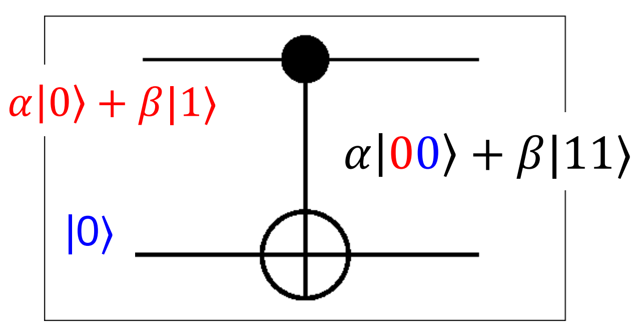
The control-phase gate



$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Columns: $|00\rangle$ $|01\rangle$ $|10\rangle$ $|11\rangle$; rows: $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$

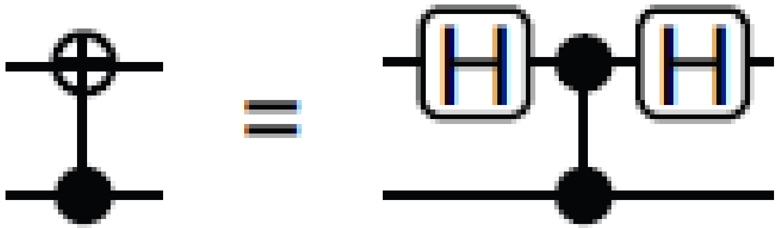
The controlled-NOT gate



$$U_{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Columns: $|00\rangle$ $|01\rangle$ $|10\rangle$ $|11\rangle$; rows: $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$

Decomposition of CNOT gate



One-qubit
Hadamard gate

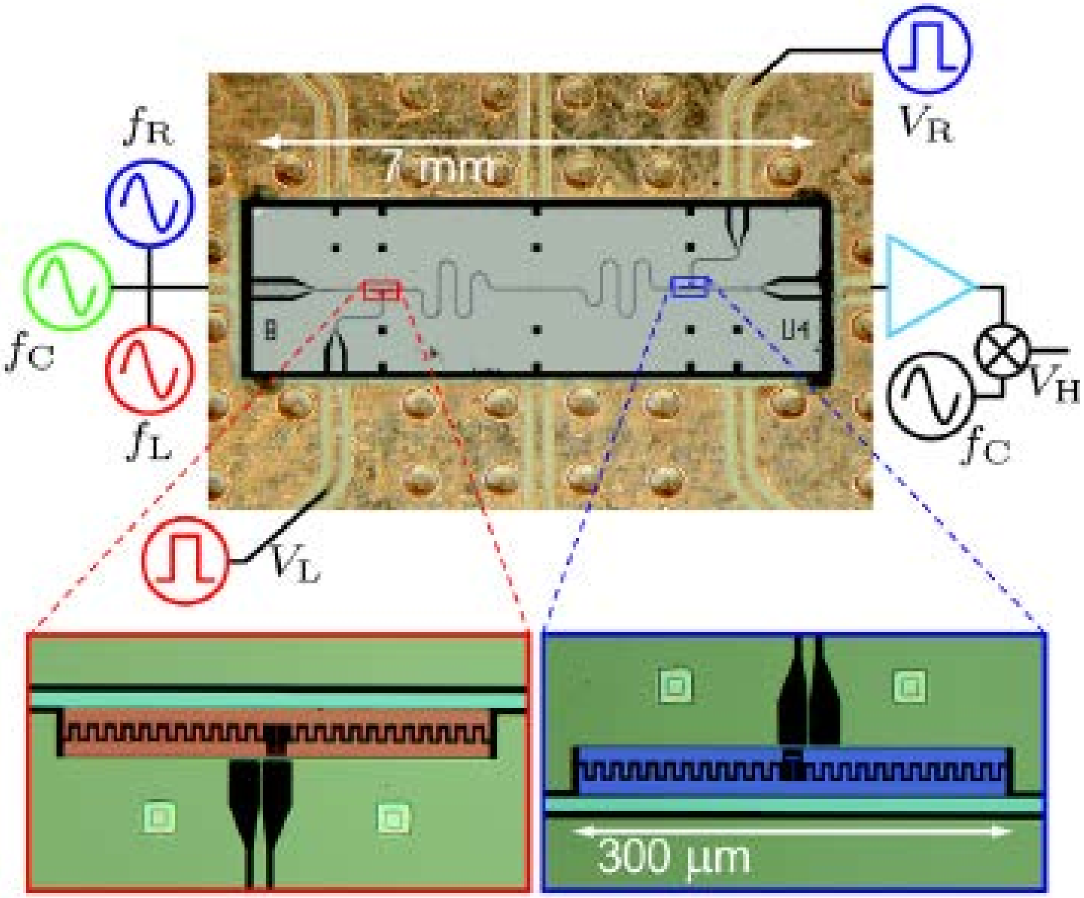
$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\hat{H}|0\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$\hat{H}|1\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

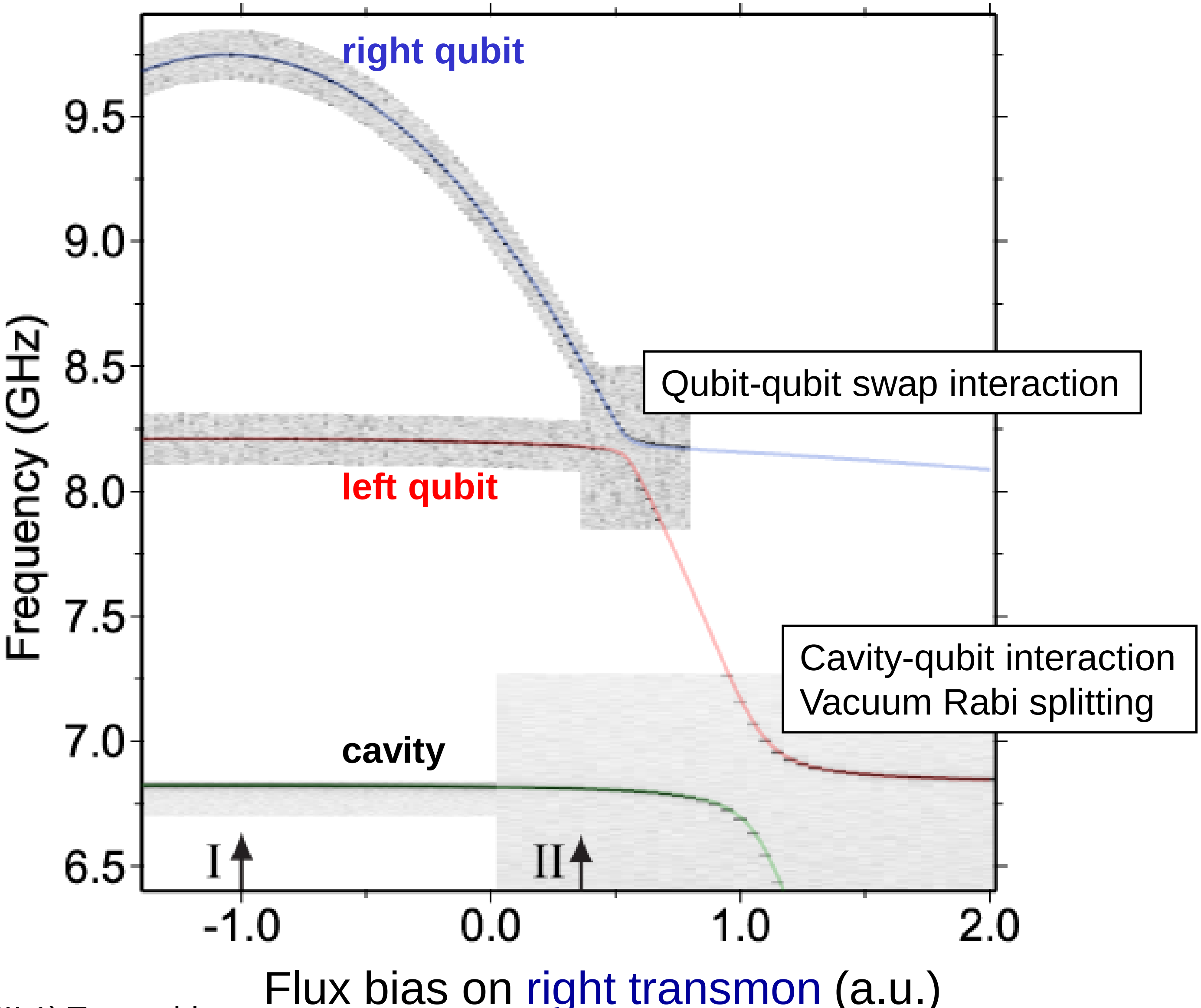
Control-Phase with two coupled transmons

DiCarlo et al., Nature 460, 240-244 (2009)



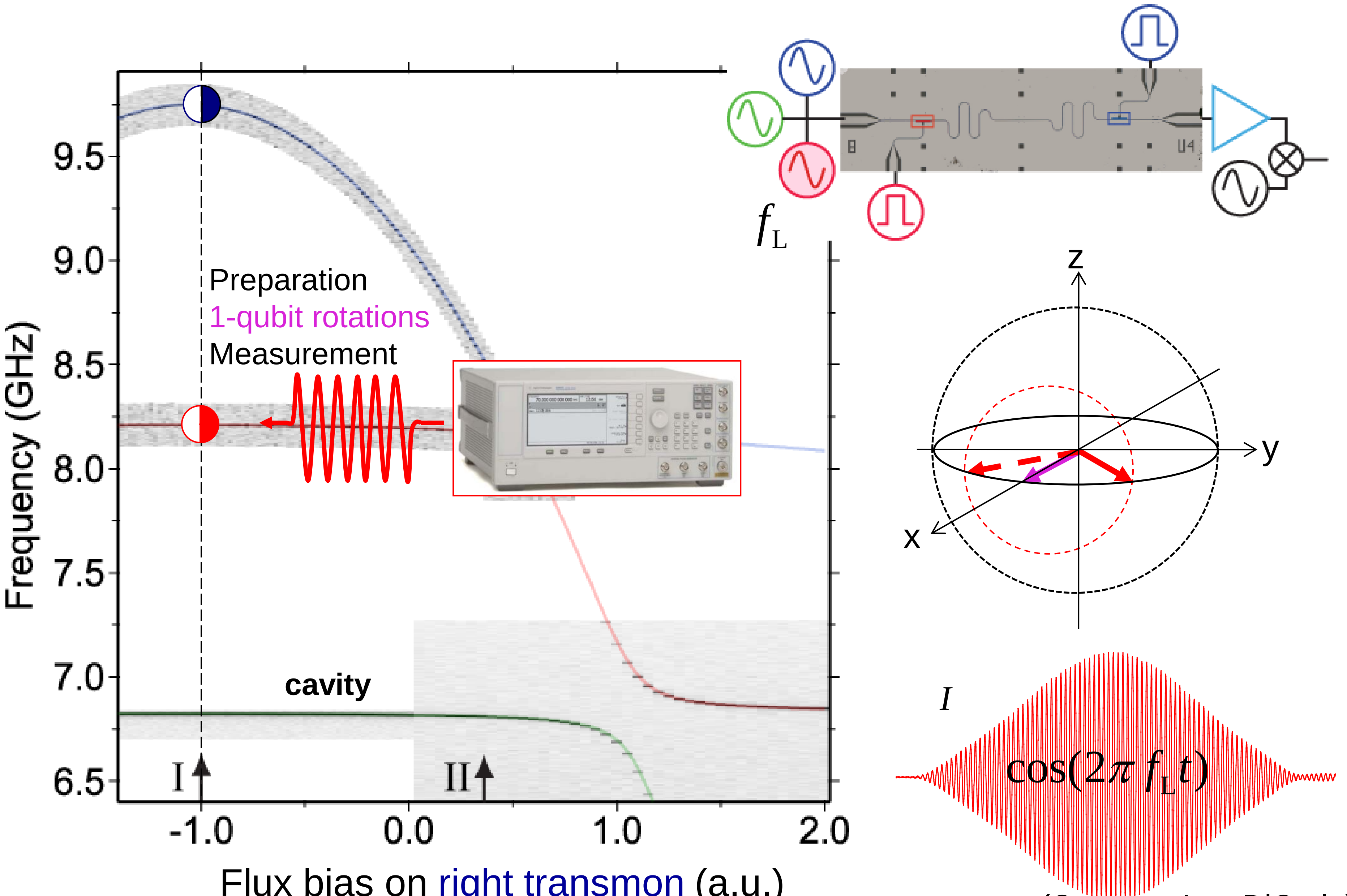
$$H_{\text{int}}/\hbar = g_{eff\,1}\left(|1_L 0_R\rangle\langle 0_L 1_R| + h.c\right) + g_{eff\,2}\left(|1_L 1_R\rangle\langle 0_L 2_R| + h.c\right)$$

Spectroscopy of two qubits + cavity



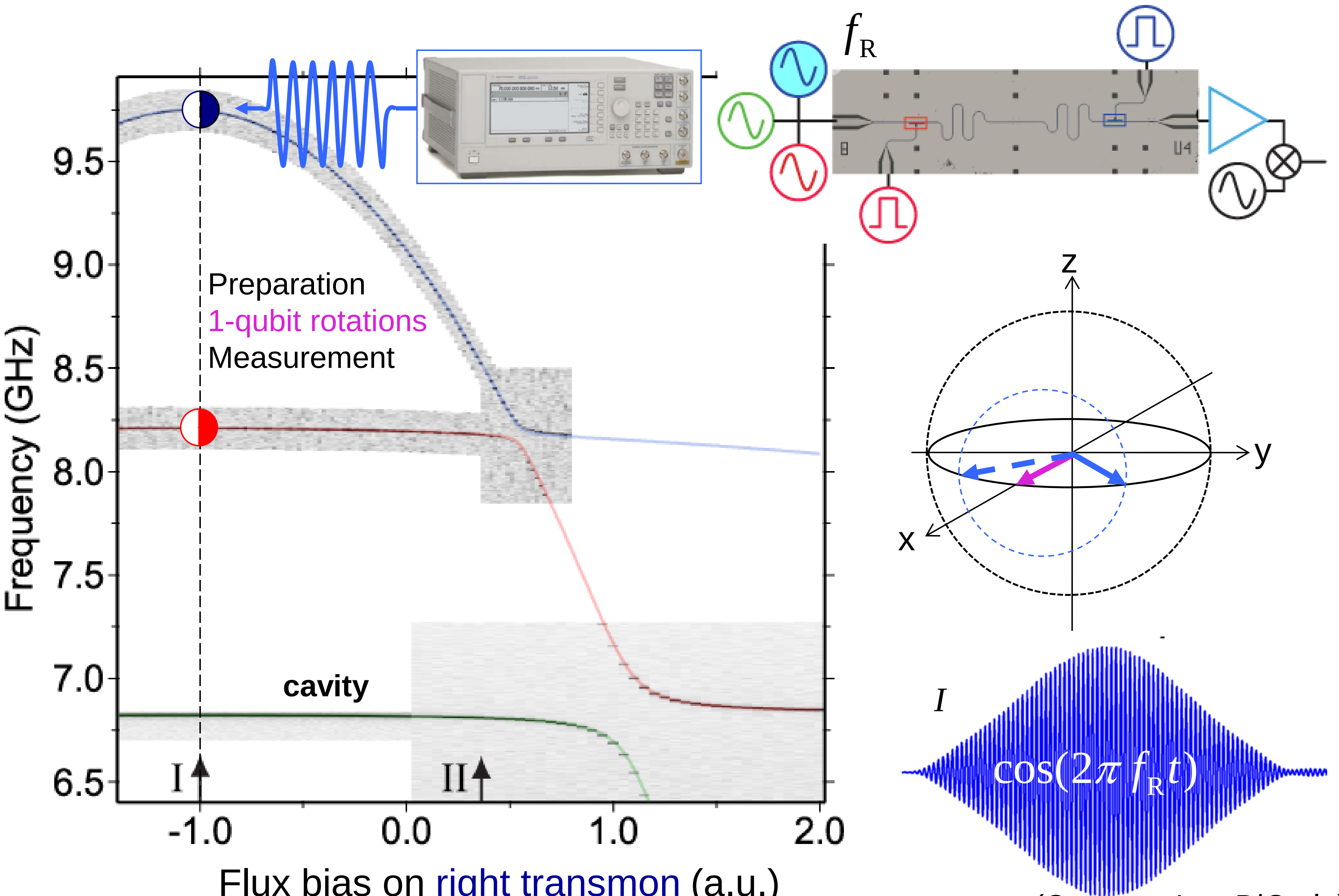
(Courtesy Leo DiCarlo)

One-qubit gates: *X* and *Y* rotations



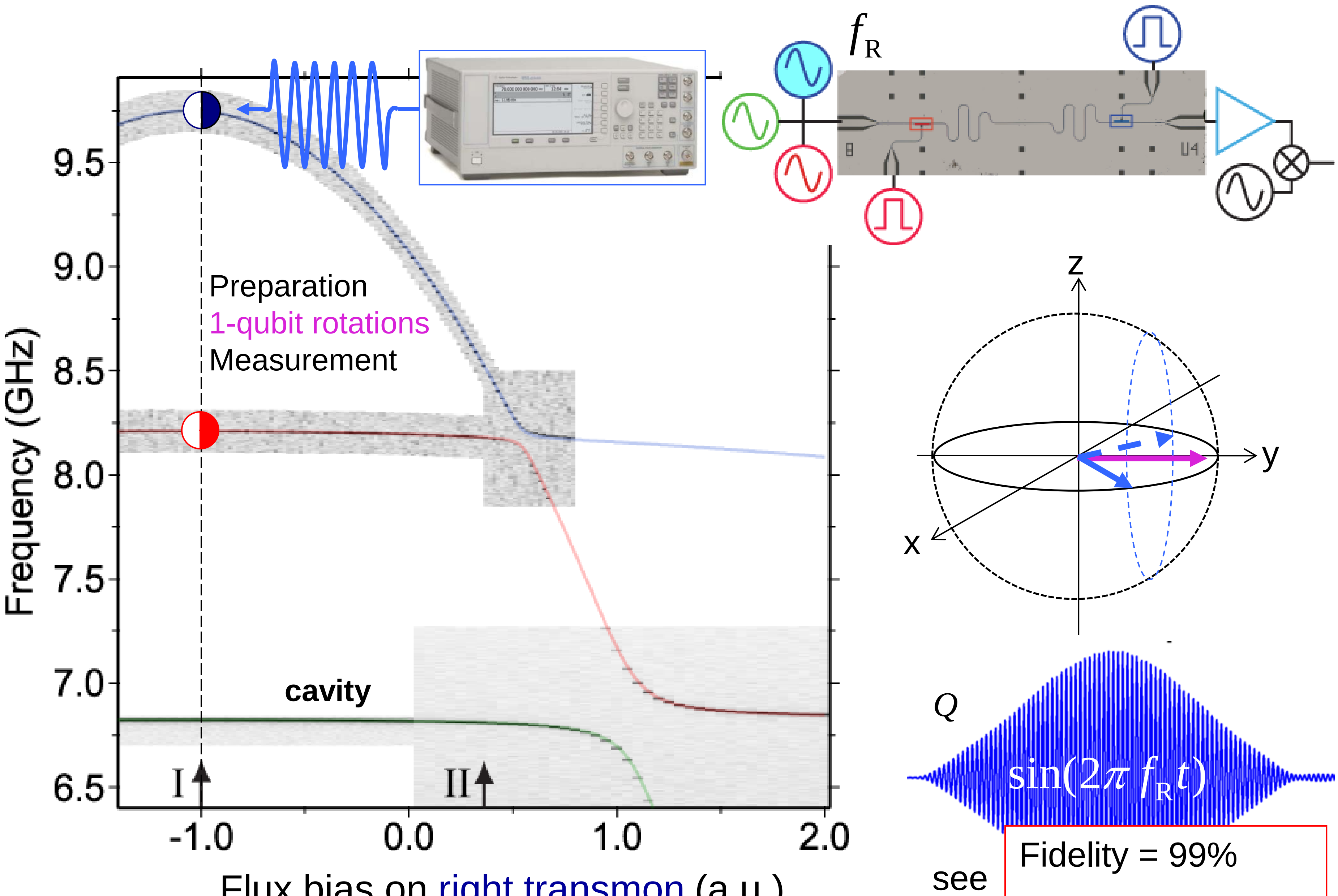
(Courtesy Leo DiCarlo)

One-qubit gates: *X* and *Y* rotations



(Courtesy Leo DiCarlo)

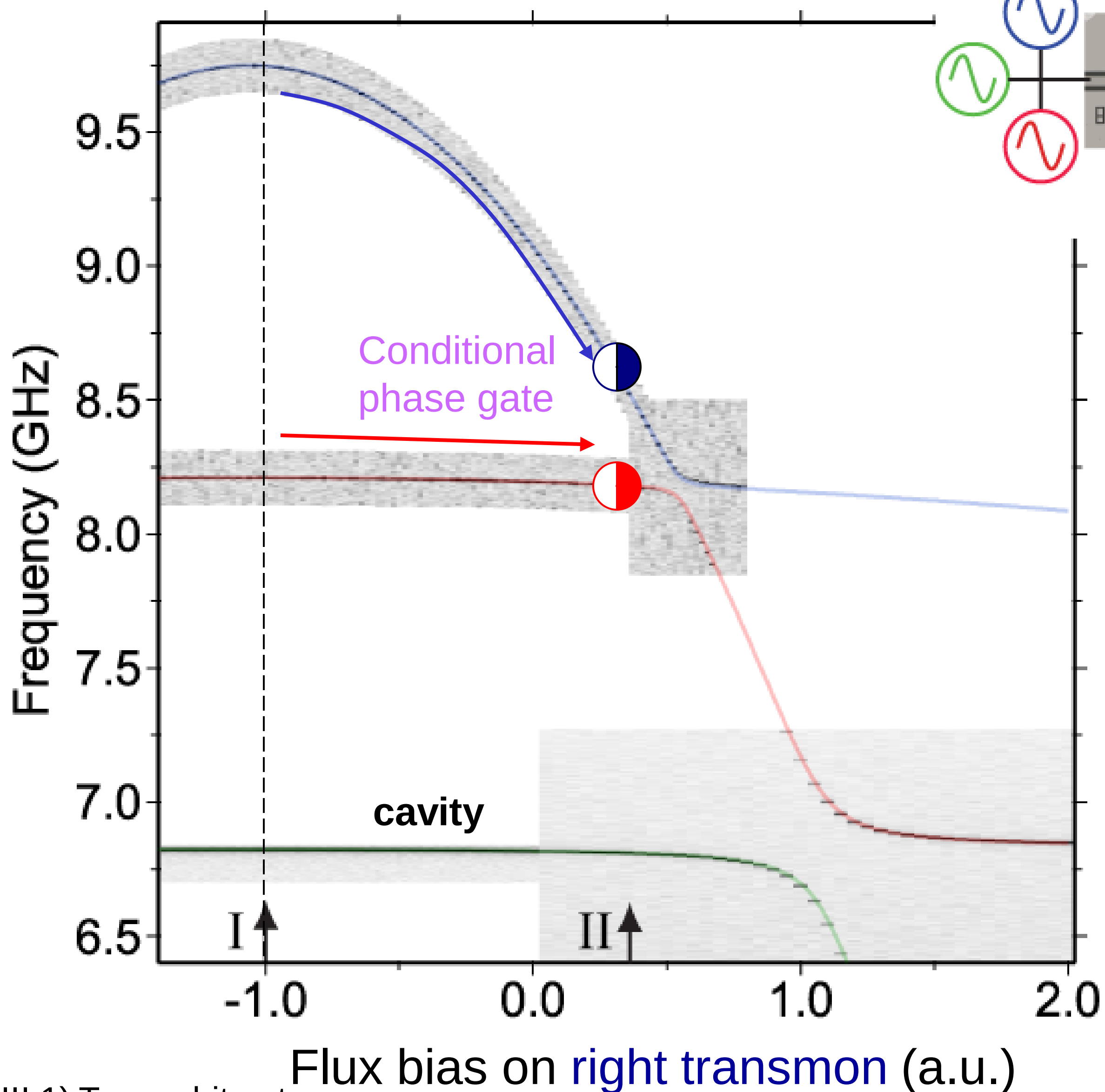
One-qubit gates: *X* and *Y* rotations

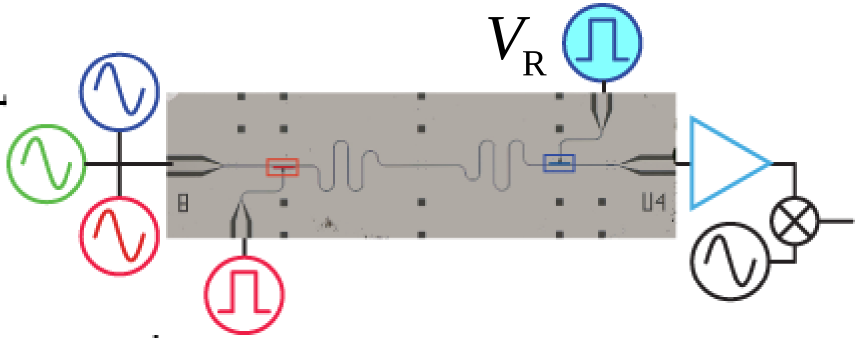


Fidelity = 99%

see J. Chow *et al.*, PRL (2009)

Two-qubit gate: turn on interactions





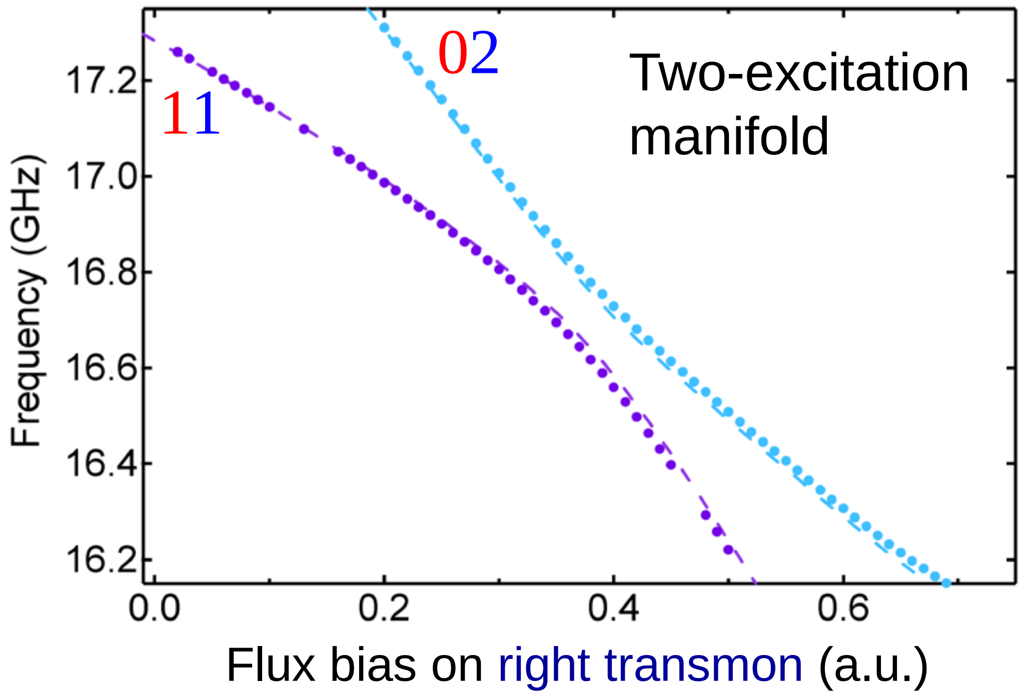
Use control lines to push qubits near a resonance

(Courtesy Leo DiCarlo)

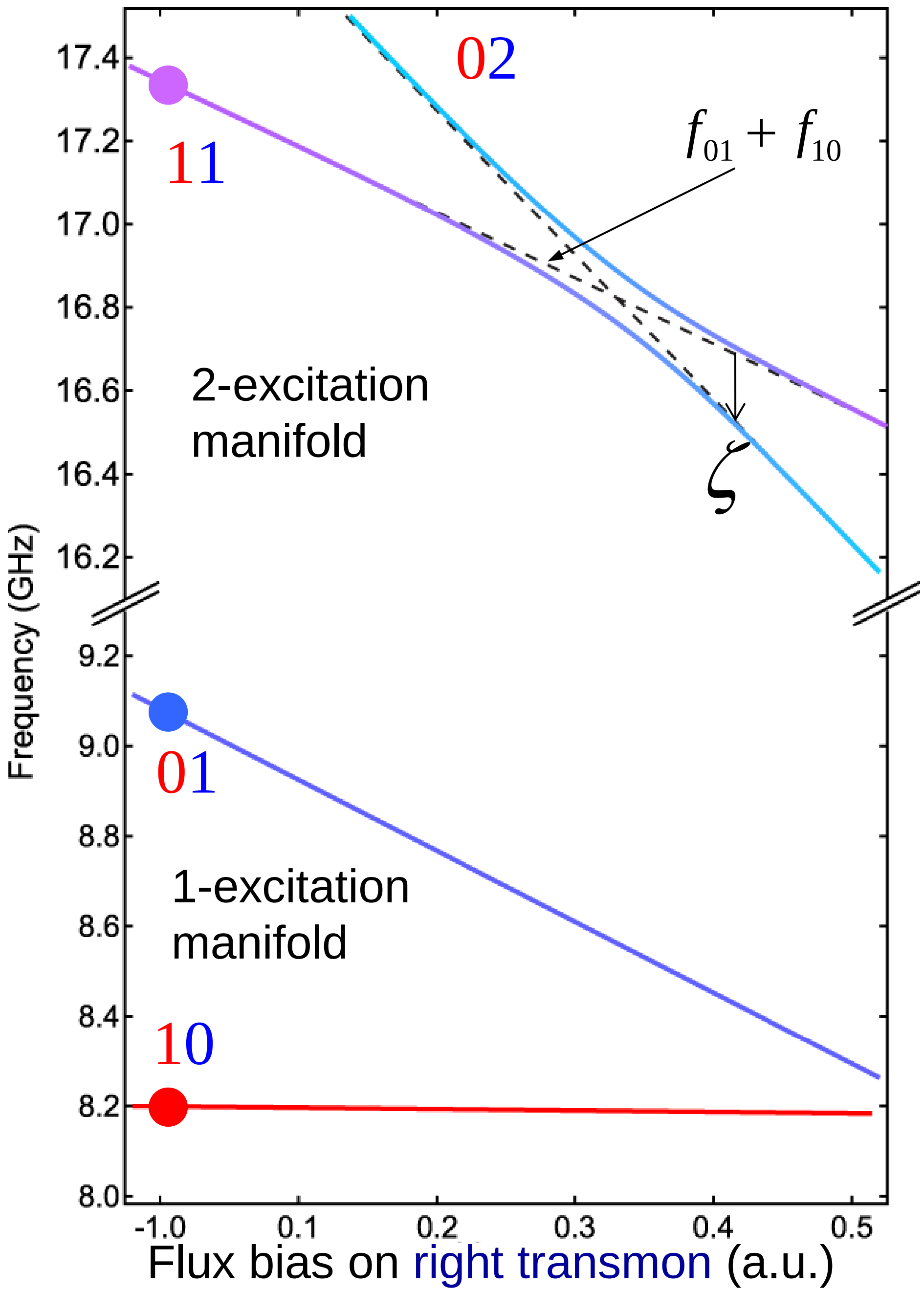
Two-excitation manifold of system

- Avoided crossing (160 MHz)

$$|11\rangle \leftrightarrow |02\rangle$$



(Courtesy Leo DiCarlo)



$$\varphi_a = -2\pi \int_{t_0}^{t_f} \delta f_a(t)\,dt$$

$$|11\rangle \rightarrow e^{i\varphi_{11}} |11\rangle$$

$$\varphi_{11} = \varphi_{10} + \varphi_{01} - 2\pi \int_{t_0}^{t_f} \zeta(t)\,dt$$

$$|01\rangle \rightarrow e^{i\varphi_{01}} |01\rangle$$

$$|10\rangle \rightarrow e^{i\varphi_{10}} |10\rangle$$

(Courtesy Leo DiCarlo)

Implementing C-Phase

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\varphi_{01}} & 0 & 0 \\ 0 & 0 & e^{i\varphi_{10}} & 0 \\ 0 & 0 & 0 & e^{i\varphi_{11}} \end{pmatrix}$$

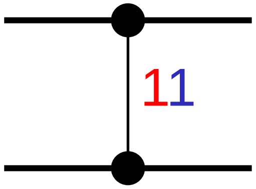
(columns labelled $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$; rows labelled $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$)

Adjust timing of flux pulse so that only quantum amplitude of $|11\rangle$ acquires a minus sign:

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

(columns labelled $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$; rows labelled $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$)

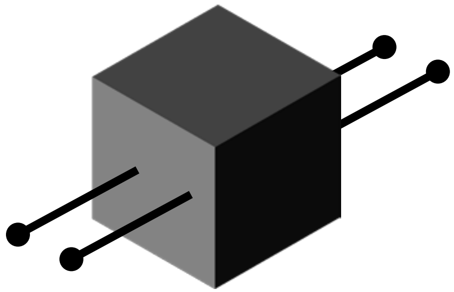
C-Phase$_{11}$



(Courtesy Leo DiCarlo)

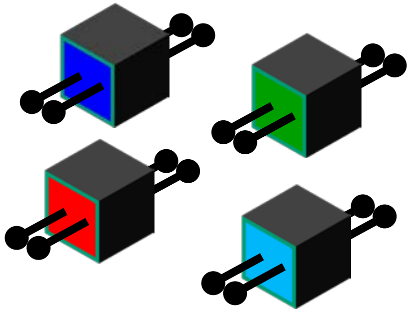
The Grover Search Algorithm: A Benchmark for Quantum Speed-Up

A **search oracle** *marks a state **i***



$$i \in \{00 \quad 01 \quad 10 \quad 11\}$$

Four possible oracles.
Which one have we got?



What is the probability to give correct answer after **one** call of the oracle ?

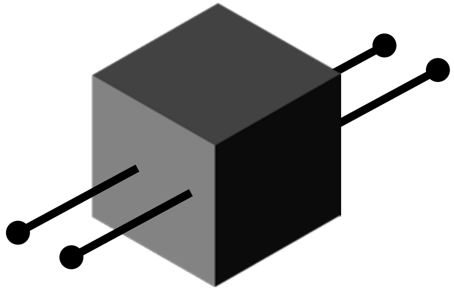
CLASSICALLY

Try one state. Probability ¼ to be the state marked by oracle.
If it is not marked, guess randomly amongst 3 remaining possible states.

Total maximal classical probability of success = ¼ + ¾*1/3 = 1/2

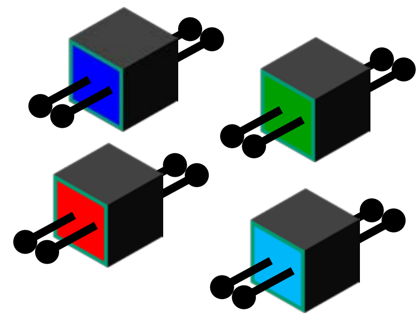
The Grover Search Algorithm: A Benchmark for Quantum Speed-Up

A **search oracle** *marks a state **i***



$$i \in \{00 \quad 01 \quad 10 \quad 11\}$$

Four possible oracles.
Which one have we got?



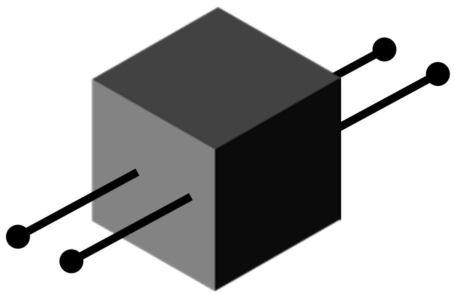
What is the probability to give correct answer after **one** call of the oracle ?

<u>QUANTUM-MECHANICALLY</u>

Grover’s search algorithm : probability can reach 1 !

The Grover Search Algorithm: A Benchmark for Quantum Speed-Up

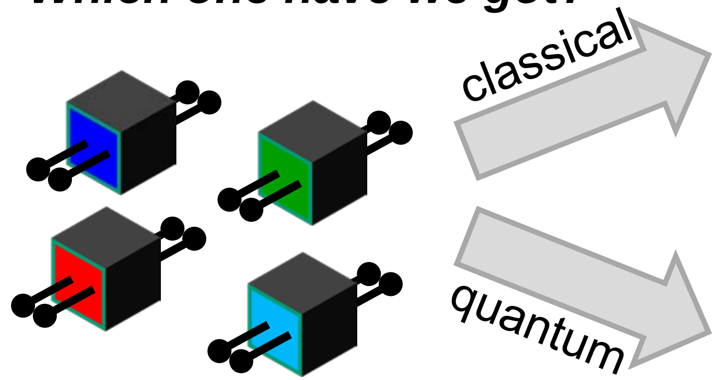
A **search oracle** *marks a state i*



$i \in \{00 \quad 01 \quad 10 \quad 11\}$

Four possible oracles.

Which one have we got?

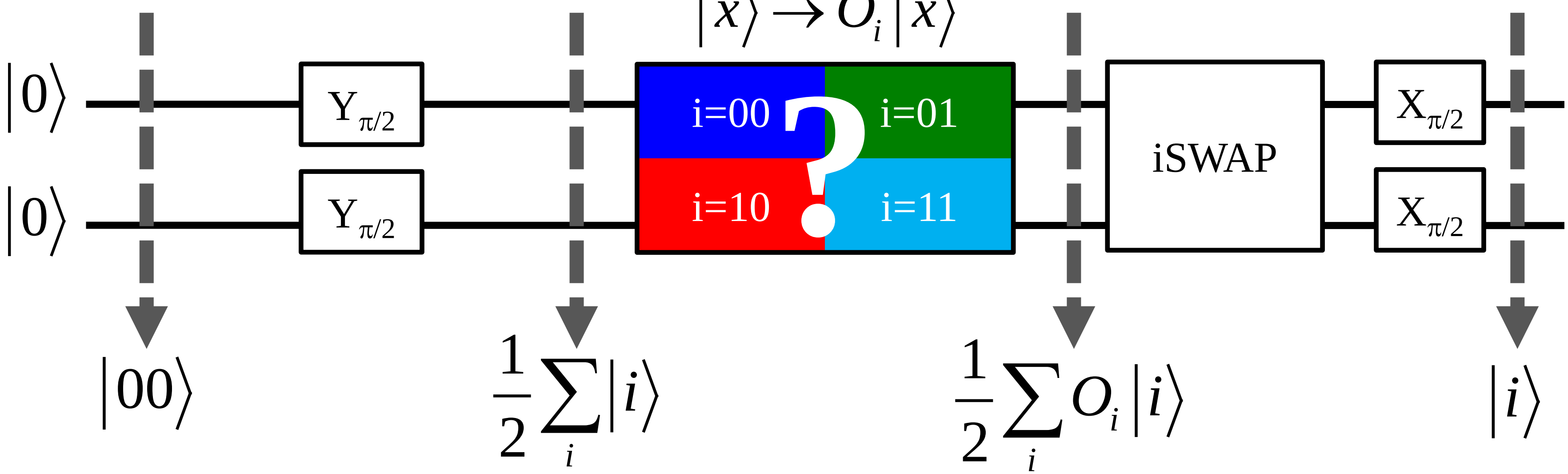


In one call : $P_{success} \leq 0.5$

In one call : $P_{success} \leq 1$

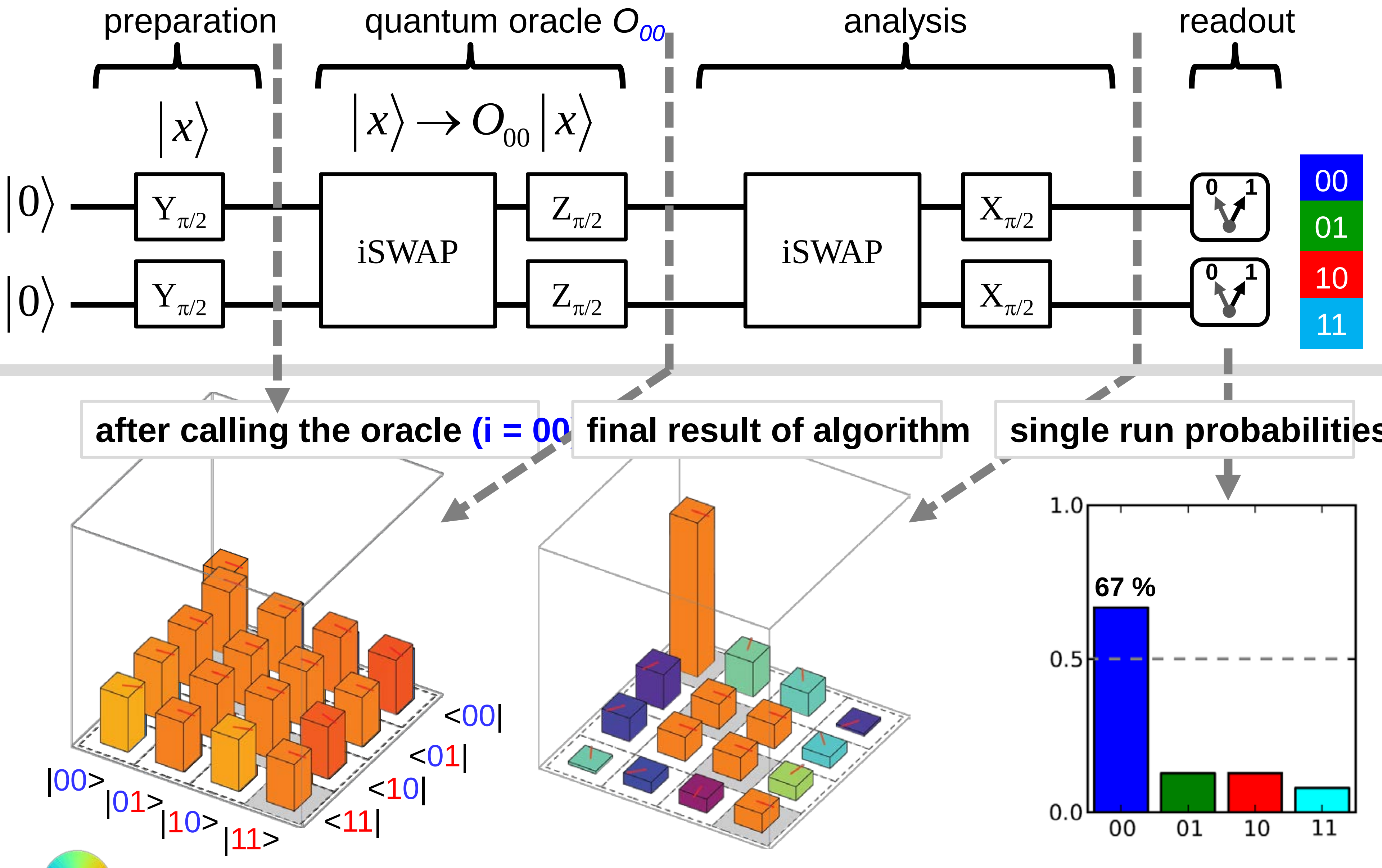
prepare a test state | **call the quantum Oracle *O*** | **analyze the result**

$|x\rangle \rightarrow O_i |x\rangle$



Grover, Proc. 28th Ann. ACM Sym. on the Theory of Computing (1996)

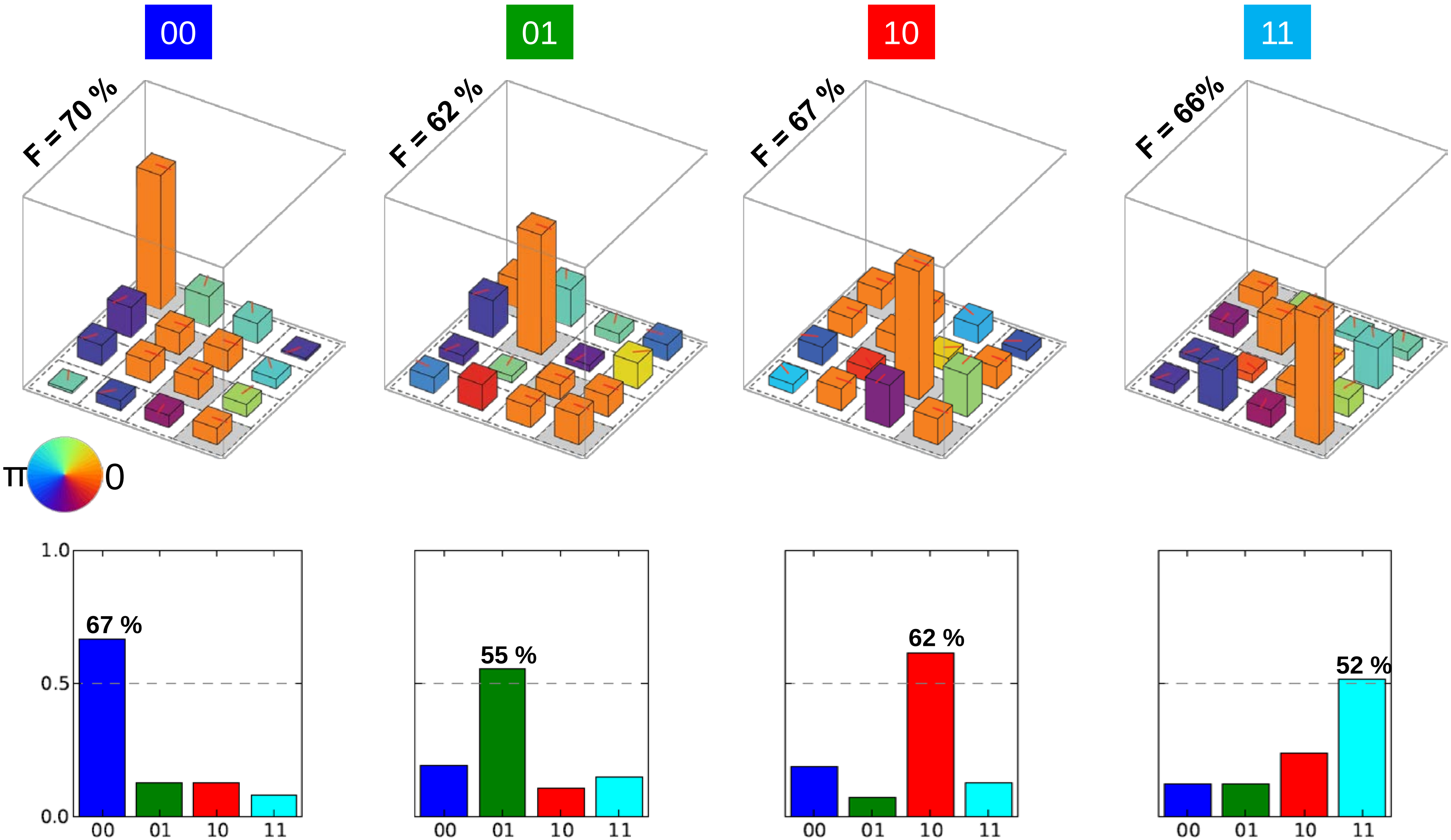
Implementation of the Grover Algorithm



fidelity F = 98 % / 87 % **fidelity F = 70 %**

Dewes et.al. **PRB (2012)**

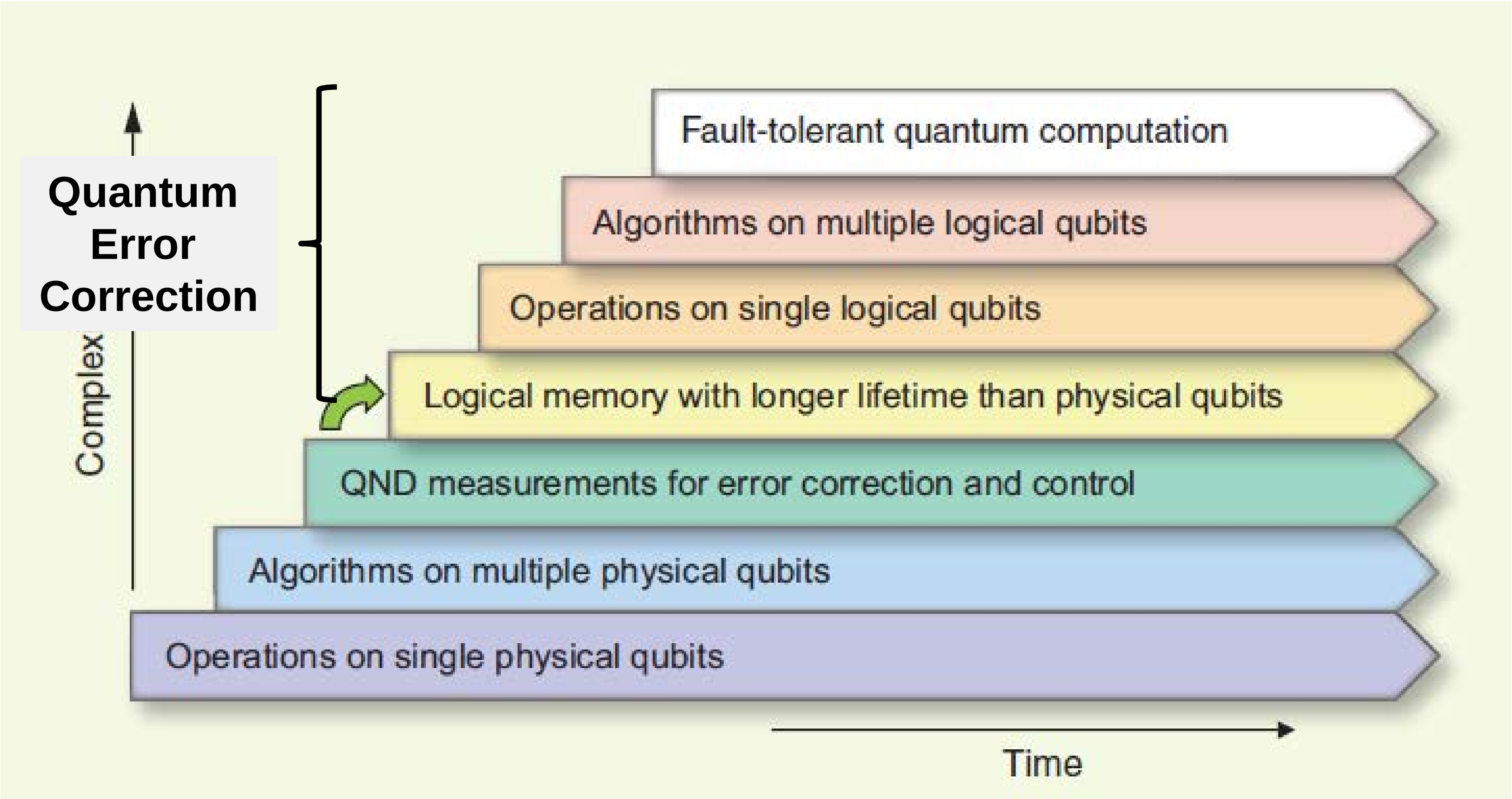
Results for Different Oracle Functions



***F_i* > 50 %** for all four oracles → <u>Demonstration of quantum speed-up</u>

Dewes et.al. **PRB (2012)**

Steps towards quantum computer



R. Schoelkopf and M. Devoret, Science (2013)

Basics of Quantum Error Correction

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Bit-flip errors

$$\alpha|1\rangle + \beta|0\rangle$$

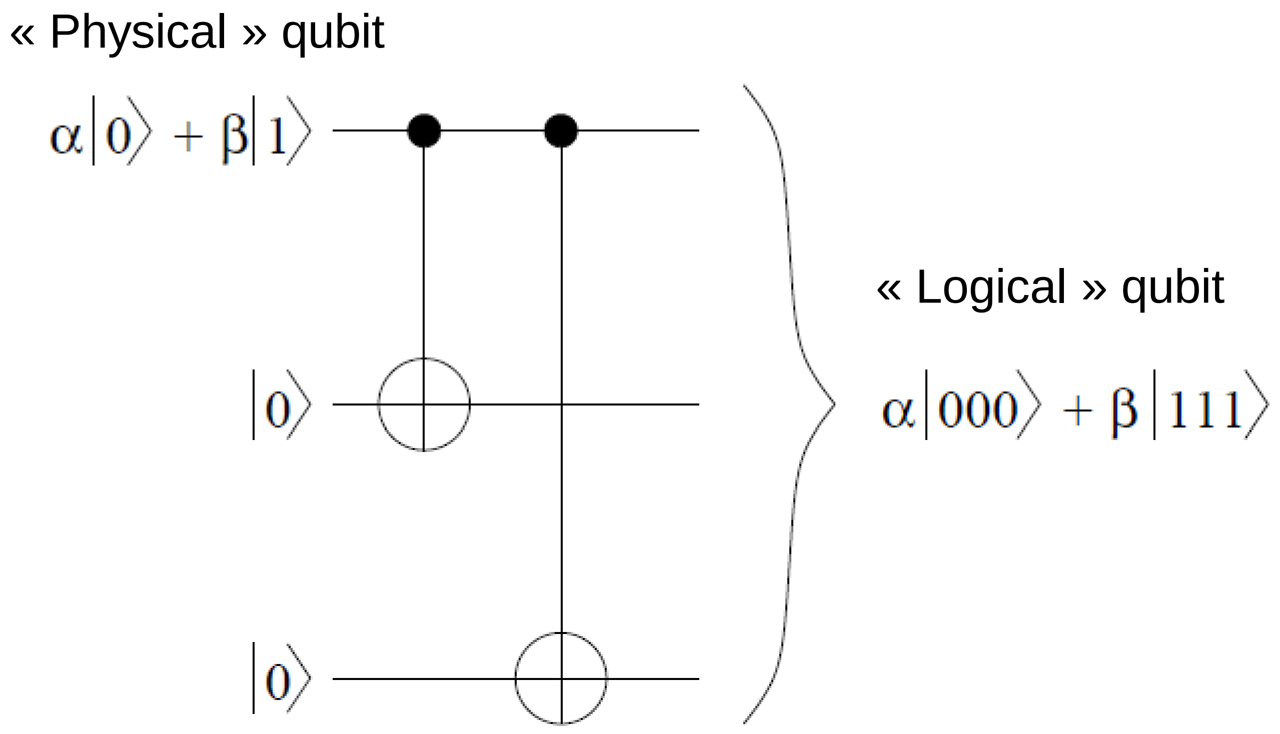
Phase-flip errors

$$\alpha|1\rangle + e^{i\phi}\beta|0\rangle$$

Detecting and correcting these errors ?

Difficulty : Quantum measurement !

Correcting bit-flip errors (1) : encoding



D. Mermin, lecture notes on quantum computation, chap. 5

Correcting bit-flip errors (2) : detecting the error

Key property of logical qubit state

$$\alpha|000\rangle + \beta|111\rangle$$

Each two-qubit pair is in an eigenstate of the parity operators $\hat{P}_{12}, \hat{P}_{23}$**,** $\hat{P}_{13}$ **with value +1 :** $P_{12} = +1, P_{23} = +1,\ P_{13} = +1$

In case of one bit-flip error, the occurrence and the position of the errors can be detected by measuring the parities of each pair.

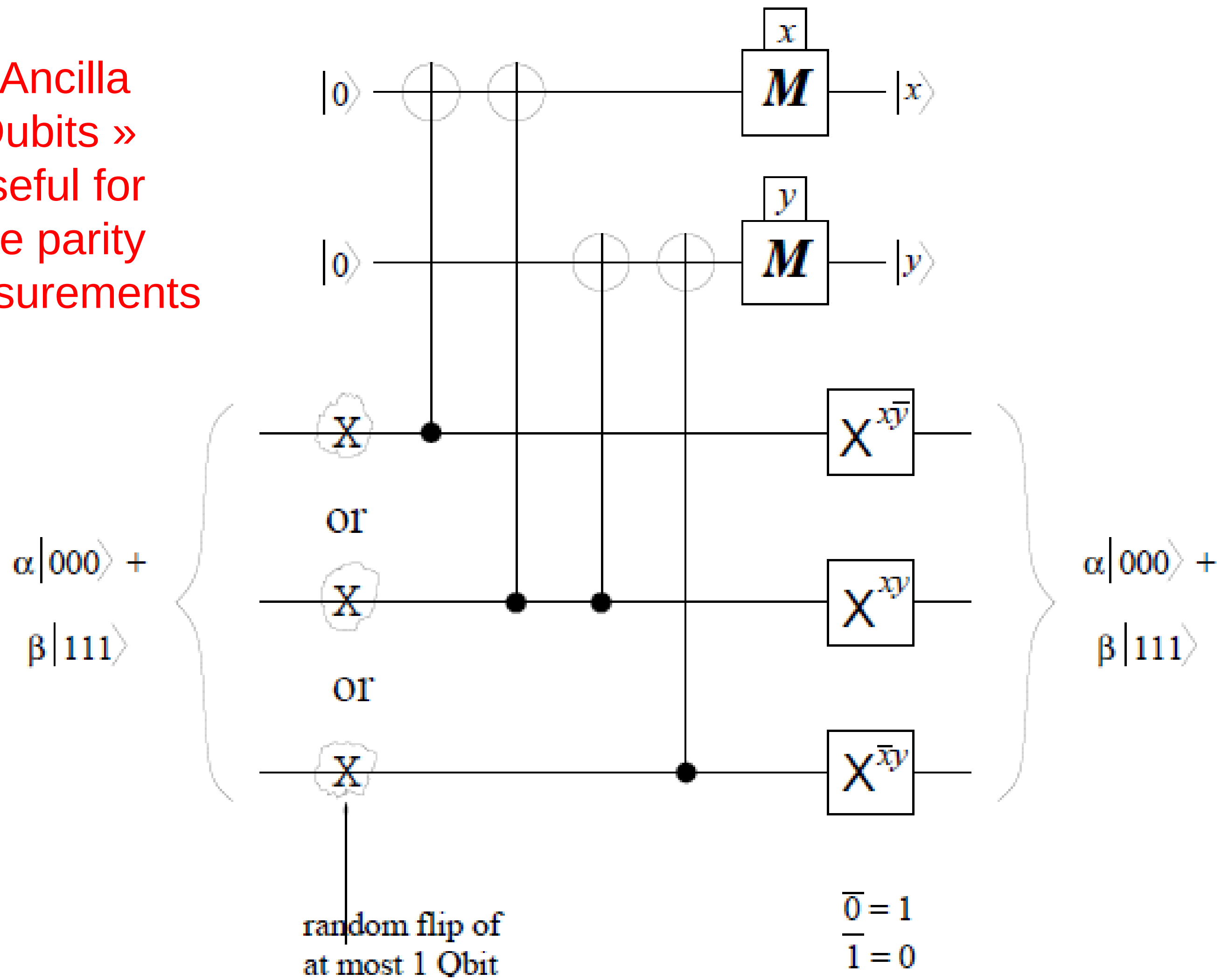
If there is no error, the parity measurements ***do not perturb the state***

But an error can be detected. For instance on qubit 2 would yield $\alpha|010\rangle + \beta|101\rangle$ which would result in $P_{12} = -1, P_{23} = -1,\ P_{13} = +1$

The challenge of QEC is thus to repititvely and non-destructively measure parity operators of pairs of qubits

Correcting bit-flip errors (2) : detecting an error

« Ancilla Qubits » useful for the parity measurements

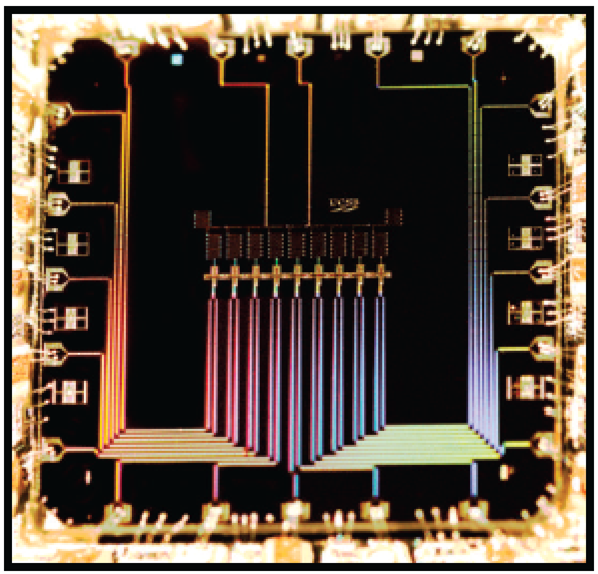


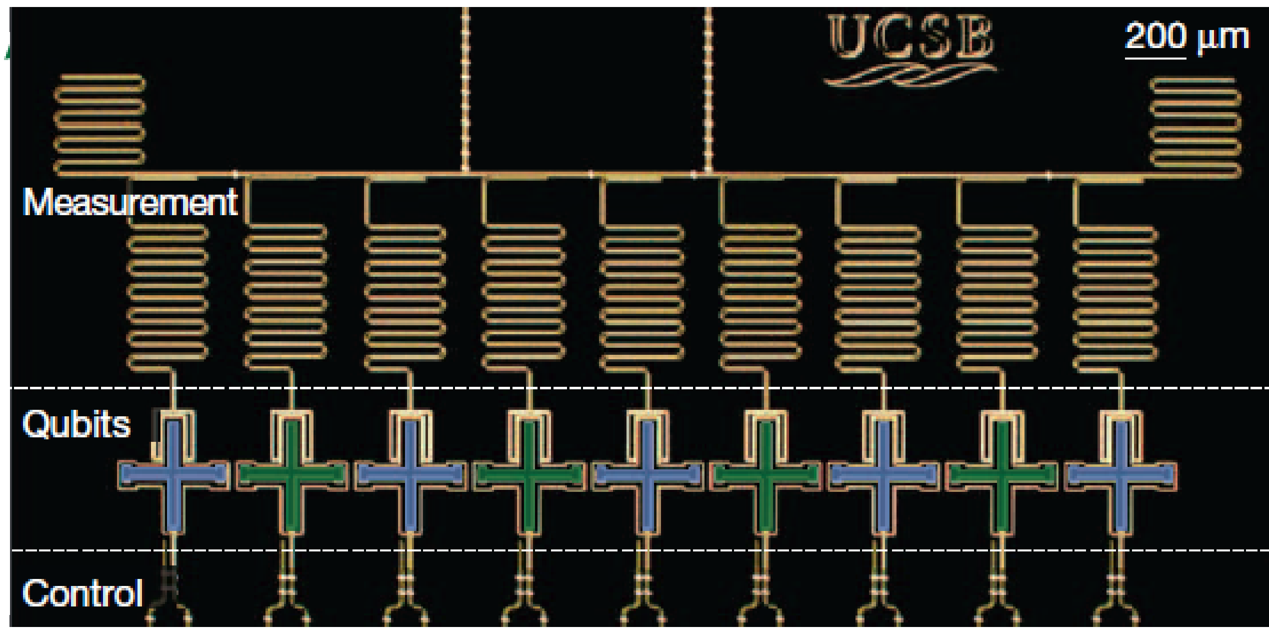
D. Mermin, lecture notes on quantum computation, chap. 5

Correcting bit-flip errors (2) : detecting an error with a 9-qubit chip

State preservation by repetitive error detection in a superconducting quantum circuit (Nature, 2015)

J. Kelly[1]*, R. Barends[1]†*, A. G. Fowler[1,2]†*, A. Megrant[1,3], E. Jeffrey[1]†, T. C. White[1], D. Sank[1]†, J. Y. Mutus[1]†, B. Campbell[1], Yu Chen[1]†, Z. Chen[1], B. Chiaro[1], A. Dunsworth[1], I.-C. Hoi[1], C. Neill[1], P. J. J. O'Malley[1], C. Quintana[1], P. Roushan[1]†, A. Vainsencher[1], J. Wenner[1], A. N. Cleland[1] & John M. Martinis[1]†

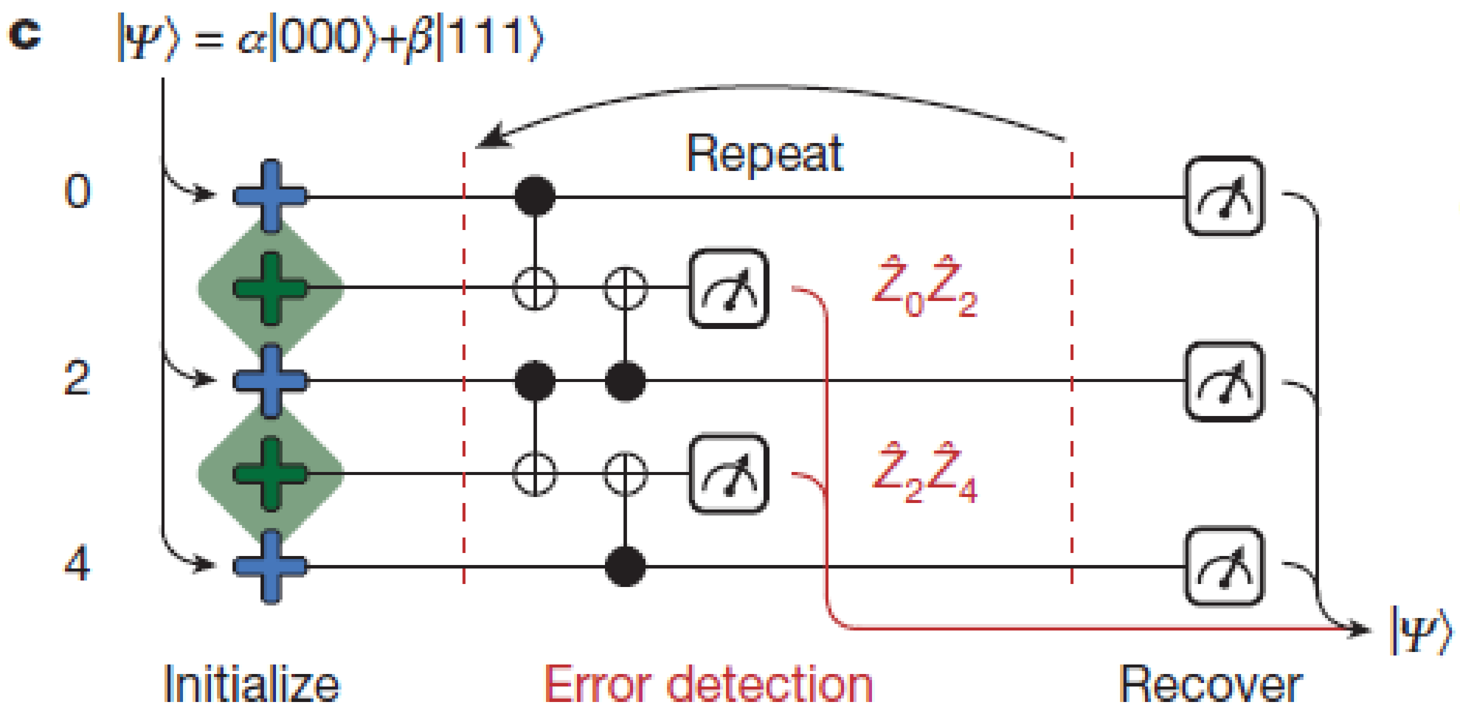




Measurement qubit (« Ancilla »)

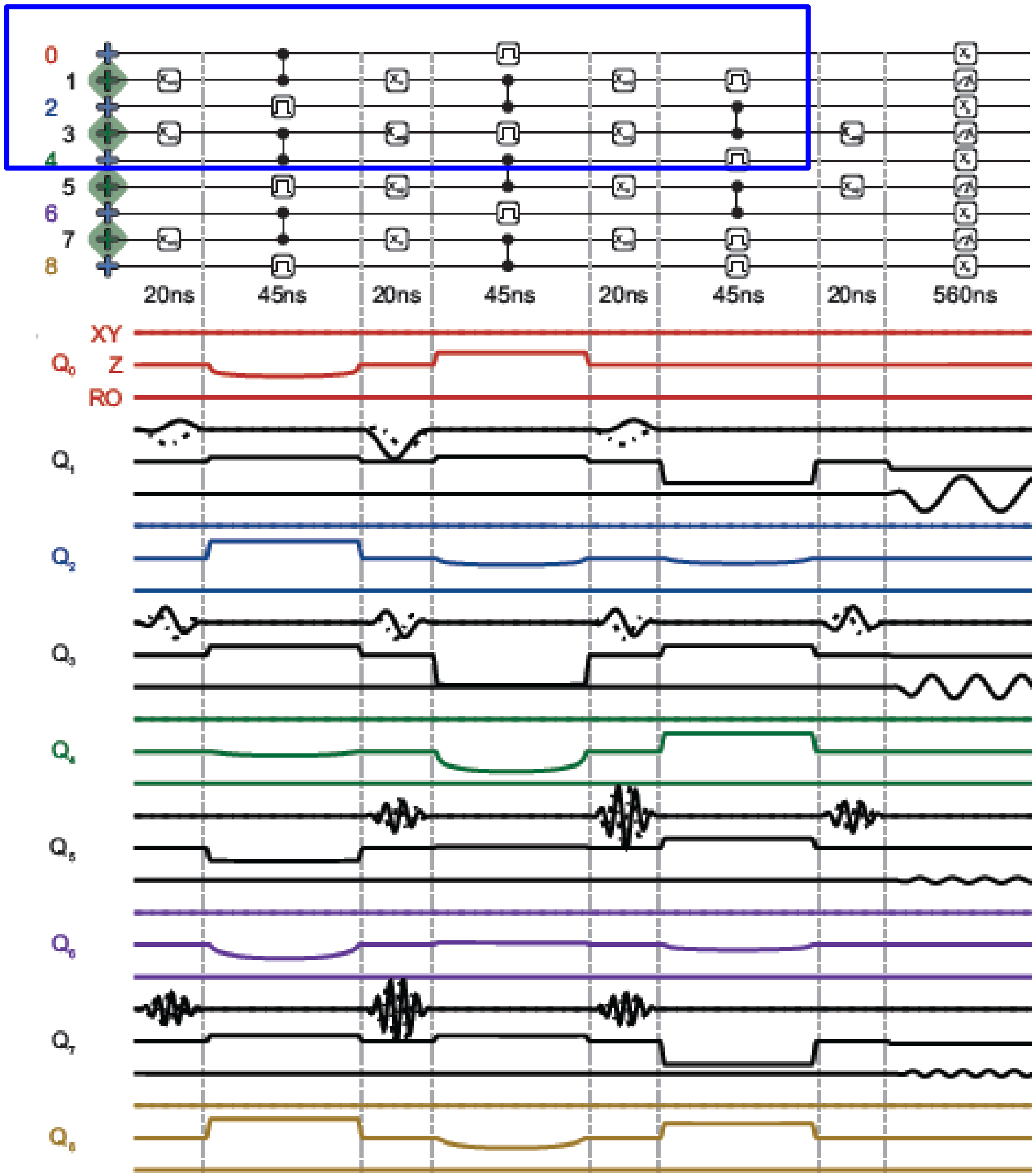
Data qubit

Correcting bit-flip errors (2) : detecting an error with a 9-qubit chip

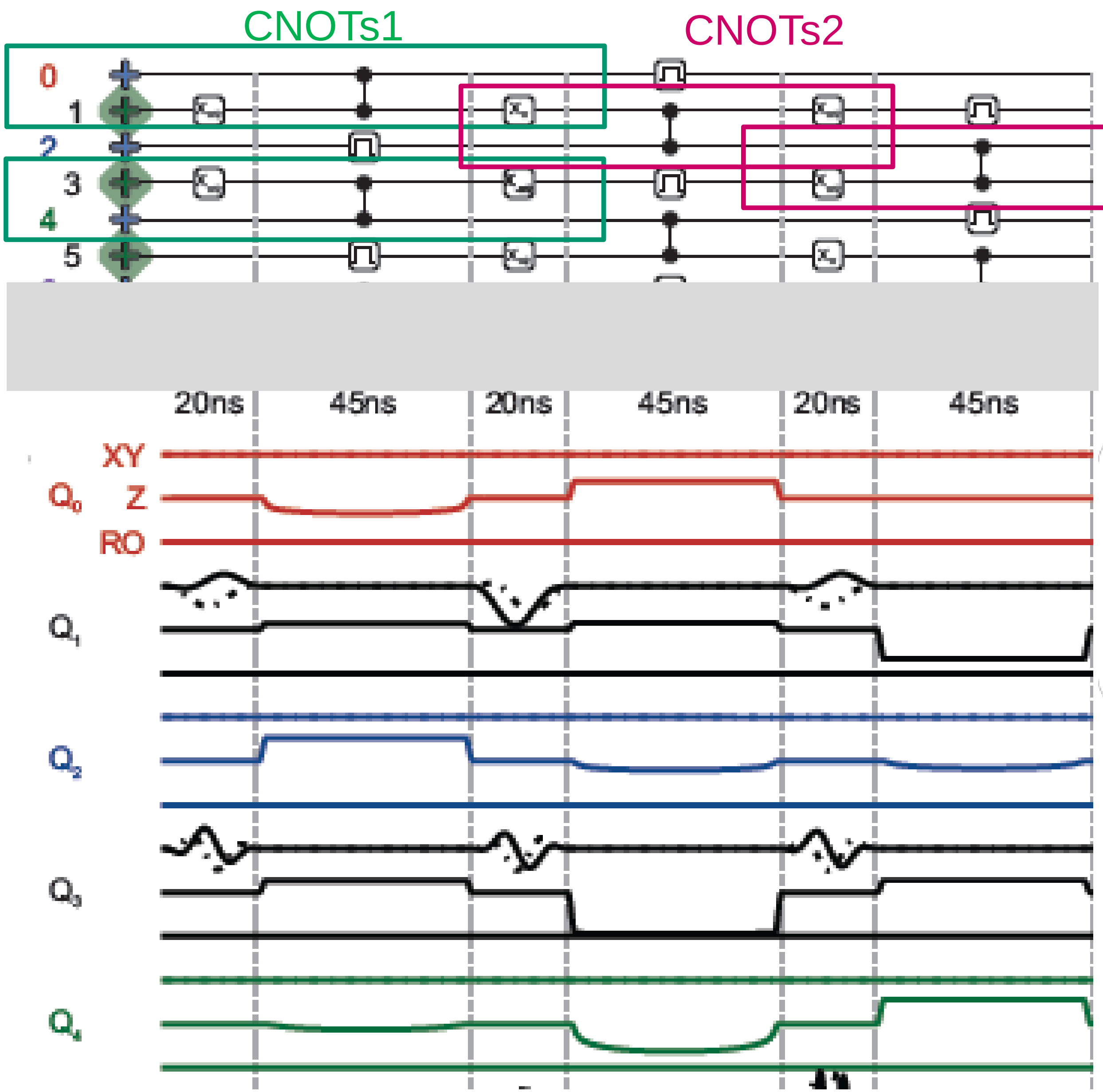


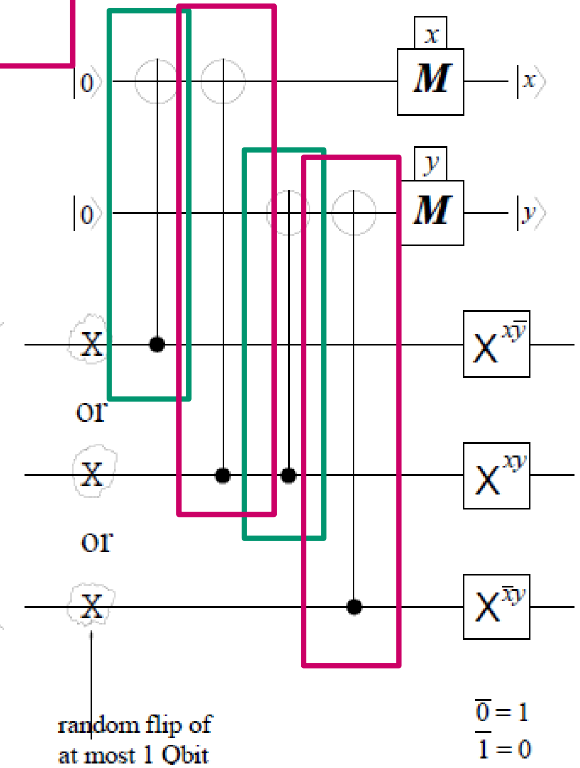
Q_A	$\hat{Z}_A\hat{Z}_B$	Q_B
0	0	0
0	1	1
1	1	0
1	0	1

Correcting bit-flip errors (2) : implementation

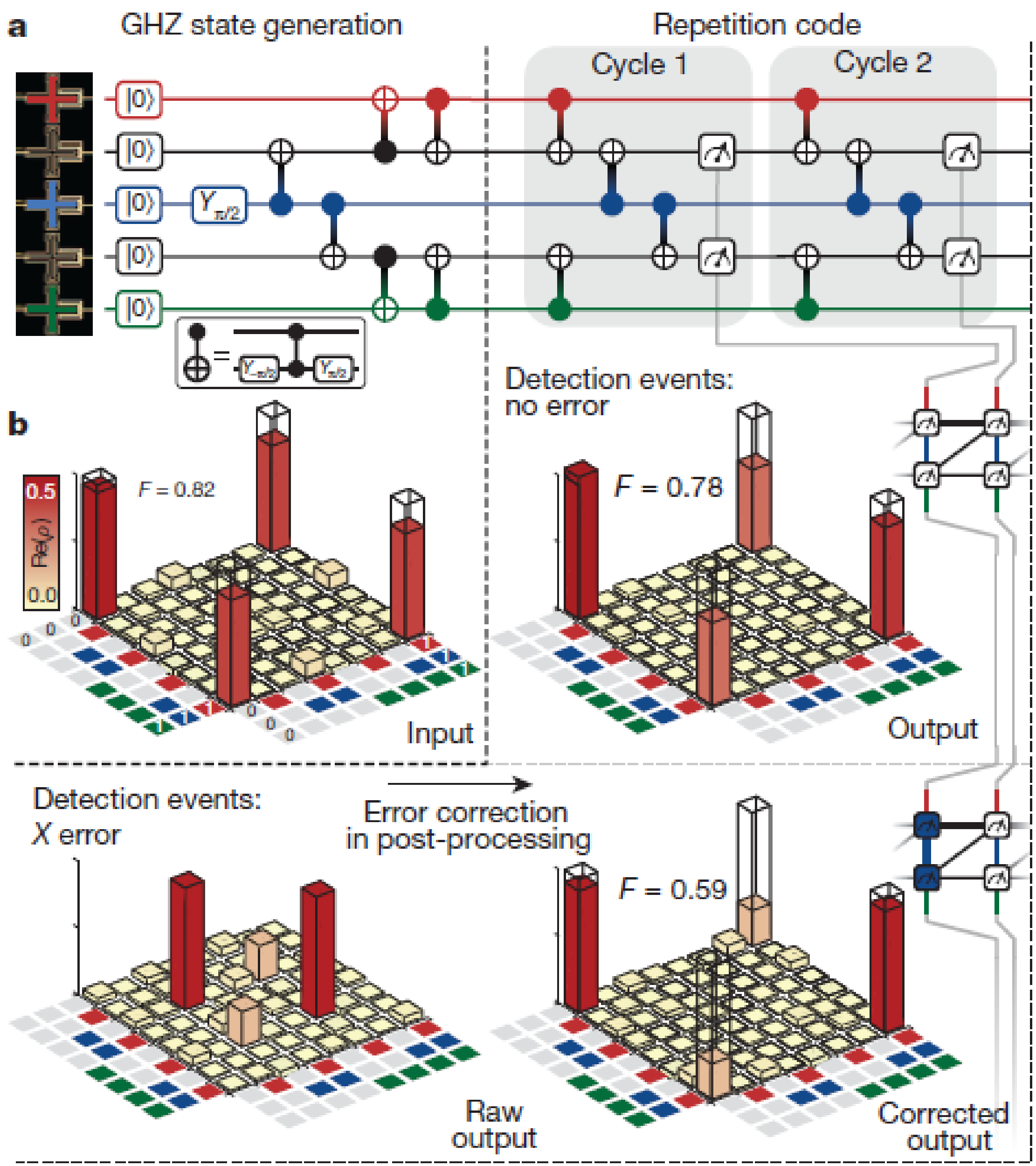


Correcting bit-flip errors (2) : implementation

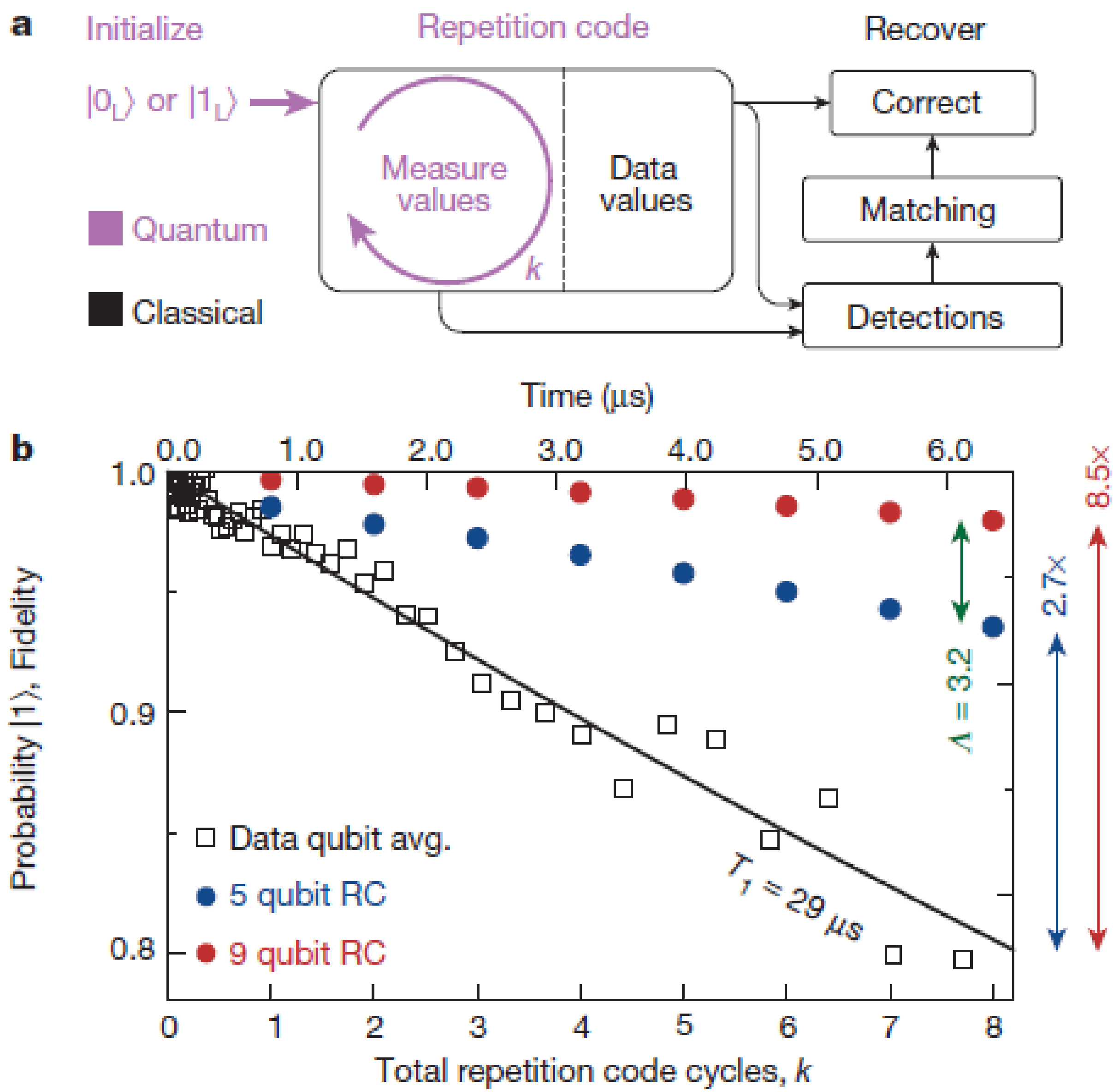




Correcting bit-flip errors (2) : detecting an error with a 9-qubit chip



Correcting bit-flip errors (2) : detecting errors with a 9-qubit chip



Conclusions

- Multi-qubit operations possible thanks to improvements in coherence times and high-fidelity single-qubit gates and readout

- First implementations of algorithms, and even elementary Quantum Error Correction schemes (only bit-flip errors)

- Real quantum error correction still requires major improvements in gate fidelity, coherence times, control electronics, fabrication, …

- But … Still very far from a working quantum processor