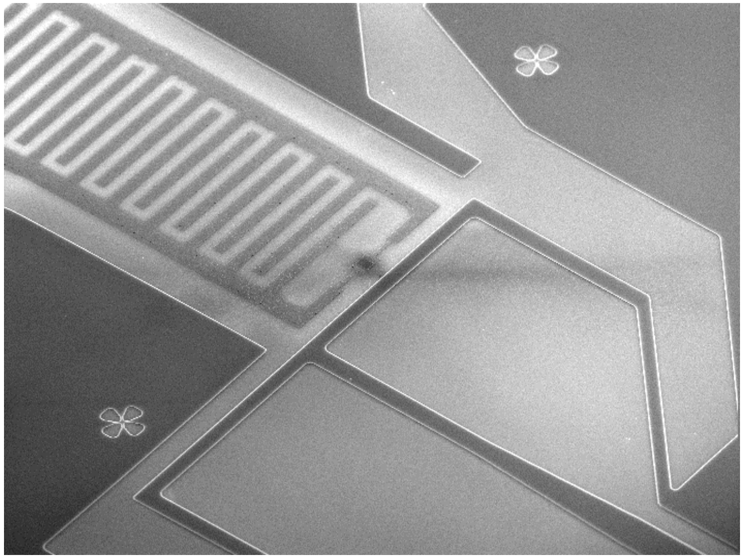
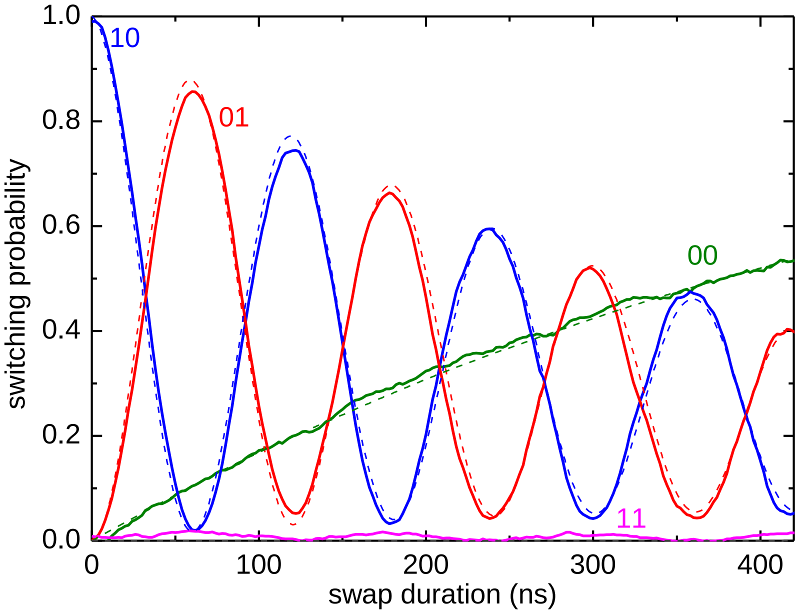





# Electrical Quantum Engineering with Superconducting Circuits

R. Heeres & P. Bertet

SPEC, CEA Saclay (France),
Quantronics group

# Outline

Lecture 1: Basics of superconducting qubits

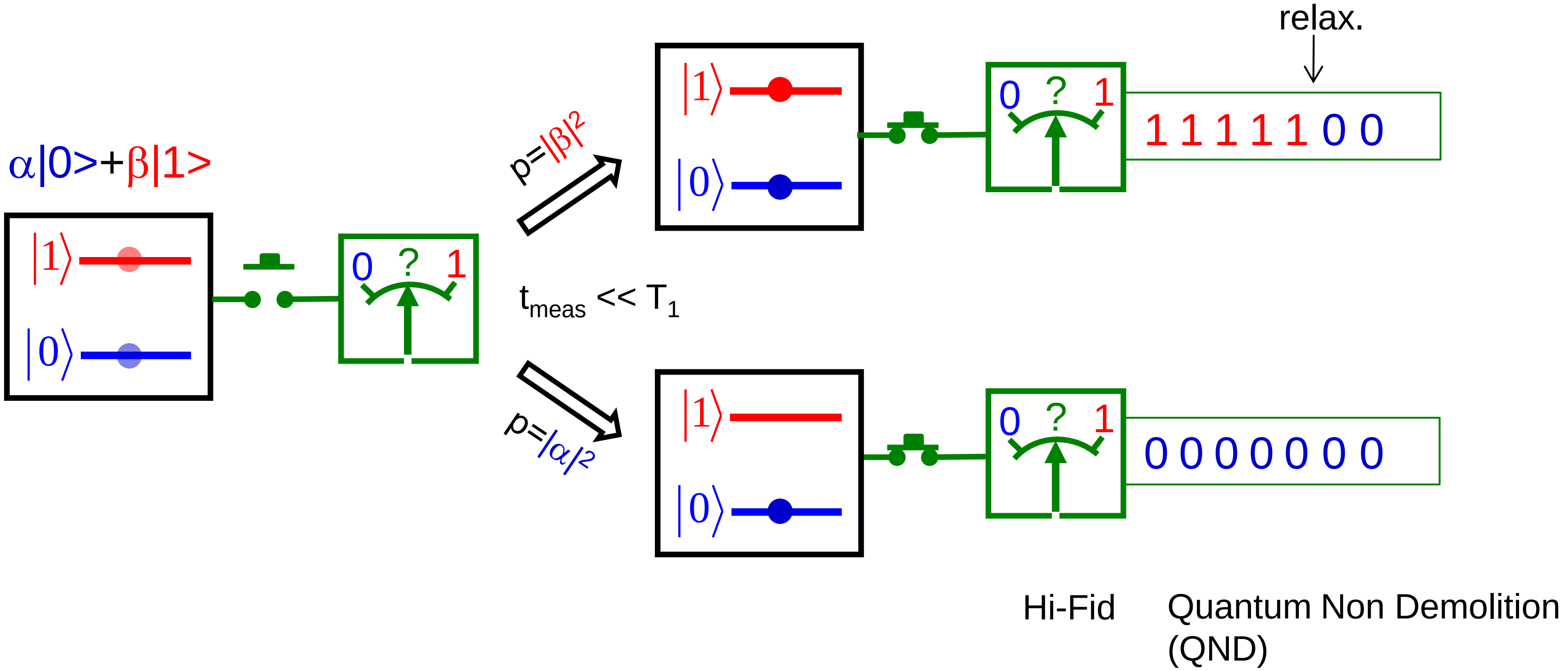
Lecture 2: Qubit readout and circuit quantum electrodynamics

1) Readout using a resonator
2) Amplification & Feedback
3) Quantum state engineering

Lecture 3: Multi-qubit gates and algorithms

Lecture 4: Introduction to Hybrid Quantum Devices

# Ideal Qubit Readout




**BUT… HOW ???**

**SURPRISINGLY DIFFICULT AND INTERESTING QUESTION FOR SUPERCONDUCTING QUBITS**

# The Readout Problem

1) Readout should be FAST:

$t_{meas} \ll T_1 \sim 40\mu s$ for high fidelity $(F \leq 1 - \frac{t_{meas}}{T_1})$

Ideally $t_{meas} <$ few hundred ns

2) Readout should be NON-INVASIVE (Quantum Non-Destructive, QND):

Unwanted transition caused by readout process → errors
(if we find $|0\rangle$ as the answer, the state should remain $|0\rangle$)

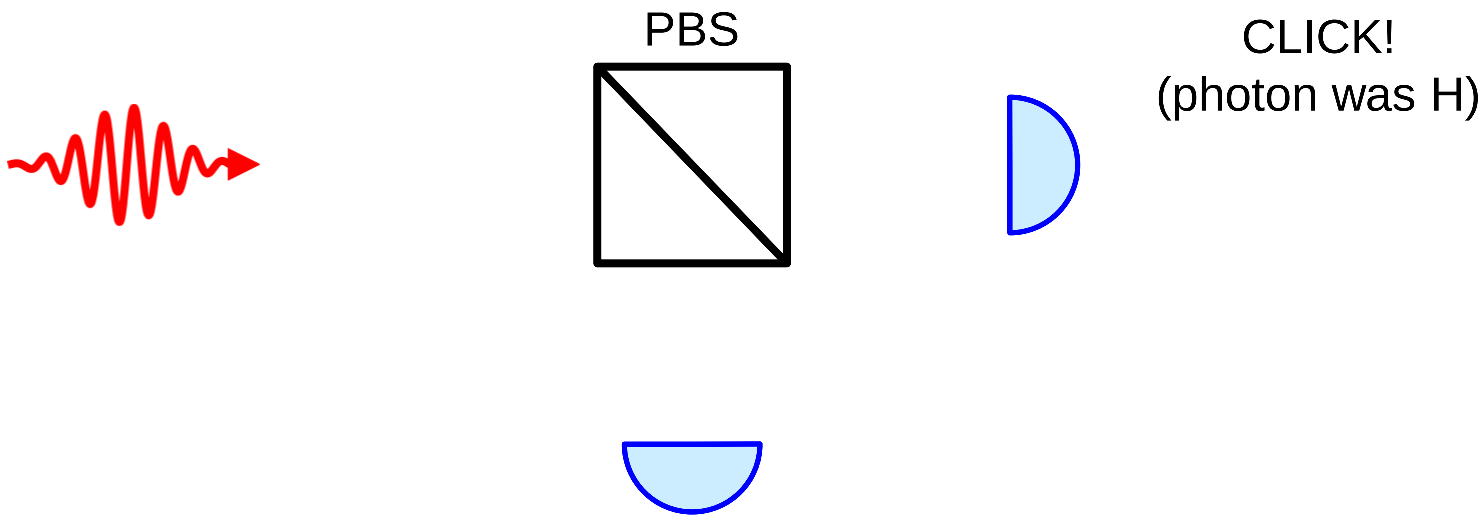
3) Readout should be COMPLETELY OFF during manipulations:

(i.e. have no interaction with environment → decoherence)

# Measurement of Photonic Qubits

- Measure a photon:


- Measure polarization:



# Readout in Different Systems

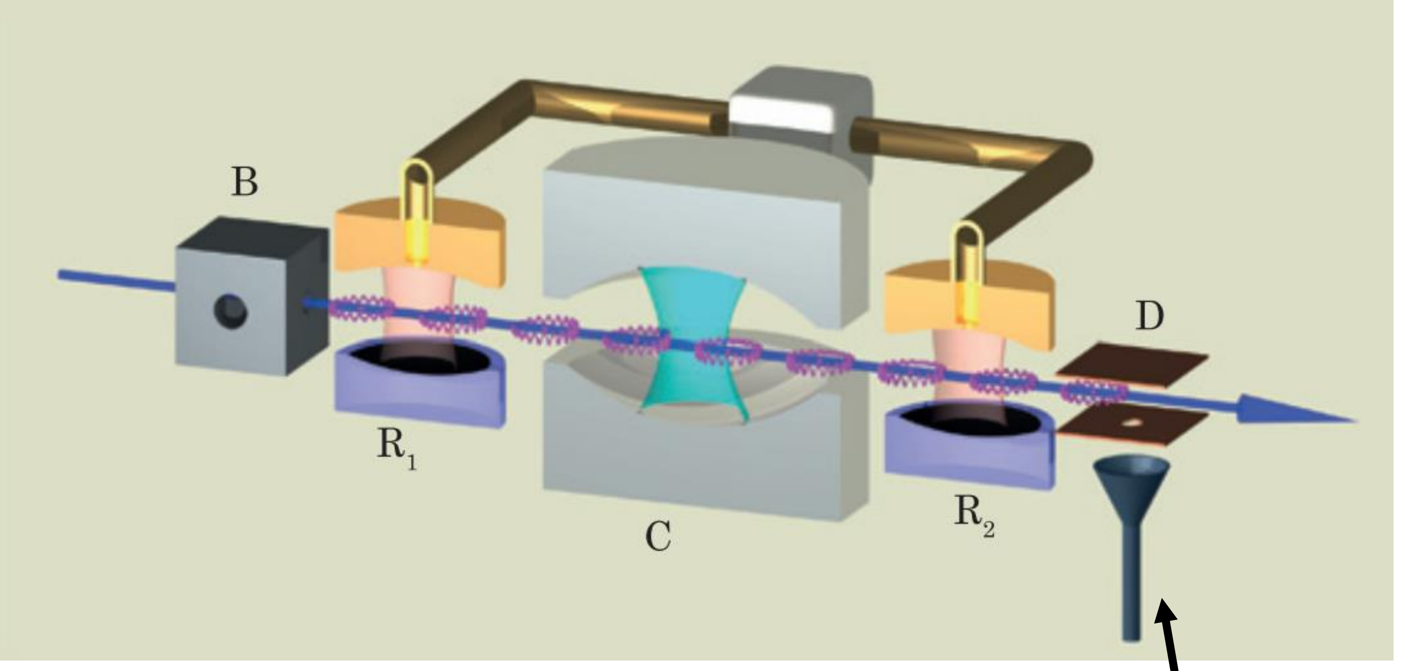
| Qubit | readout fidelity | readout time | QND | coherence time $T_1$ | $T_2$ | ref. | notes |
|---|---|---|---|---|---|---|---|
| **Trapped ions** | 99.990(1)% | 145 $\mu$s | ✓ | 1.168 s | $\sim$ 1 ms | [this work] | $^{40}Ca^+$ optical qubit. |
| | 99.965% | 11.9 ms | ✓ | 21 s | – | [42] | Adaptive $^{27}Al^+$ measurement via $^9Be^+$ ancilla. |
| | 99.925% | 3 ms | ✓ | 1.168 s | $\sim$ 1 ms | [29] | $^{40}Ca^+$ optical qubit. |
| | 99.87(4)% | 10 ms | ✓ | 35 s | 120 $\mu$s | [40] | $^{137}Ba^+$ optical qubit. |
| | 99.77(3)% | 2.4 ms | ✓ | $\sim 10^3$ s | 1.2(2) s | [41, 23] | $^{43}Ca^+$ ground-state qubit. |
| | $>$ 99.4% | $\sim$ 15 ms | ✓ | $\sim 10^3$ s | – | [39] | $^{111}Cd^+$ hyperfine ground-state qubit (single ion ICCD readout). |
| | 99% | 200 $\mu$s | ✓ | $\sim 10^3$ s | – | [38] | $^9Be^+$ hyperfine ground-state qubits |
| | 97.9(2)% | 1 ms | ✓ | $\sim 10^3$ s | 2.5(3) s | [125] | $^{171}Yb^+$ hyperfine ground-state qubit. |
| | 97% | 200 $\mu$s | ✓ | $\sim 10^3$ s | – | [35] | $^9Be^+$ hyperfine ground-state qubits |
| **Neutral atoms** | 99.921(3)% | 100 $\mu$s | ✓ | – | – | [73] | $^{87}$Rb cavity transmission/reflection. |
| | 99.4(1)% | 85 $\mu$s | ✓ | – | – | [72] | $^{87}$Rb cavity-enhanced fluorescence. |
| | 99.19(5)% | 802(17) ns | × | – | – | [70] | $^{87}$Rb photoionisation measurement. |
| | $>$ 99% | $\sim$ 500 ms | × | – | – | [66] | Cs "push-out" measurement. |
| | $>$ 98% | 10 ms | × | – | – | [65] | $^{87}$Rb "push-out" measurement. |
| | 98(1)% | $\sim$ 130 ms | × | $\sim$ 220 s | 16.7 ms | [64] | Cs "push-out" measurement. |
| **Cavity photon** | $\sim$ 99.92% | 7.8 ms | ✓ | – | – | [74] | Microwave cavity; Rydberg atom interferometer. |

| Qubit | readout fidelity | readout time | QND | decoherence time $T_1$ | $T_2$ | ref. | notes |
|---|---|---|---|---|---|---|---|
| **Superconducting** | $\sim$ 98% | 5 ns* | ? | $\sim$ 20 ns | – | [81] | Flux qubit. |
| | $\sim$ 97% | $<$ 30 ns* | ? | $\sim$ 400 ns | $\sim$ 140 ns | [78] | Two coupled phase qubits. |
| | $\sim$ 96% | 250 ns*, 700 ns | ✓ | 0.5 $\mu$s | 0.4–0.6 $\mu$s | [94] | Transmon (charge); latched JBA measurement. |
| | $\sim$ 95.1(4)% | 150 ns | 88% | $\sim$ 470 ns | – | [80] | Flux qubit. |
| | $\sim$ 92% | 25 ns | ? | $\sim$ 10 ns | – | [77] | Phase qubit. |
| | $\sim$ 90% | 300 ns* | ?✓ | – | – | [89] | Cooper-pair box (charge); latched rf-SET measurement. |
| | $\sim$ 69% | 250 ns | $\sim$ 67% | $\sim$ 1.3 $\mu$s | – | [93] | Quantronium (charge); latched JBA measurement. |
| | $\sim$ 65% | 7.3 $\mu$s | ✓ | $\sim$ 7.3 $\mu$s | $\sim$ 500 ns | [86] | Cooper-pair box (charge); continuous measurement. |
| **Quantum dots** | $\gtrsim$ 95% | $\sim$ 6 $\mu$s | †? | 34 $\mu$s | 27 ns | [105] | Two-electron double quantum dot qubit; charge measurement. |
| | $\sim$ 91% | $\sim$ 70 $\mu$s | × | 2.58(9) ms | – | [103] | Two-electron quantum dot qubit at 0.02 T; charge measurement. |
| | $\sim$ 87% | $\sim$ 20 $\mu$s | 91%† | 1.8(1) ms | – | [104] | Two-electron quantum dot qubit; charge measurement. |
| | $\sim$ 83% | $\sim$ 0.11 ms | × | $<$ 0.55(7) ms | – | [102] | Single-electron quantum dot at 10 T; charge measurement. |
| **Semicond. spins** | $\sim$ 96% | $\sim$ 100 $\mu$s | × | $\sim$ 39 ms | – | [106] | Electron in P-doped Si; charge measurement at $B = 5$ T. |
| | $\sim$ 95% | 5 ms | ? | $\sim$ 1 s | $<$ 350 $\mu$s | [107] | Electron in N-V centre at 2 K; optical measurement. |

PhD Thesis, A. Burrell (2010)

# Cavity Quantum Electrodynamics

- Interaction between a cavity and an atom

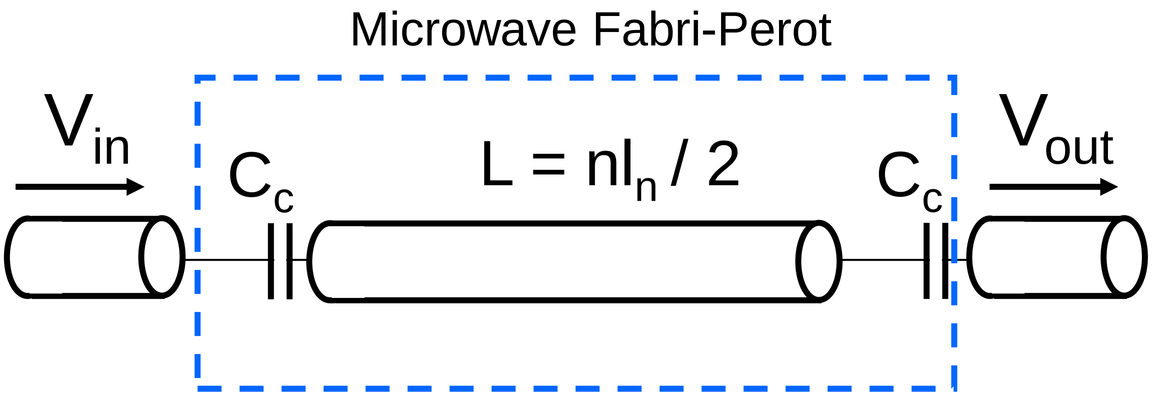


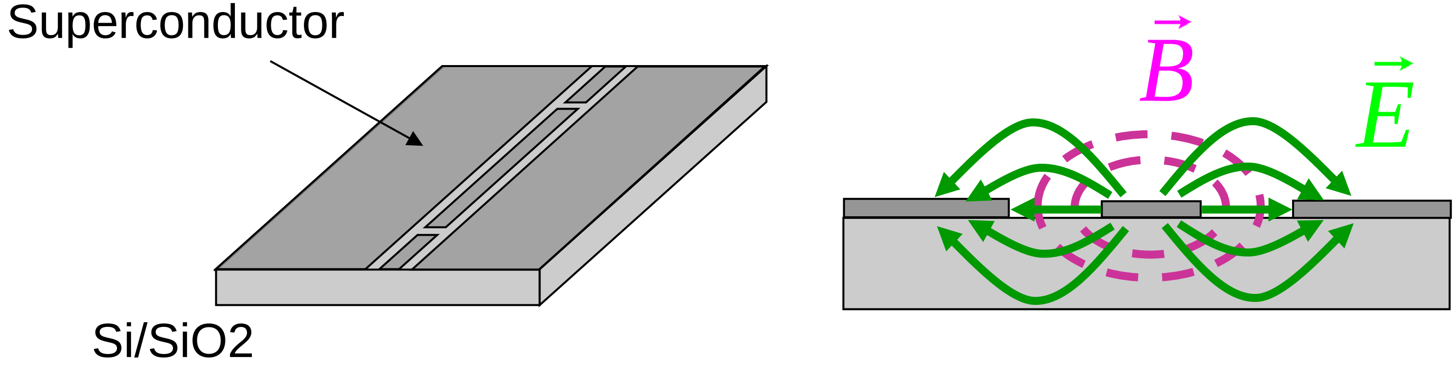

State-selective detector
(Measure P(e) by selective ionization)

Key work by Haroche (Nobel 2012)

# Qubit – Resonator Interaction

- "2D" geometry

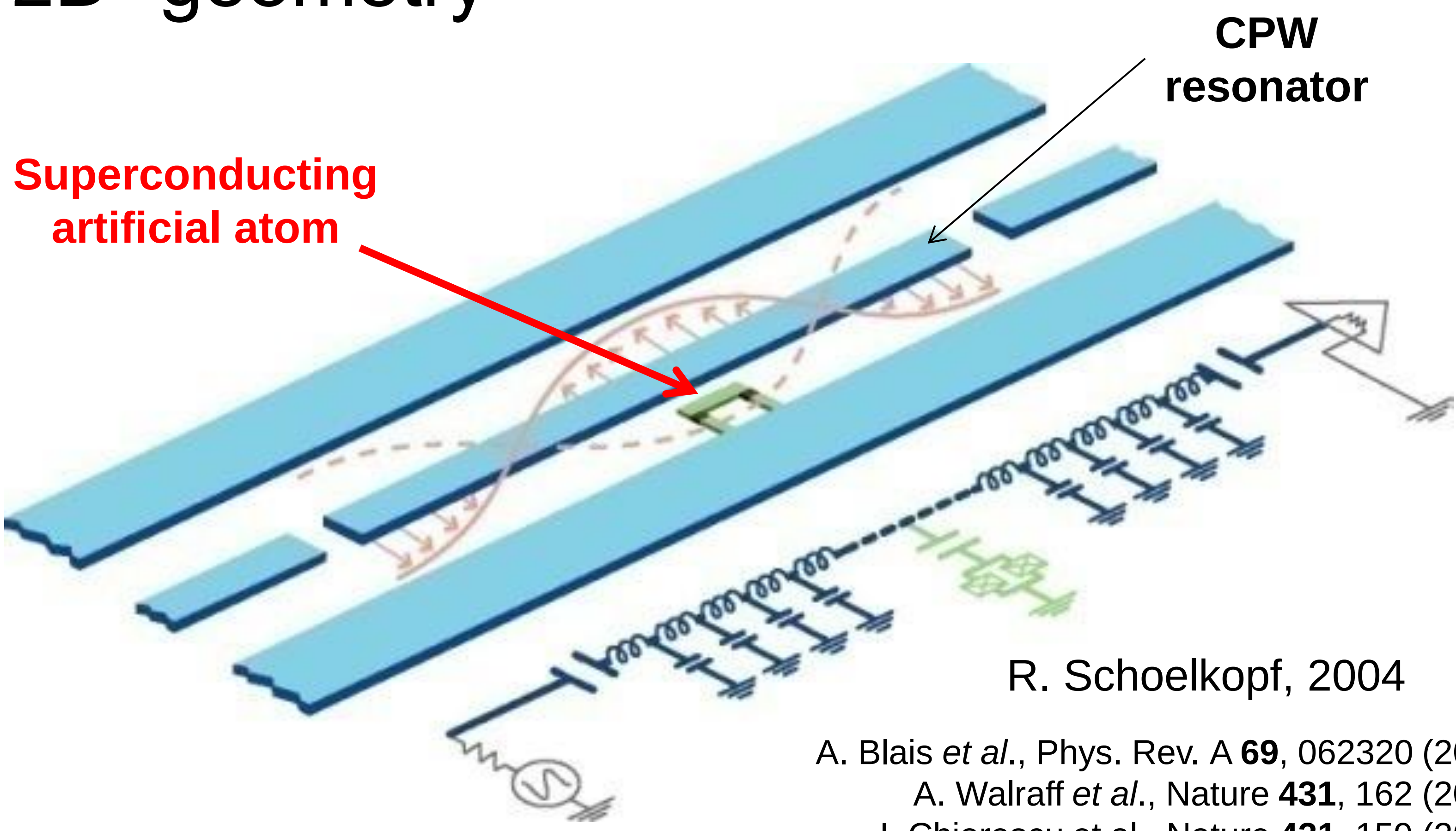



**Coplanar Waveguide (CPW) : TEM mode**

# Qubit – Resonator Interaction

- “2D” geometry



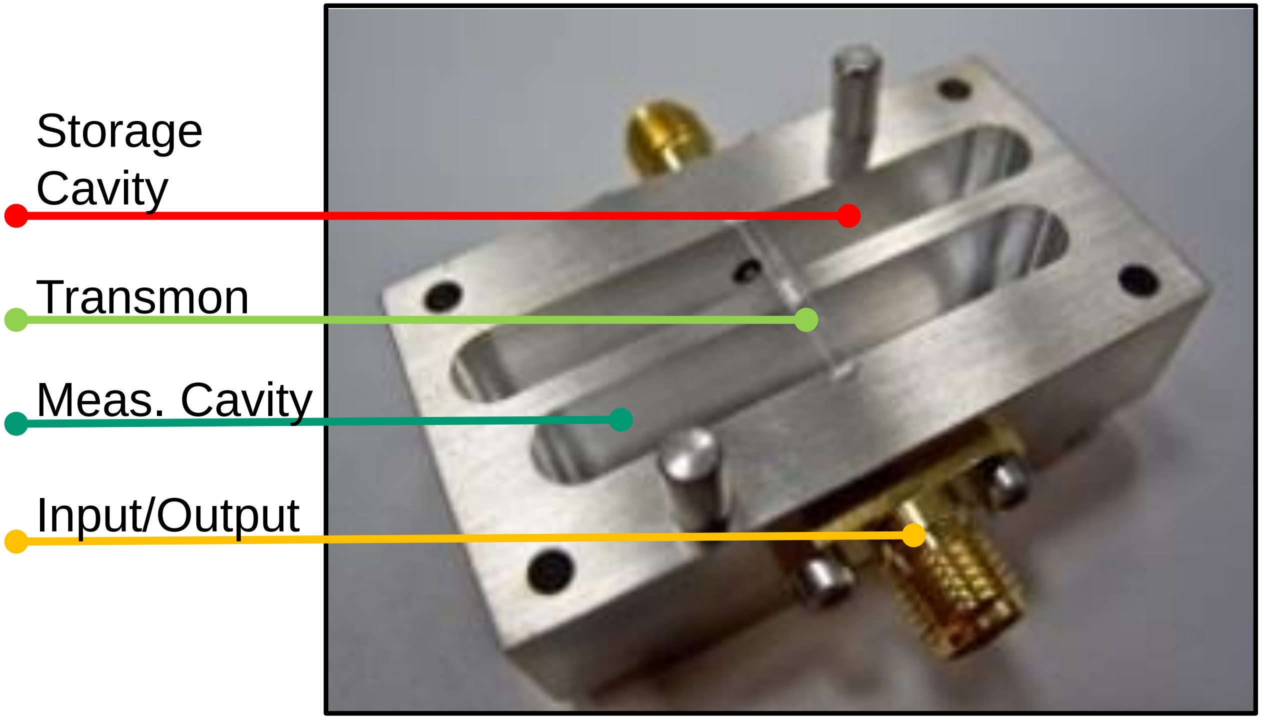

R. Schoelkopf, 2004

A. Blais *et al*., Phys. Rev. A **69**, 062320 (2004)
A. Walraff *et al*., Nature **431**, 162 (2004)
I. Chiorescu et al., Nature **431**, 159 (2004)

or: CIRCUIT QUANTUM ELECTRODYNAMICS

# Qubit – Resonator Interaction

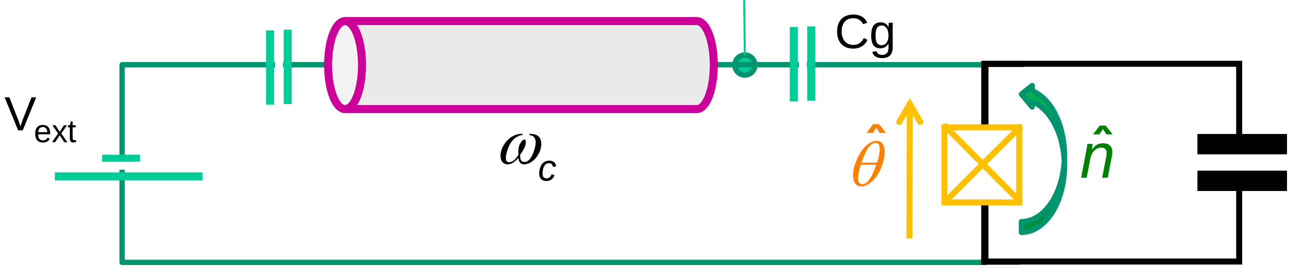
- “3D” geometry




still: CIRCUIT QUANTUM ELECTRODYNAMICS

# CPB Coupled to CPW Resonator

$$\hat{V}_g = \delta V_0\left(\hat{a}+\hat{a}^+\right)+V_{gext}$$




$$\hat{H}_{tot} = -E_J\cos\hat{\theta}+4E_C\left(\hat{n}-\hat{n}_g\right)^2+\hbar\omega_c\hat{a}^+\hat{a}$$

$$\underbrace{\hat{H}_{tot} = -E_J\cos\hat{\theta}+4E_C\left(\hat{n}-n_{gext}\right)^2}_{\hat{H}_q}+\underbrace{\hbar\omega_c\hat{a}^+\hat{a}}_{\hat{H}_{cav}}+\underbrace{8(C_g\delta V_0E_C/2e)\hat{n}(a+a^+)}_{\hat{H}_{\text{int}}}$$

2-level approximation + Rotating Wave Approximation

$$H_{tot} = -\frac{\omega_{ge}}{2}\sigma_z+\omega_c a^\dagger a+g(\sigma^+a+\sigma^-a^\dagger)$$

**Jaynes-Cummings Hamiltonian**

A. Blais *et al*., PRA **69**, 062320 (2004)

# Strong Coupling Regime

$$g = 2e\delta V_0 (C_g / C)\langle 0|\hat{n}|1\rangle$$

GEOMETRICAL dependence of g

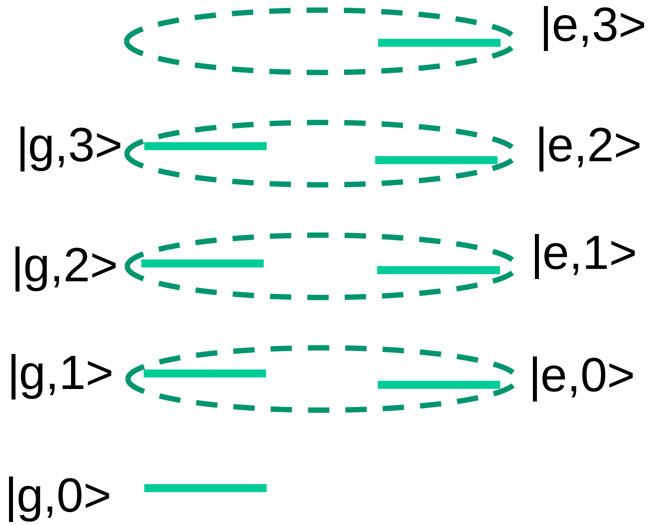
→ Easily tuned by circuit design
Can be made very large !
(Typically : 0 – 200MHz)

$$g \approx 200MHz >> \gamma, \kappa \approx 100 - 500kHz$$

**Strong coupling condition naturally fulfilled with superconducting circuits**

(Q=100 enough for strong coupling !!)

# The Jaynes-Cummings Model




$$H_{J-C} = -\frac{\omega_{ge}}{2}\sigma_z + \omega_c(a^+a + 1/2) + g(\sigma^+a + \sigma^-a^+)$$

$H_{J-C}$ couples only level doublets

$\{|g,n+1\rangle , |e,n\rangle\}$

⟹ Exact diagonalization possible

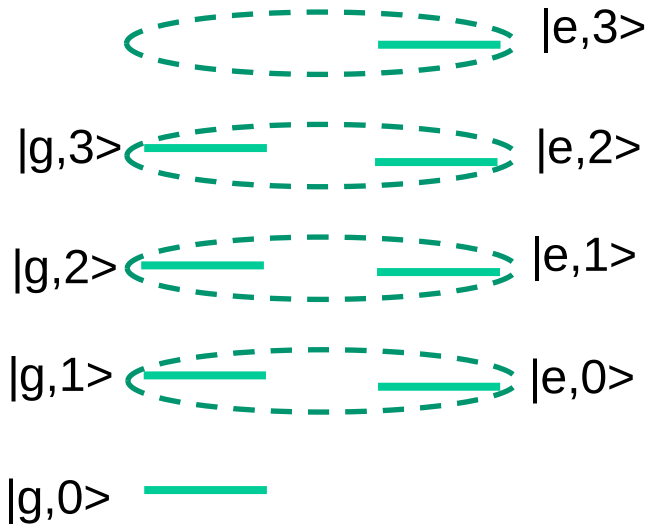
Restriction of $H_{J\text{-}C}$ to $\{|g,n+1\rangle , |e,n\rangle\}$

$$(\delta = \omega_{ge} - \omega_c)$$

$$\begin{array}{c|cc} & |g,n+1\rangle & |e,n\rangle \\ \hline |g,n+1\rangle & (n+1)\omega_c - \delta/2 & g\sqrt{n+1} \\ |e,n\rangle & g\sqrt{n+1} & (n+1)\omega_c + \delta/2 \end{array}$$

Note : |g,0> state is left unchanged by $H_{J\text{-}C}$ with $E_{g,0} = -\delta/2$

# The Jaynes-Cummings Model




$$H_{J-C} = -\frac{\omega_{ge}}{2}\sigma_z + \omega_c(a^+a + 1/2) + g(\sigma^+a + \sigma^-a^+)$$

$H_{J-C}$ couples only level doublets

$\{$ |g,n+1> , |e,n> $\}$

Exact diagonalization possible

Coupled states

$$|+,n\rangle = \cos\theta_n |e,n\rangle + \sin\theta_n |g,n+1\rangle$$

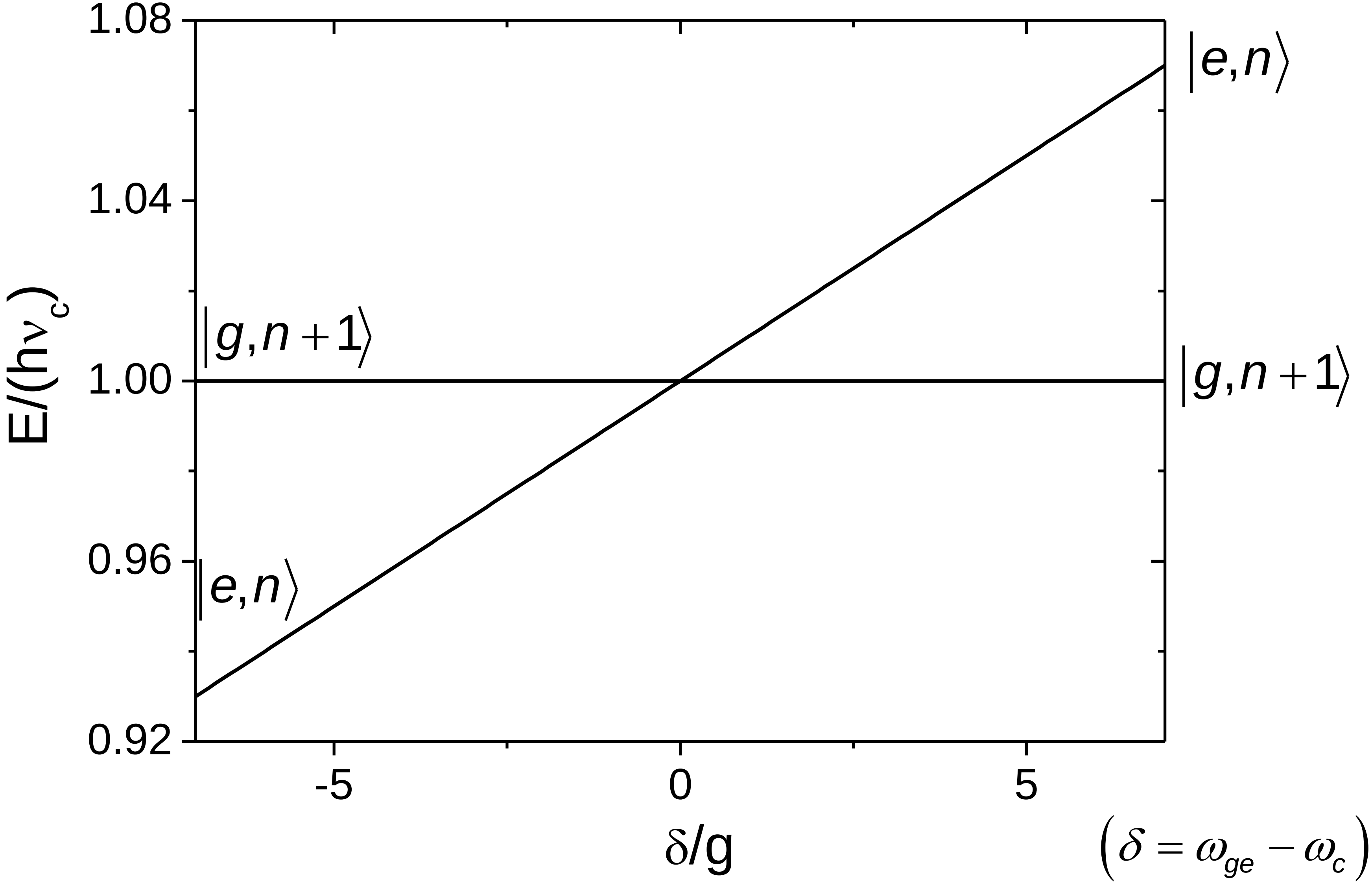
$$|-,n\rangle = -\sin\theta_n |e,n\rangle + \cos\theta_n |g,n+1\rangle$$

$$E_{+,n} = (n+1)\hbar\omega_c + \frac{\hbar}{2}\sqrt{4g^2(n+1) + \delta^2}$$

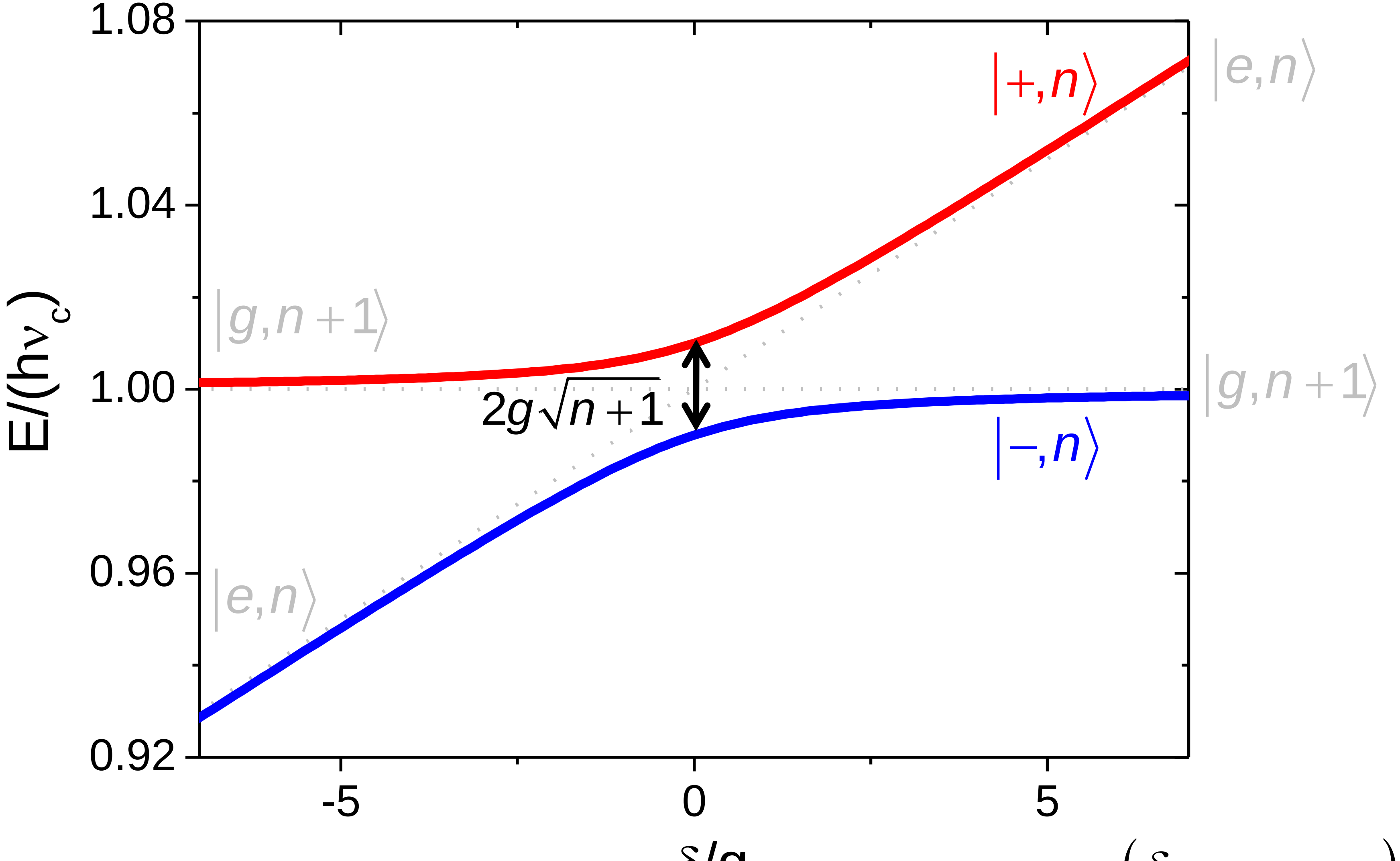
$$E_{-,n} = (n+1)\hbar\omega_c - \frac{\hbar}{2}\sqrt{4g^2(n+1) + \delta^2}$$

$$\theta_n = \frac{1}{2}\tan^{-1}\left(\frac{2g\sqrt{n+1}}{\delta}\right)$$

# The Jaynes-Cummings model

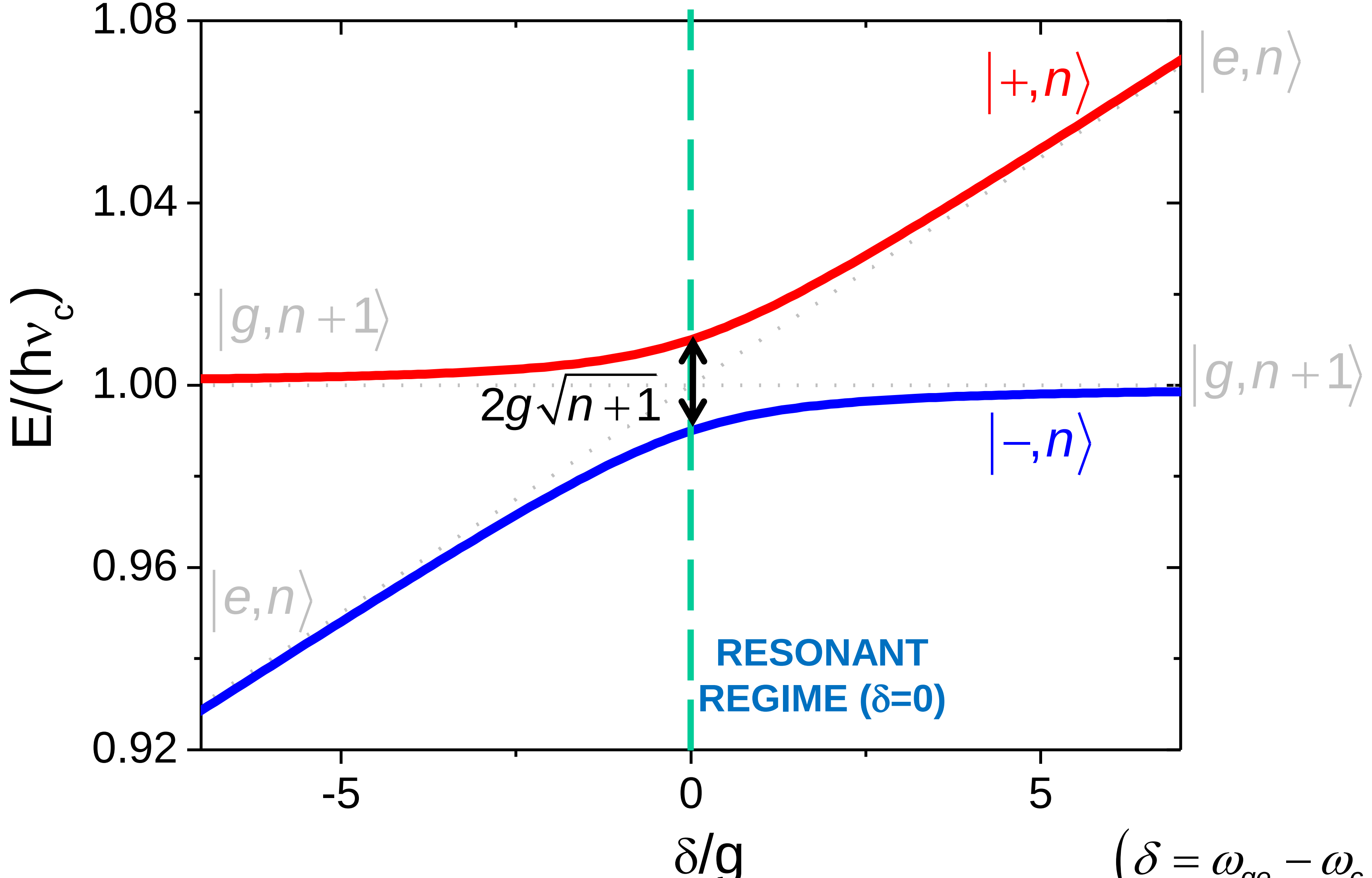


# The Jaynes-Cummings model



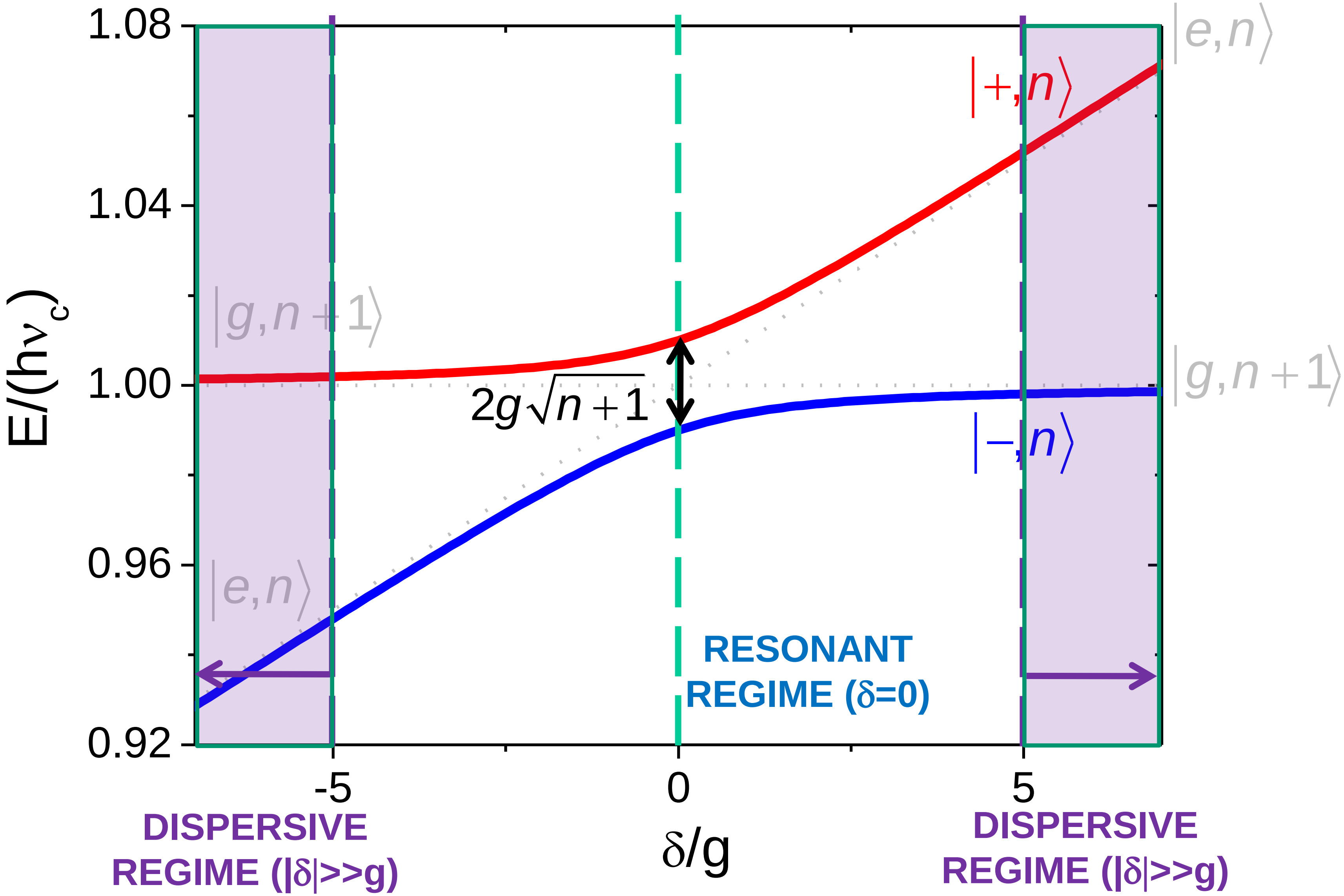

$\left(\delta = \omega_{ge} - \omega_c\right)$

# Two Interesting Limits

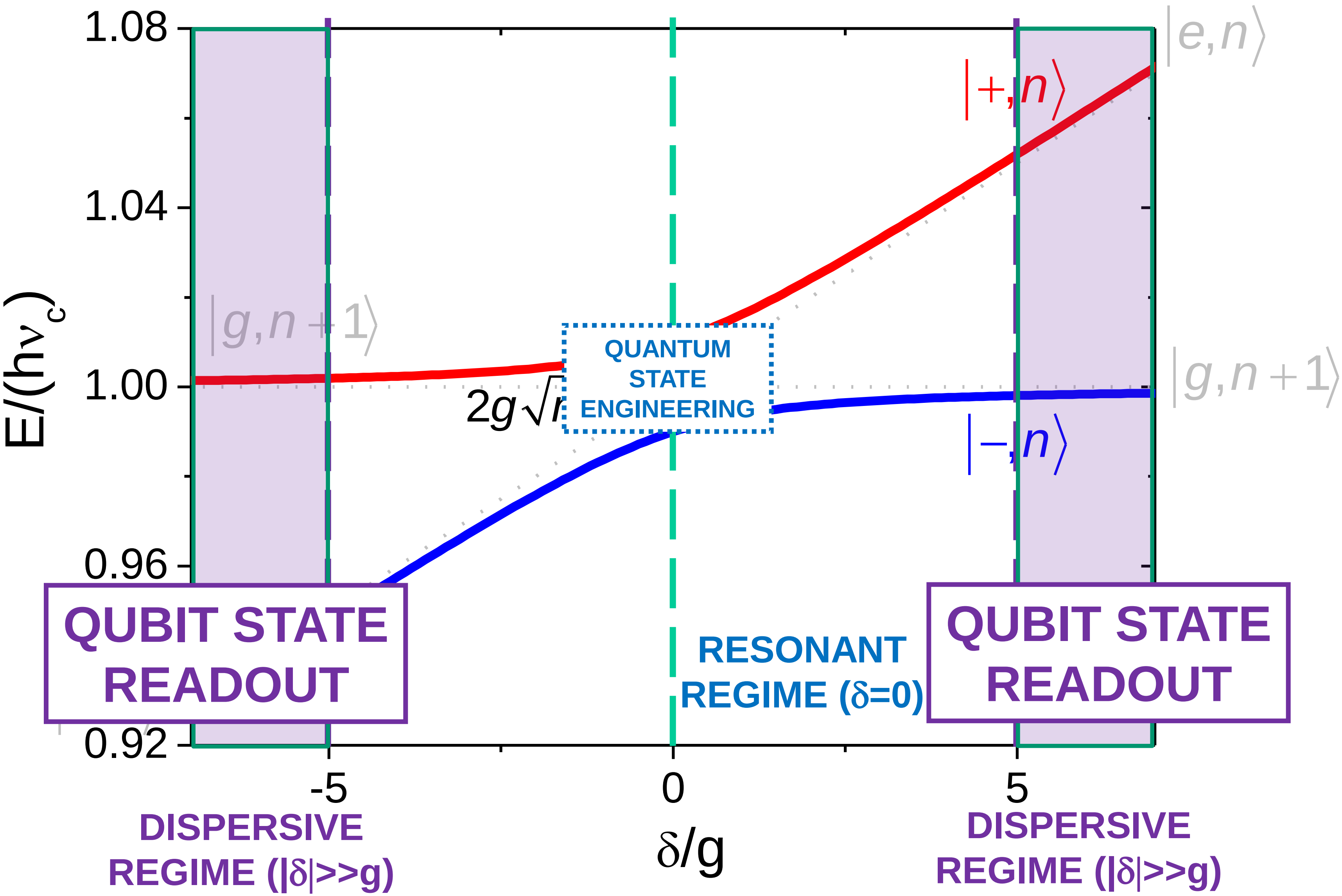



$\left(\delta = \omega_{ge} - \omega_c\right)$

# Two Interesting Limits



# Two Interesting Limits



# JC: Dispersive Regime ($\delta \gg g$)

$$H_{J-C}/\hbar \approx -\frac{\omega_{ge}+\chi}{2}\sigma_z + (\omega_c + \chi\sigma_z)a^+a = -\frac{\omega_{ge}+2\chi(a^+a+1/2)}{2}\sigma_z + \omega_c a^+a$$

with $\chi = \frac{g^2}{\delta}$ the dispersive coupling constant

1) Qubit state-dependent **shift of the cavity frequency**

$$\tilde{\omega}_c = \omega_c + \chi\sigma_z$$

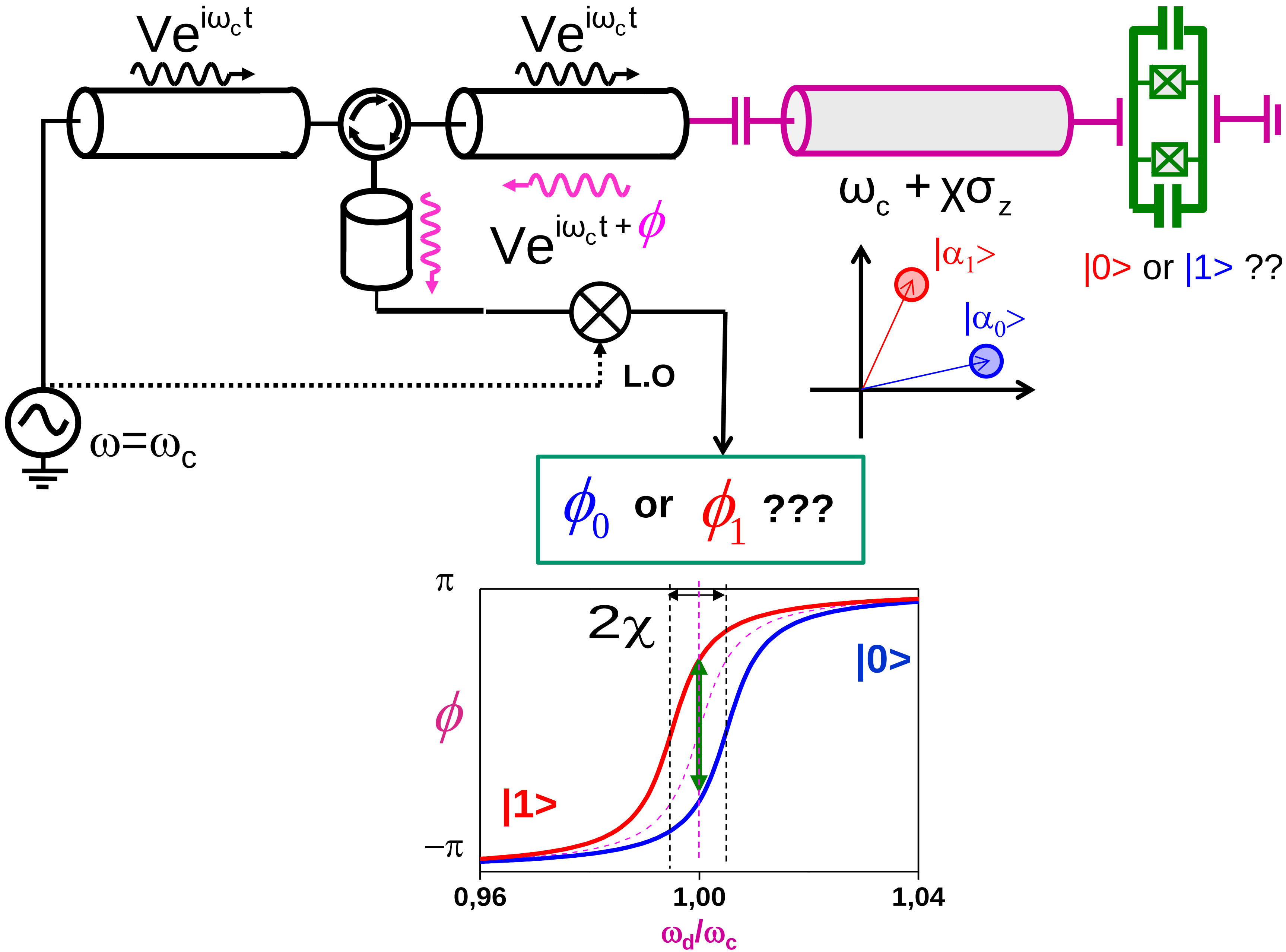
⟹ **Cavity can probe the qubit state non-destructively**

2) **Light shift** of the qubit transition in the presence of n photons

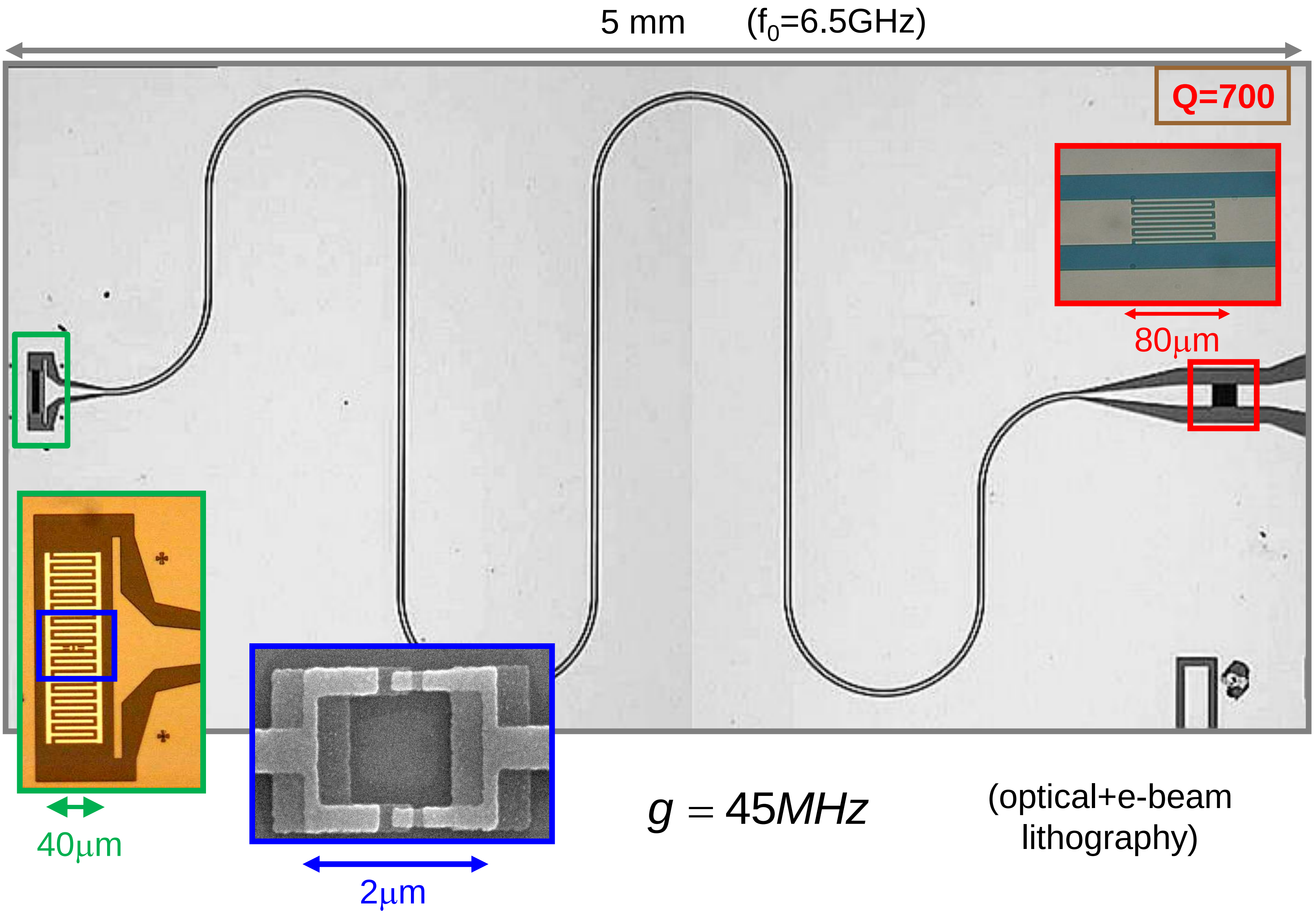
$$\delta\omega_{ge} = -2\chi n$$

⟹ **Field in the resonator causes qubit frequency shift and decoherence (if fluctuating)**

# Dispersive Readout Principle



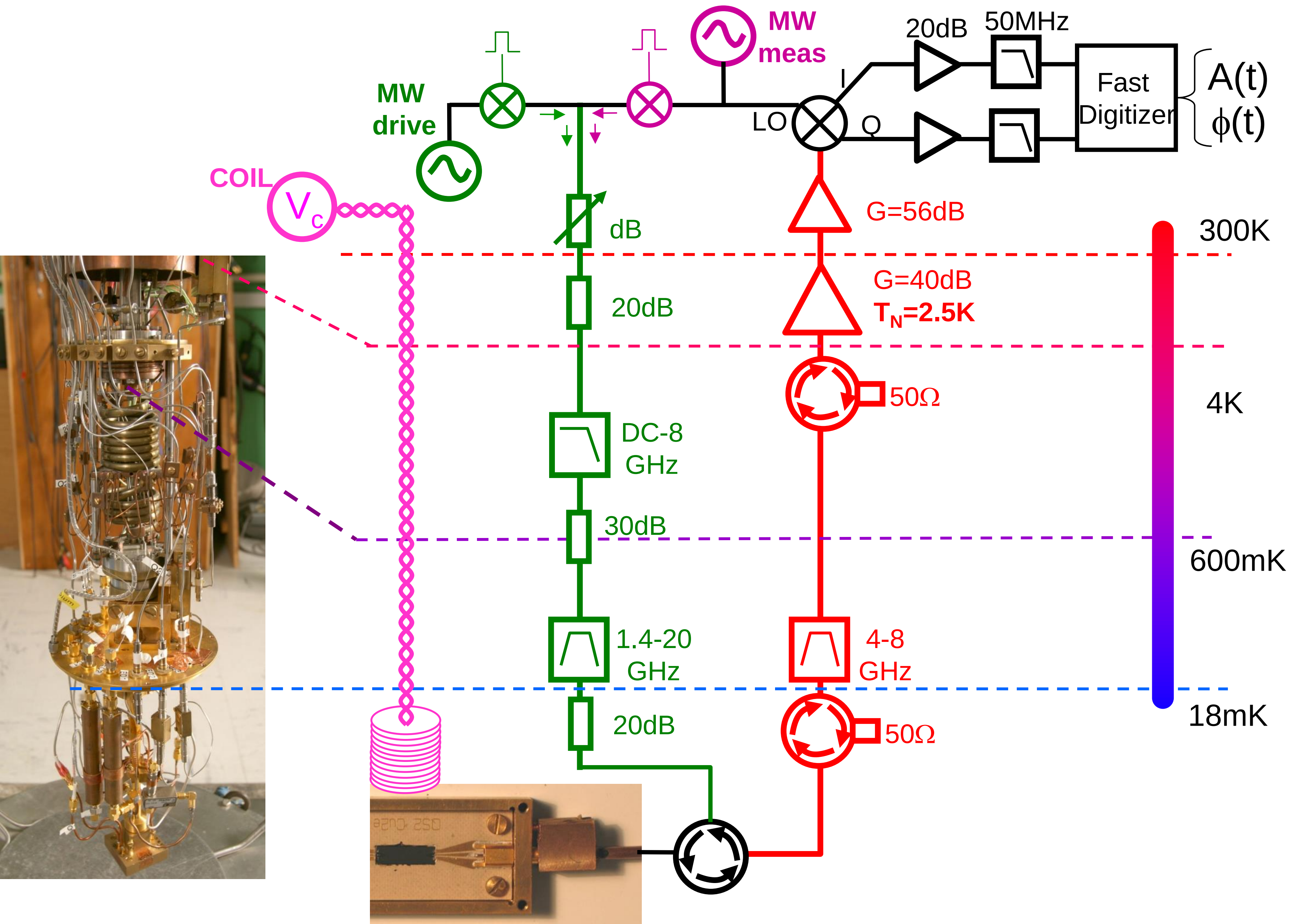
# Typical 2D Implementation




$g = 45MHz$

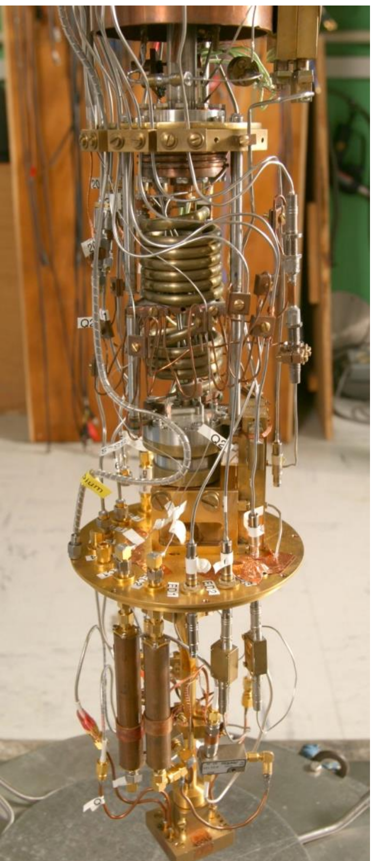
(optical+e-beam lithography)

# Typical Setup



# Typical Setup

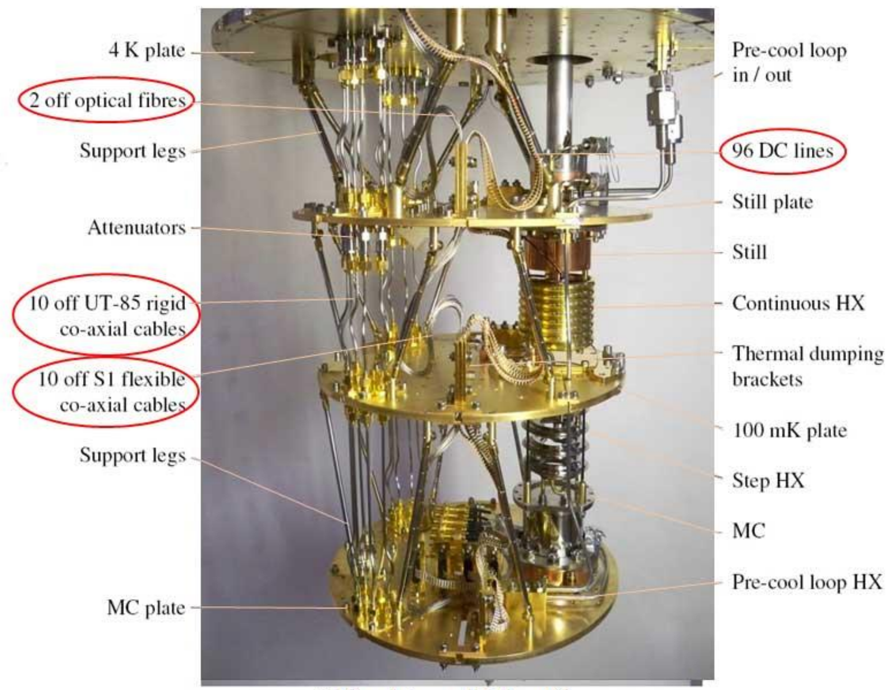
"Wet fridge"



Last 5-10 years

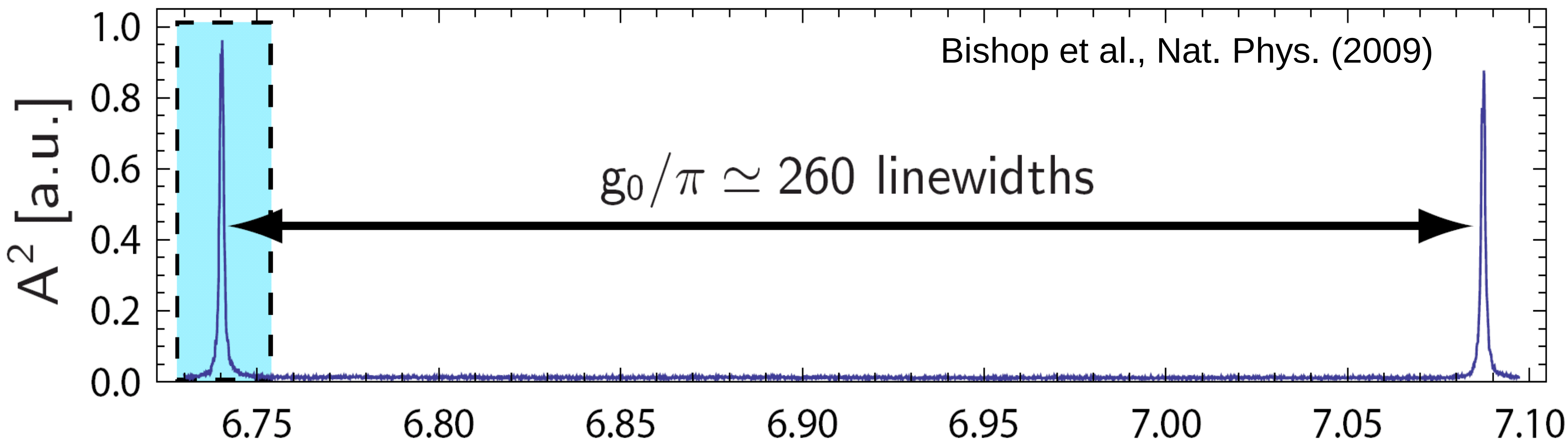
- Much easier to operate (no liquid He)
- Much more space

"Dry fridge"




Still achieved 10 mK!

# JC Hamiltonian in Action




**cavity QED**

R.J. Thompson et al., PRL 68, 1132 (1992)

I. Schuster et al. Nature Physics 4, 382-385 (2008)

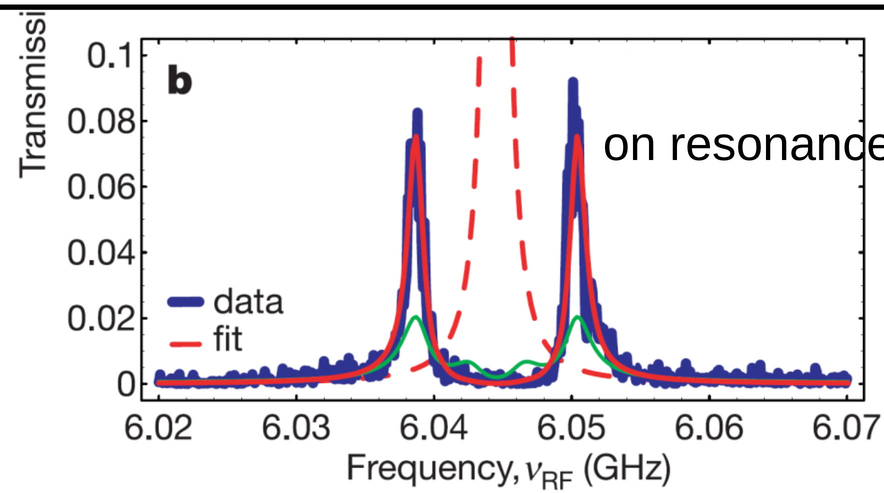
**circuit QED**

A. Wallraff et al., Nature 431, 162 (2004)

**quantum dot systems**

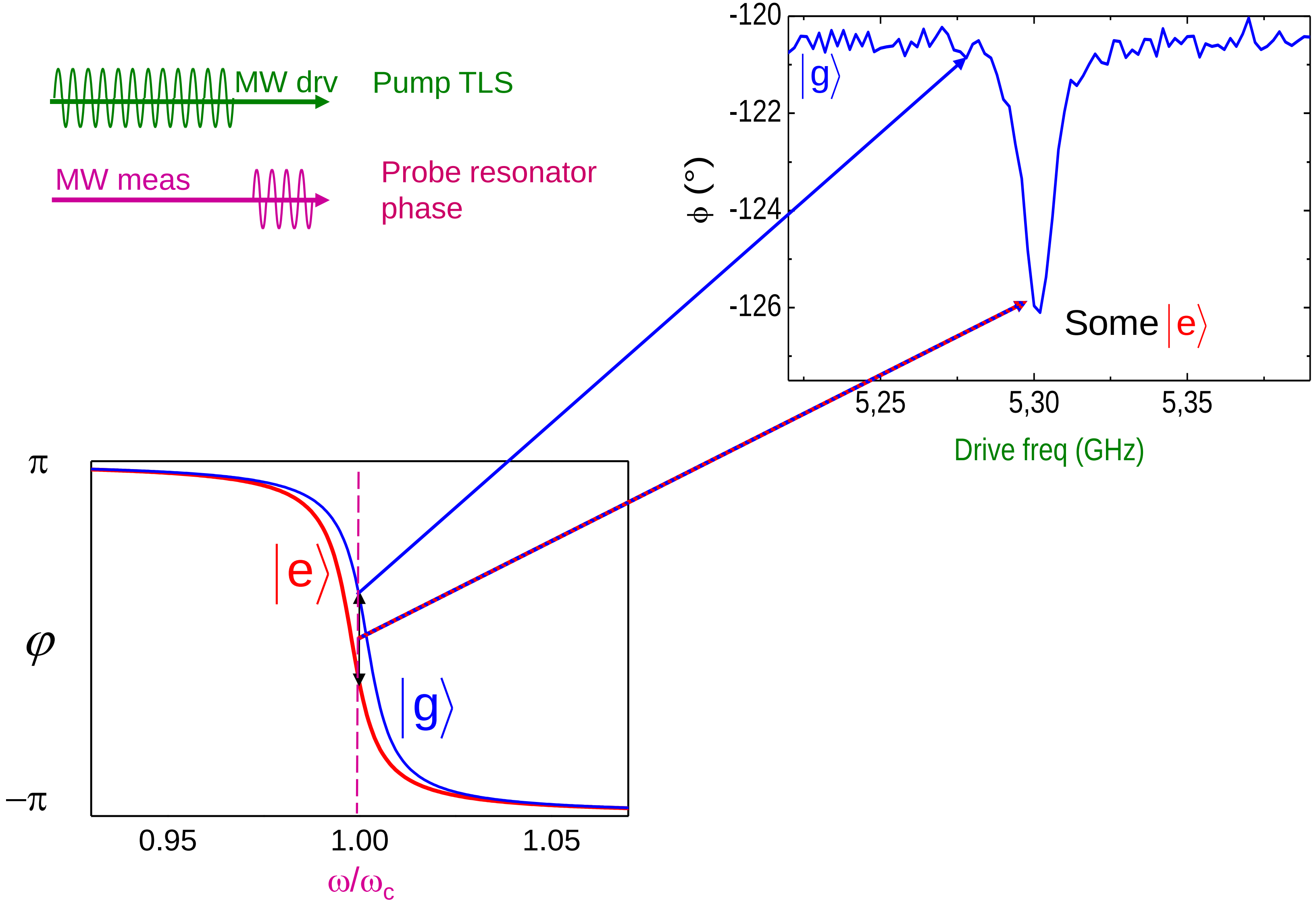
J.P. Reithmaier et al., Nature 432, 197 (2004)

T. Yoshie et al., Nature 432, 200 (2004)

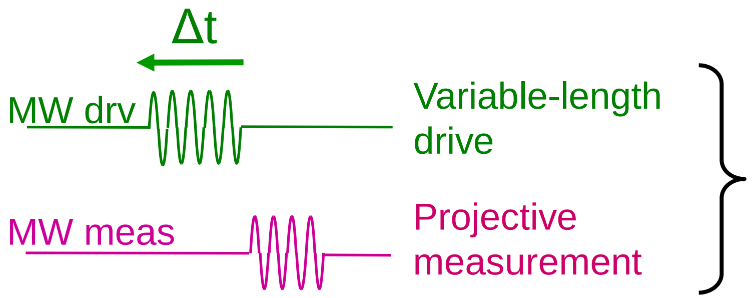



A. Wallraff et al., Nature 431, 162 (2004)

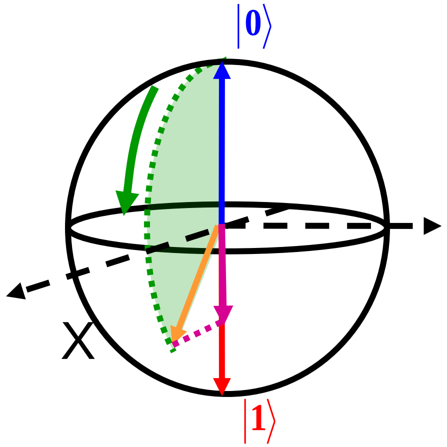
# Qubit Spectroscopy

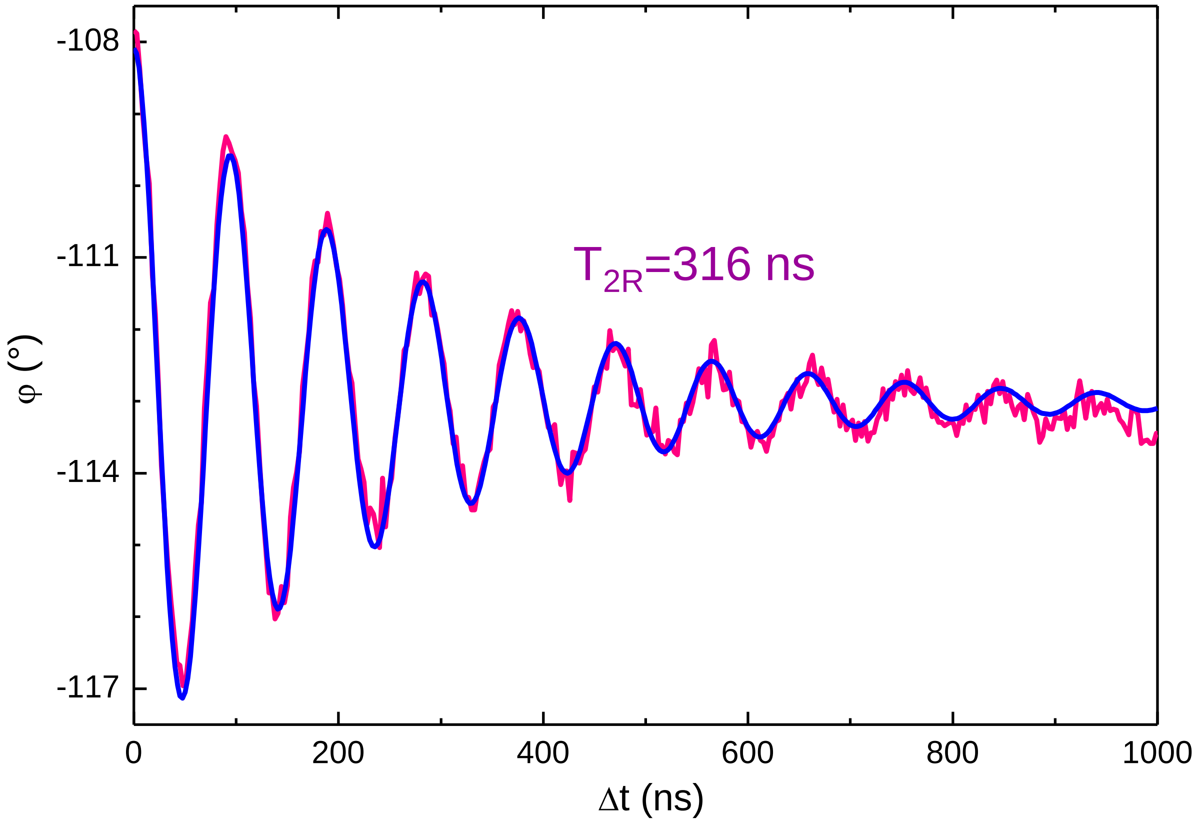


# Detecting Rabi Oscillations



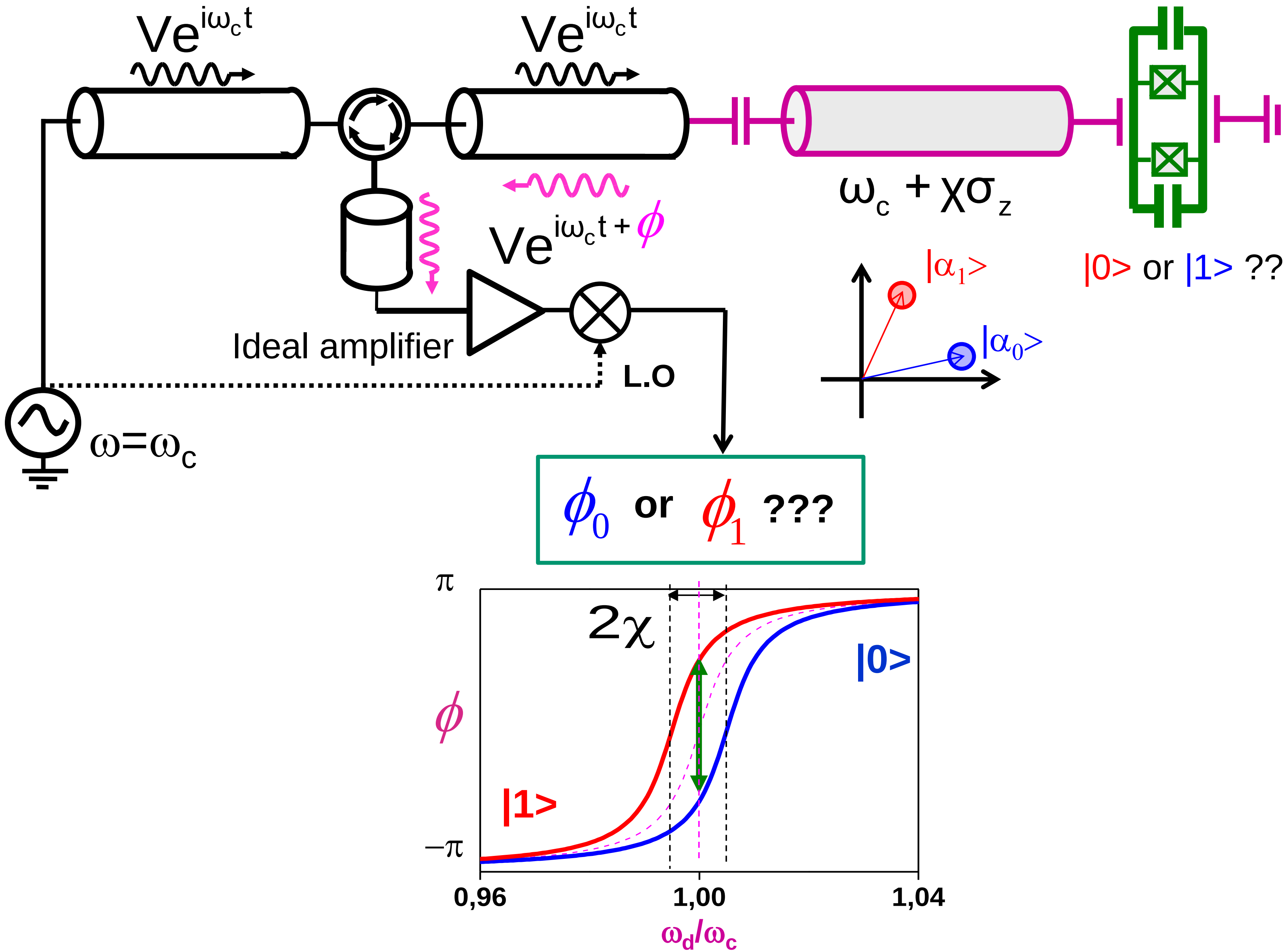

x 10000
➔ **Ensemble averaging**



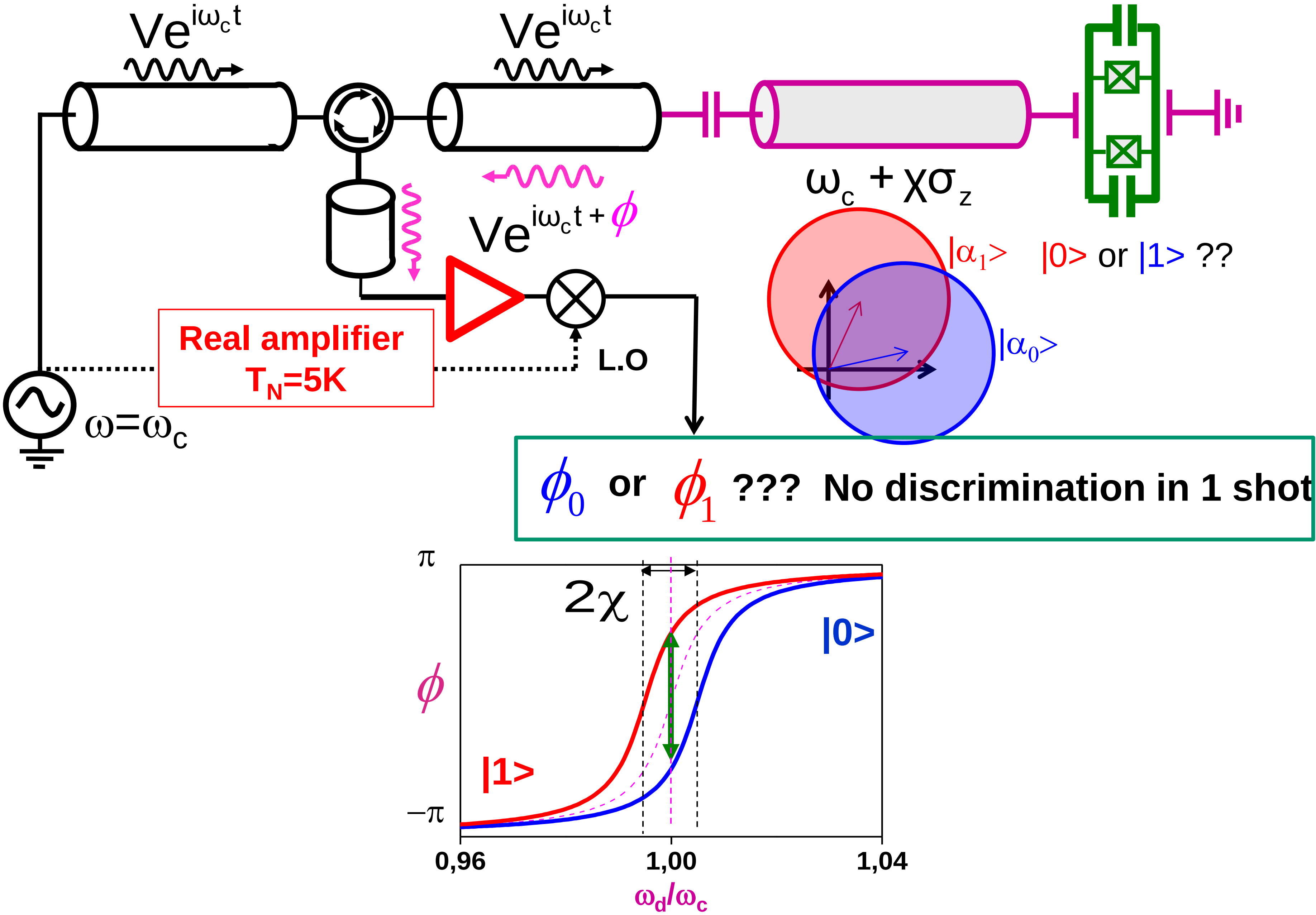



# PART II: AMPLIFICATION & FEEDBACK

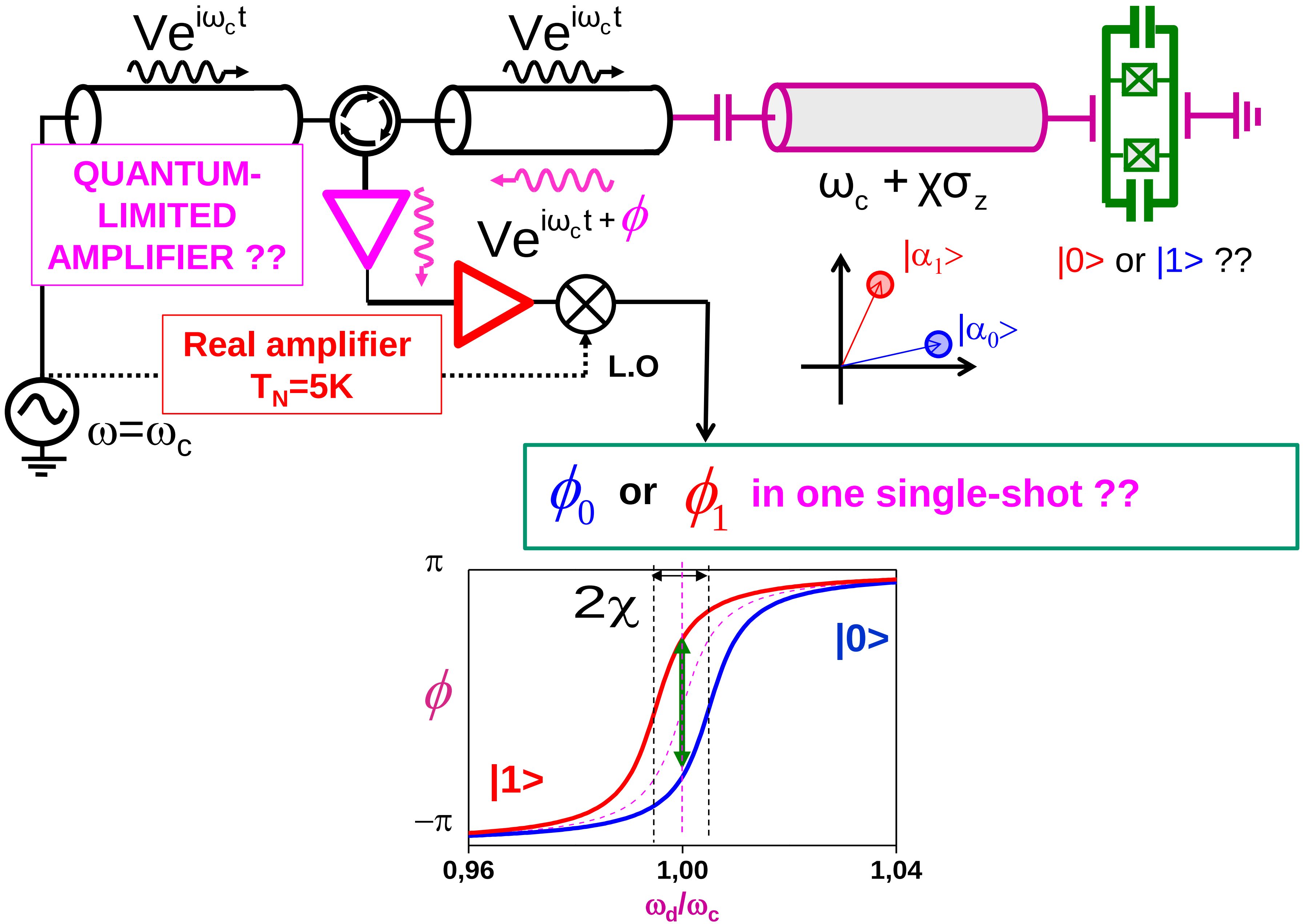
# Readout: Signal-to-Noise



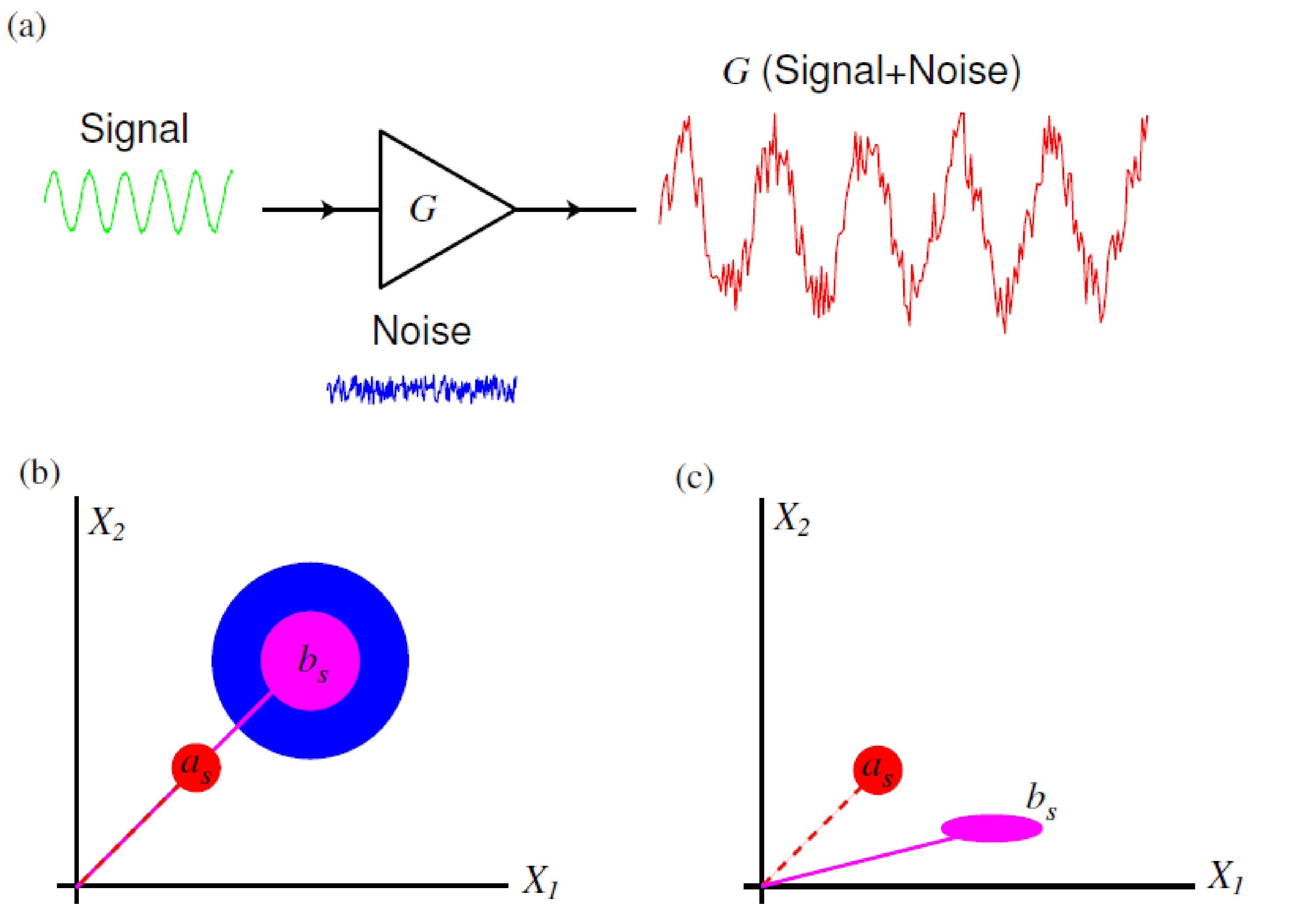
# Readout: Signal-to-Noise



# Readout: Signal-to-Noise

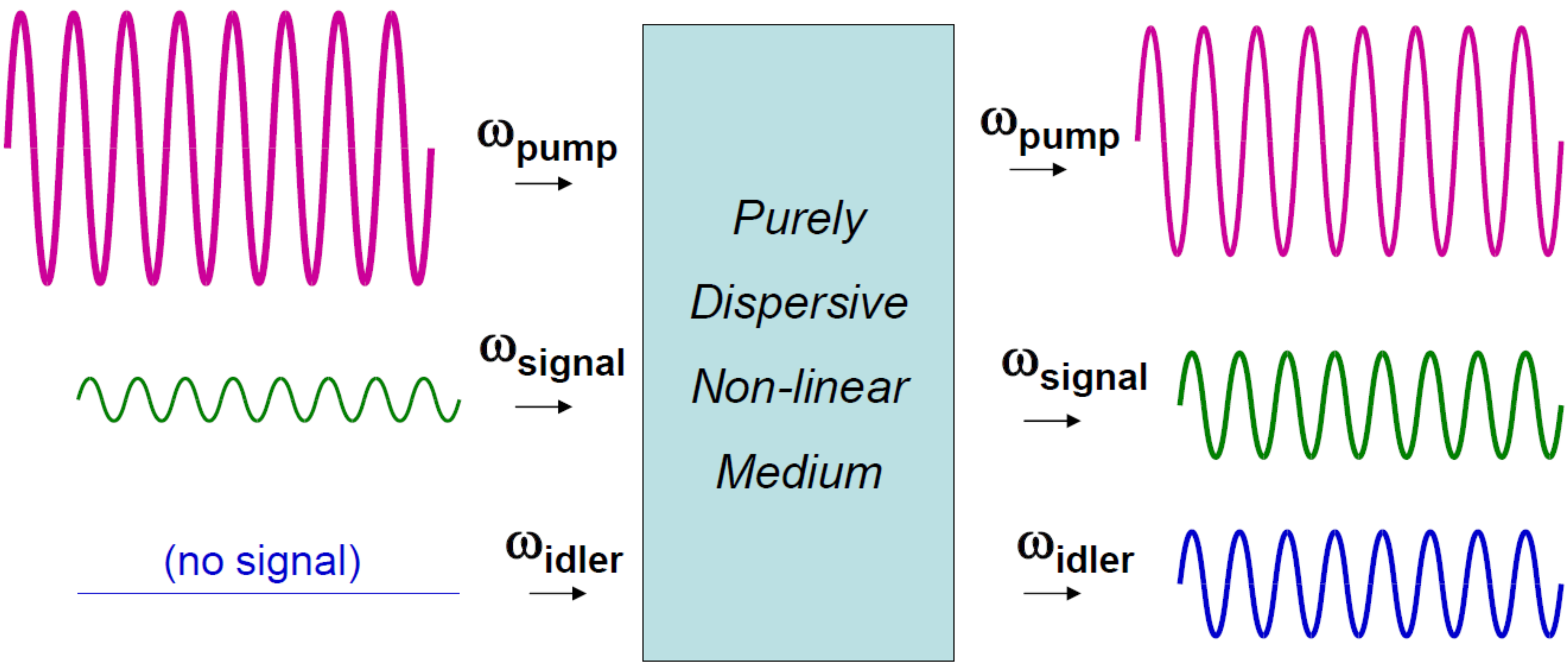


# Parametric Amplification




Castellanos, PhD thesis (2010)

# Parametric Amplification




$$\omega_{signal} + \omega_{idler} = \omega_{pump}$$ "3-wave process"

$$\omega_{signal} + \omega_{idler} = 2\omega_{pump}$$ "4-wave process"

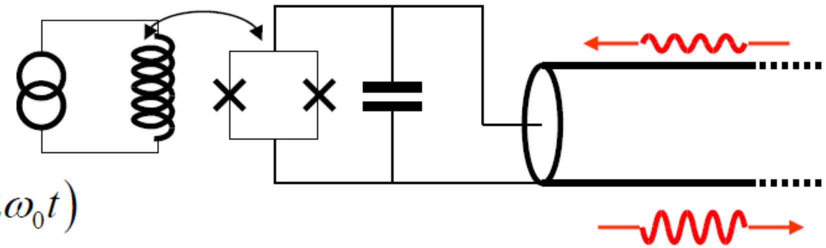
(Slide by Michel Devoret)

# Parametric Amplification

ELECTRICAL SYSTEM

Josephson pot. $\sim\sin(\phi)$
Gives all odd terms!

RF FLUX BIASED DC SQUID (3W)



(NEC, Chalmers)

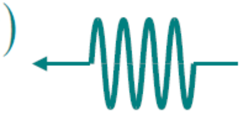
$$I_B(t) = I_B + I_P \sin(2\omega_0 t)$$

---

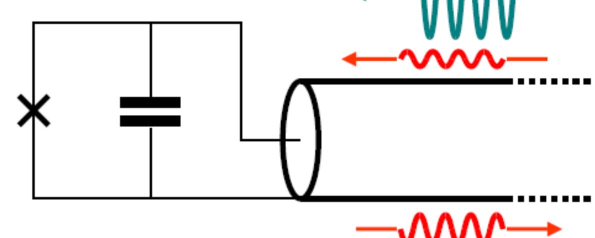
RF BIASED JUNCTION (4W)

*EFFECTIVE PARAMETRIC DRIVE*

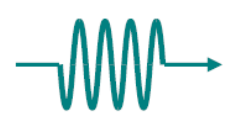
$$A^{in}(t) = A_S^{in}(t) + A_P^{in}\cos(\omega_0 t)$$



Josephson pot. $\sim\cos(\phi)$
Gives all even terms!



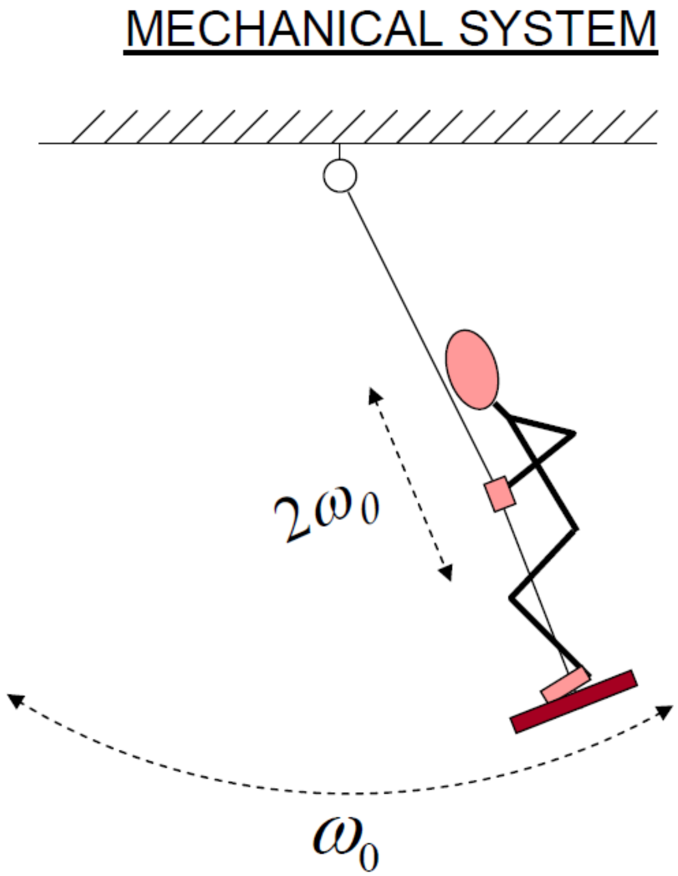
(Yale, NIST
KTH, Berkeley)



*INDIRECT PARAMETRIC PUMPING @ $2\omega_0$*

(Slide by Michel Devoret)

# Parametric Amplification



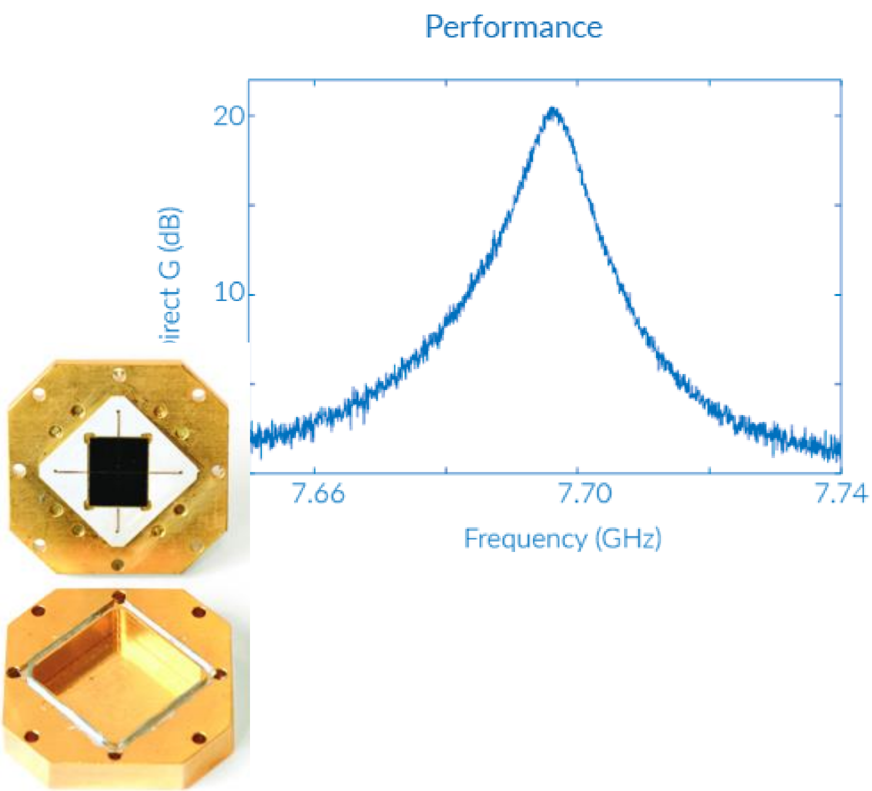

PUMP: WORK
AGAINST CENTRIFUGAL
FORCE

(Slide by Michel Devoret)

# Parametric Amplification

**JPA / JPC**

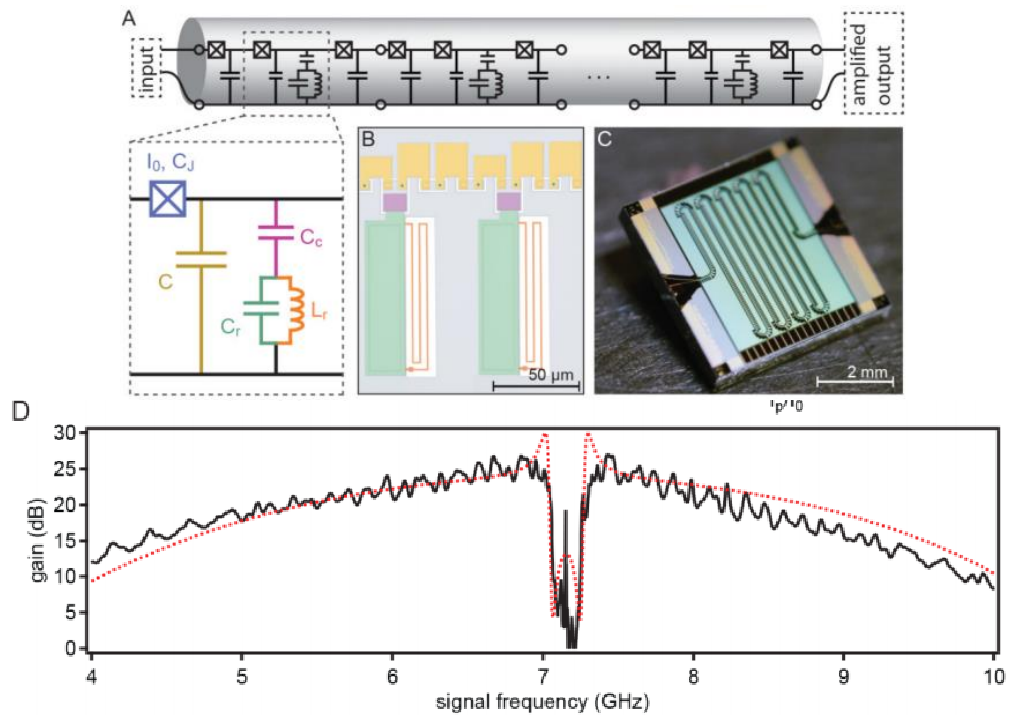
+ Very simple to make
- Narrow band
+ Ultimate efficiency




Quantumcircuits.com

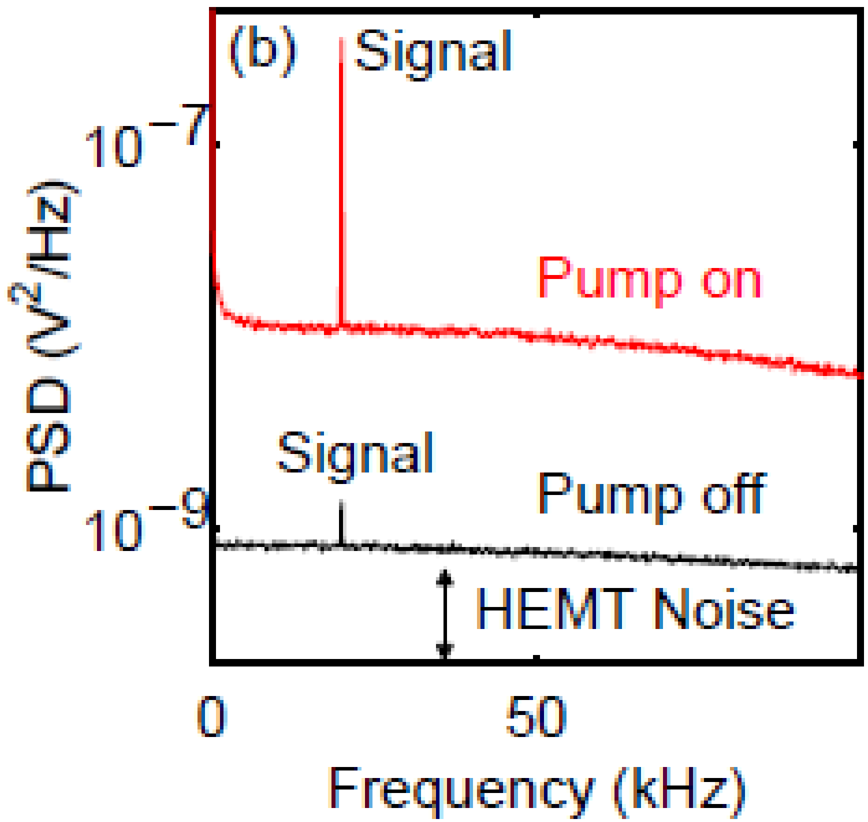
**TWPA**

- Complicated:1000s JJ
- Slightly less efficient now
+ Broadband, directional




Macklin et al., Science (2015)

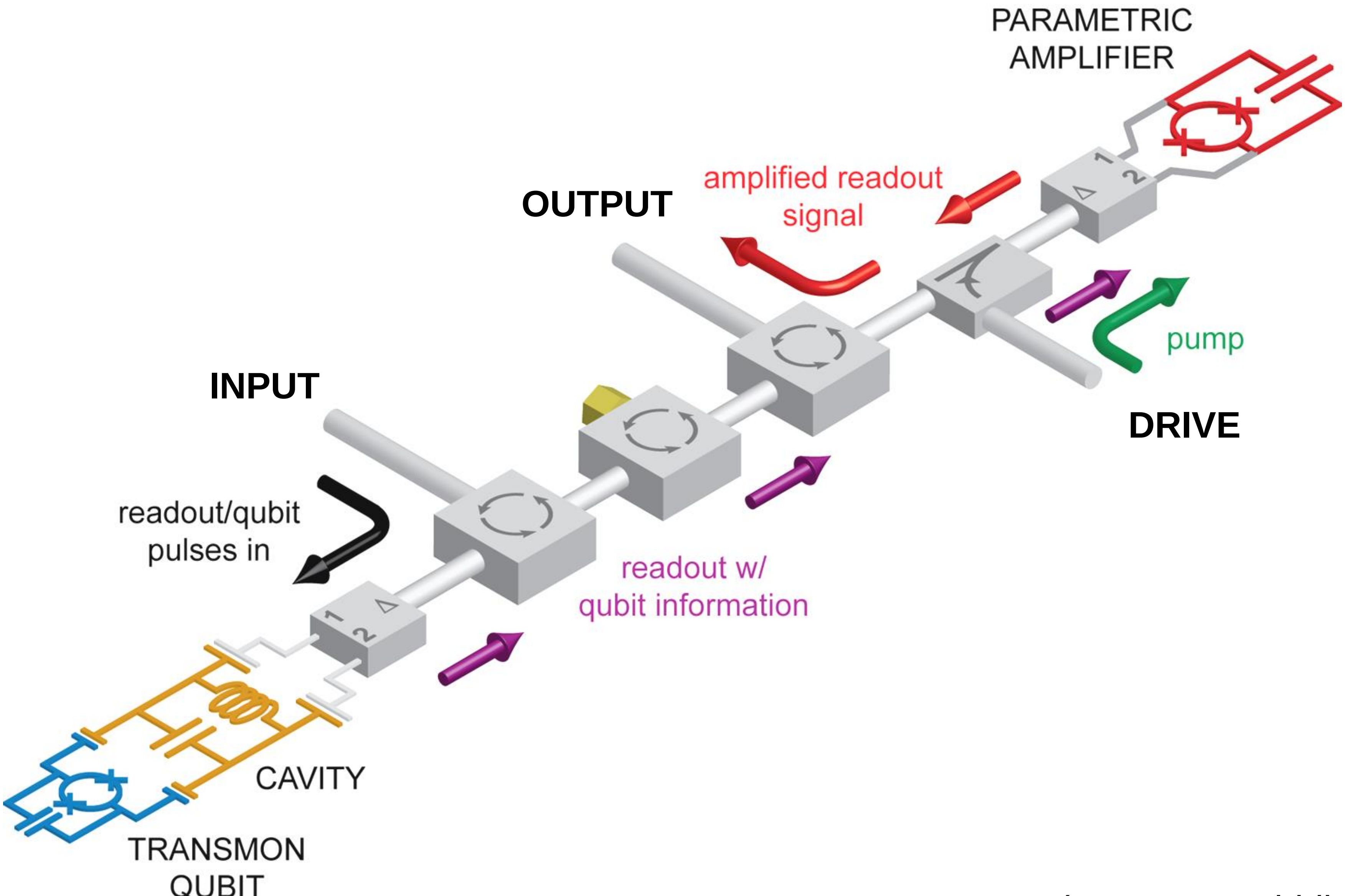
# Signal-to-Noise with Par-amp




M. Castellanos-Beltran, K. Lehnert, APL (2007)

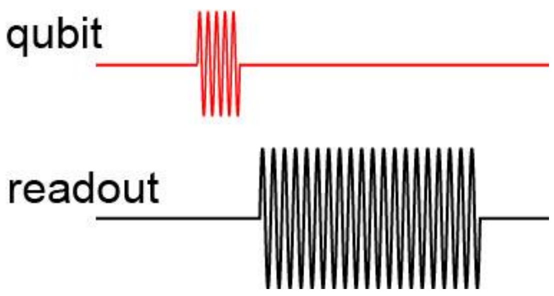
(**quantum limit** on how good an amplifier can be : ***Caves theorem***)
Actually reached in several experiments : ***quantum limited measurement***

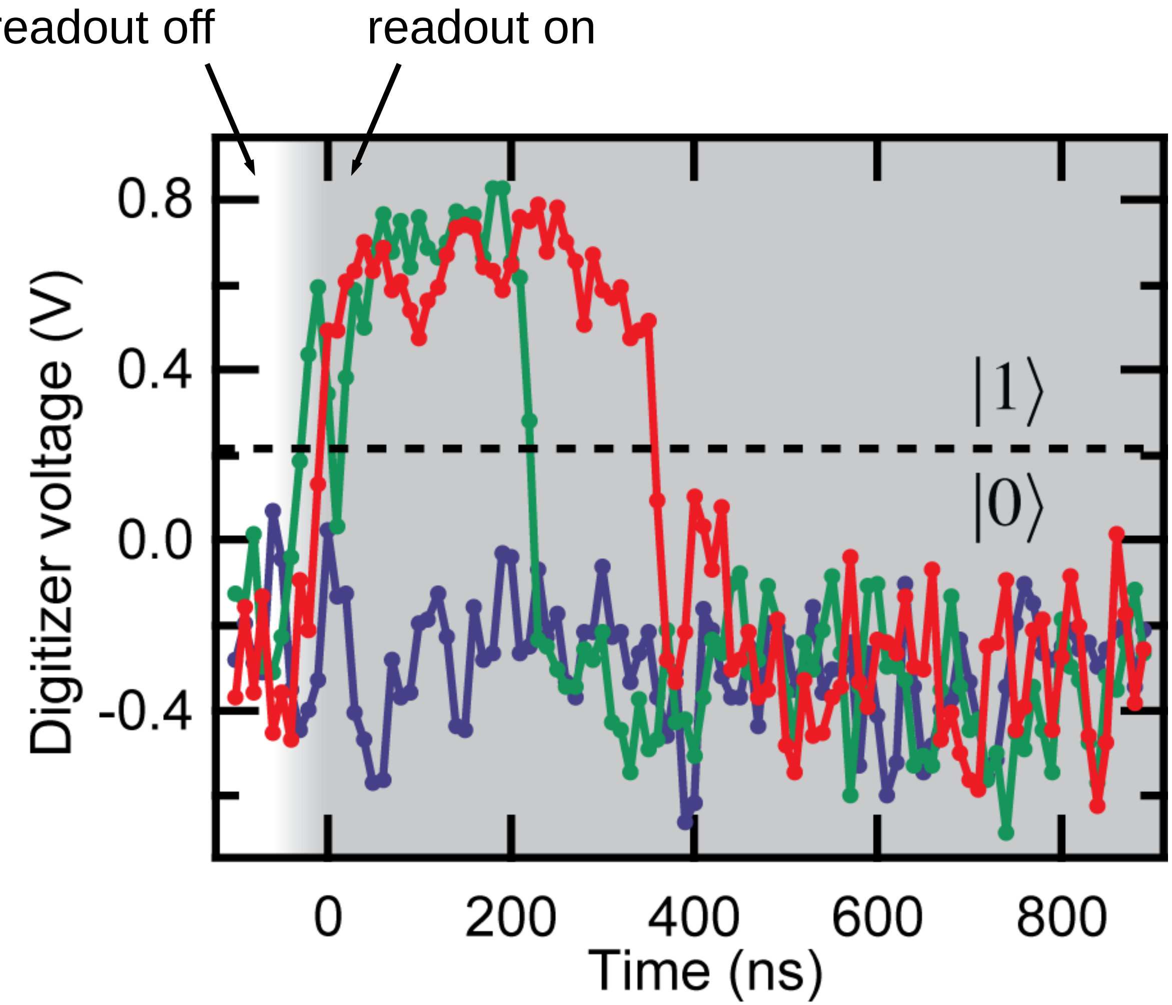
# Typical Measurement Chain




(courtesy I. Siddiqi)

# Pulsed Readout



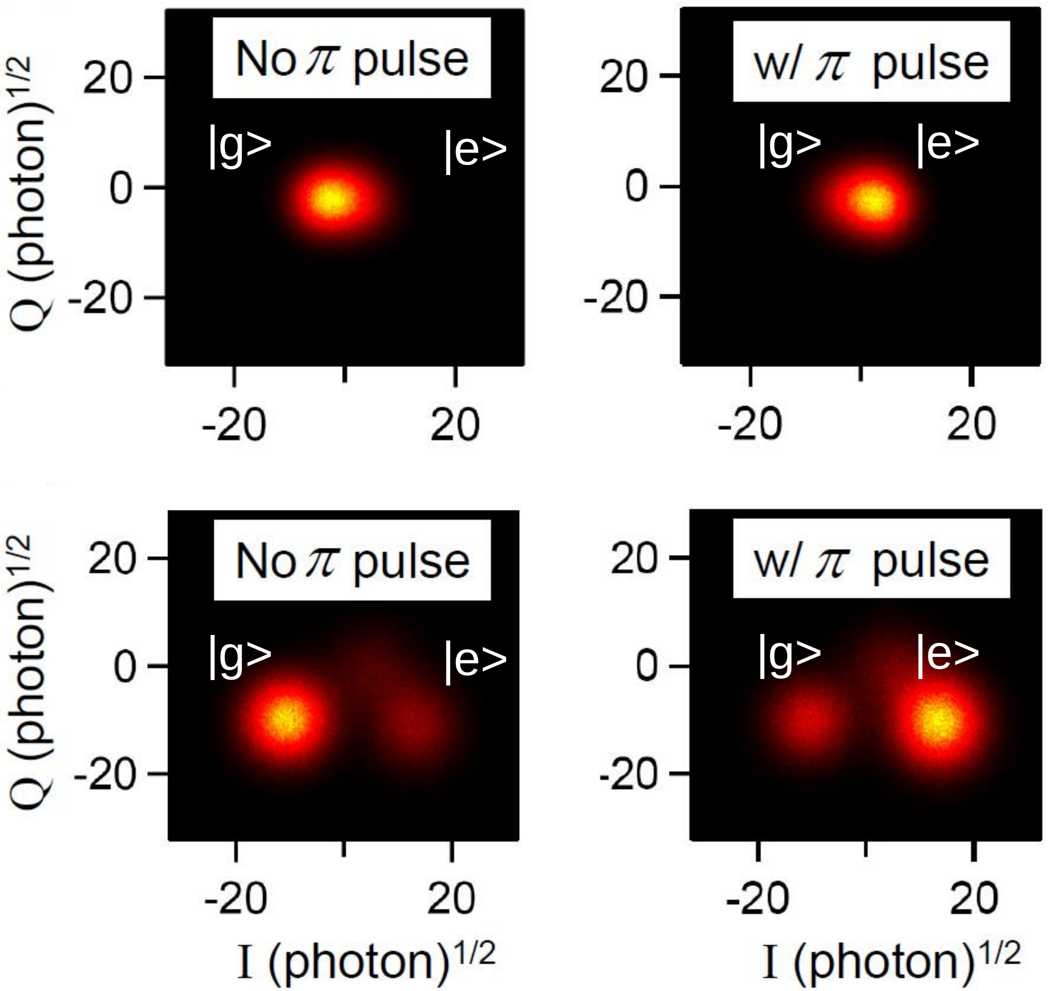

R. Vijay, D.H. Slichter, and I. Siddiqi, PRL **106**, 110502 (2011)

(courtesy I. Siddiqi)

# Single-Shot Histograms

No Par-amp

With Par-amp



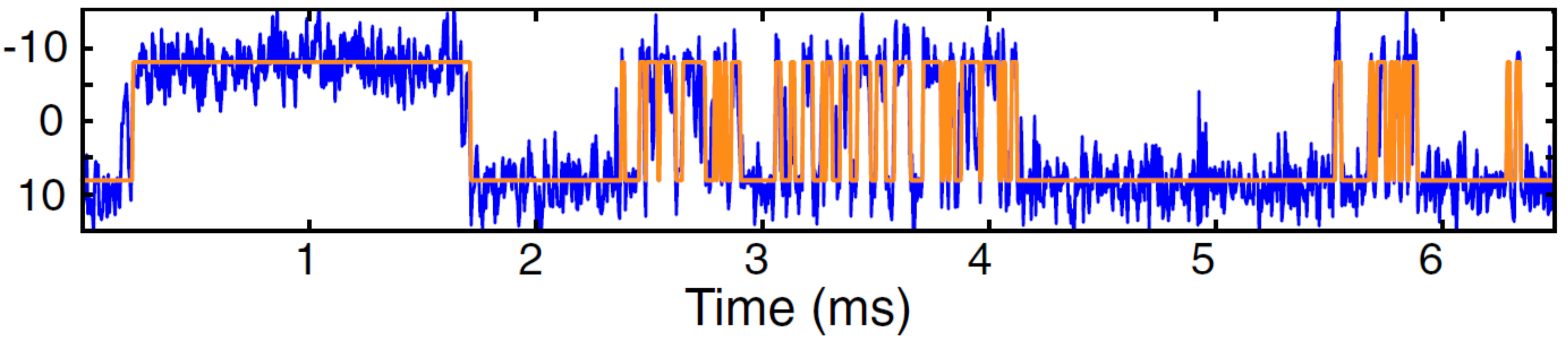

→ Single-shot discrimination of qubit state

Abdo *et al.*, PRL (2014)

# Cont. Readout: Quantum Jumps

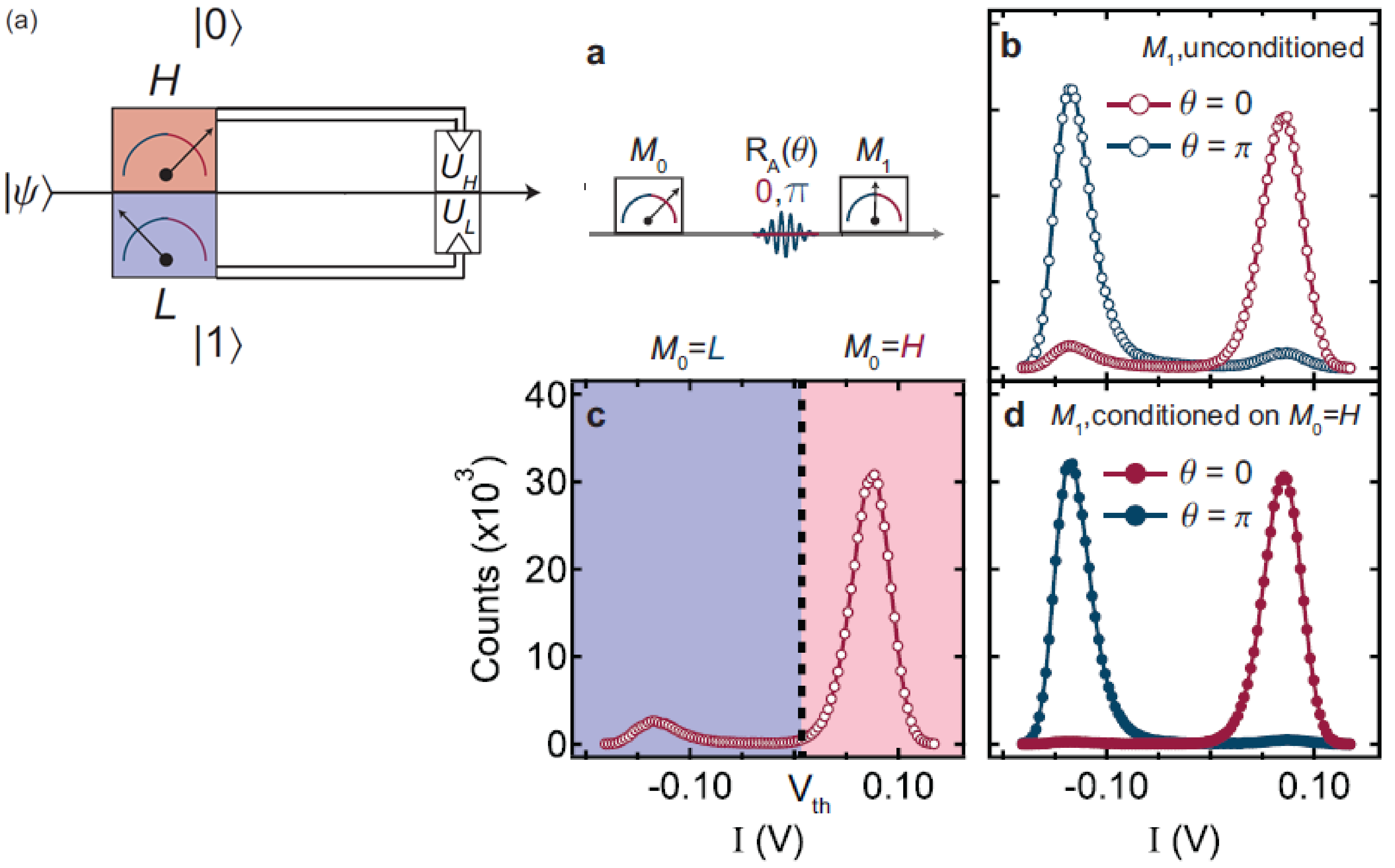
- You can observe transitions in real-time!

  (in this case for a fluxonium qubit)



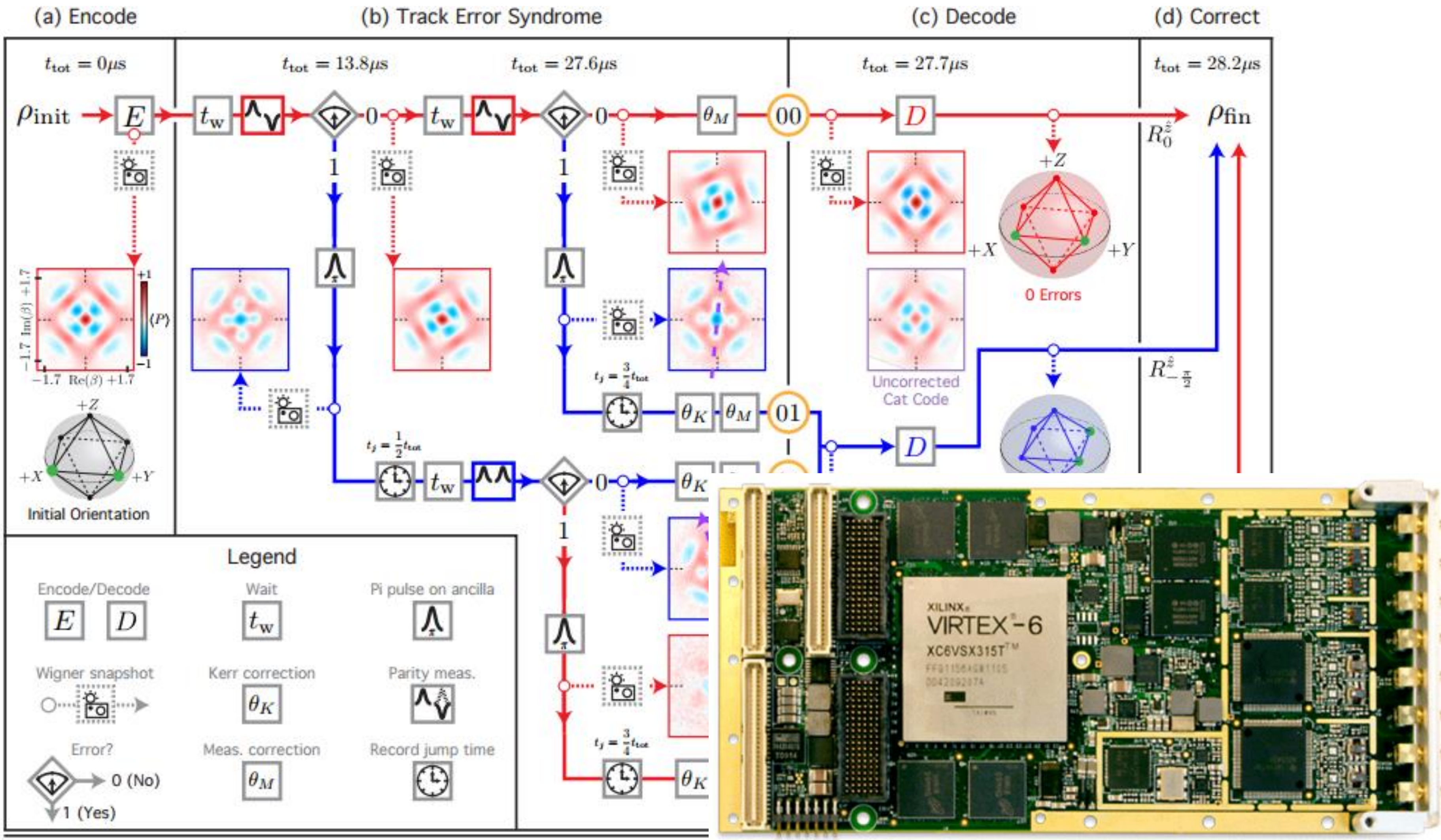

Vool *et al.*, PRL (2014)

# Single-Shot Readout → Feedback



# Real-Time Feedback

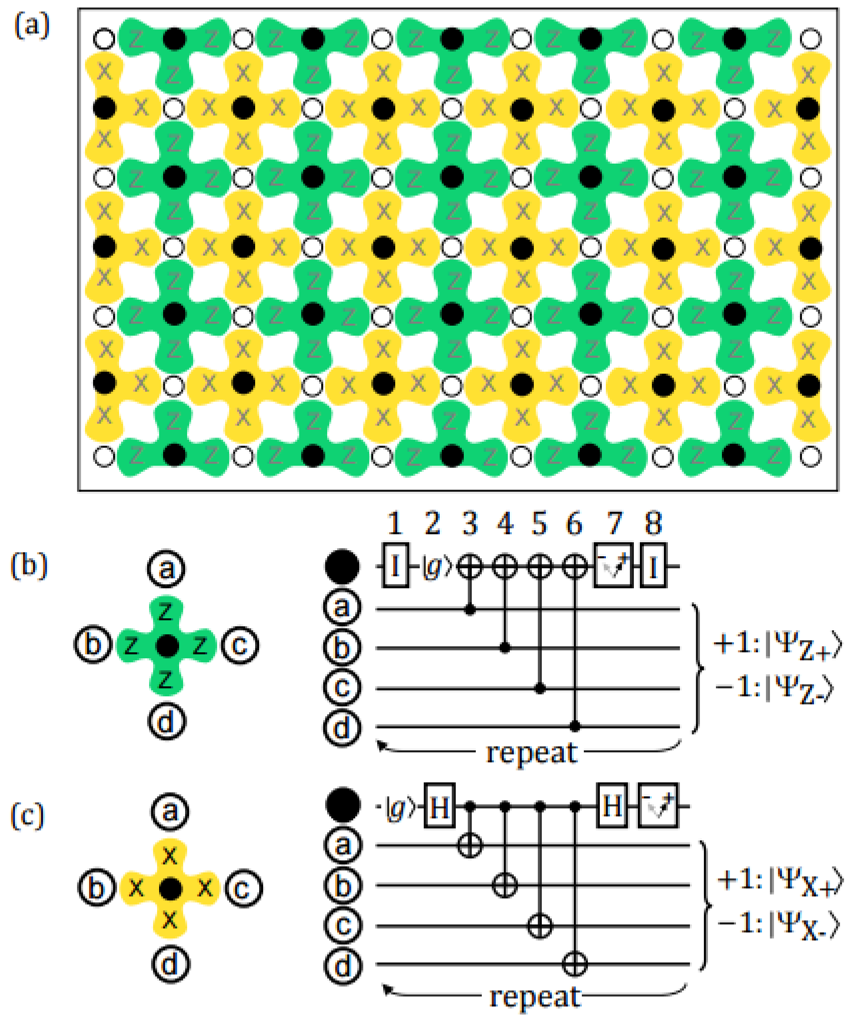
- Many use FPGAs for real-time feedback




Ofek *et al.*, Nature 2016

# Quantum Error Correction

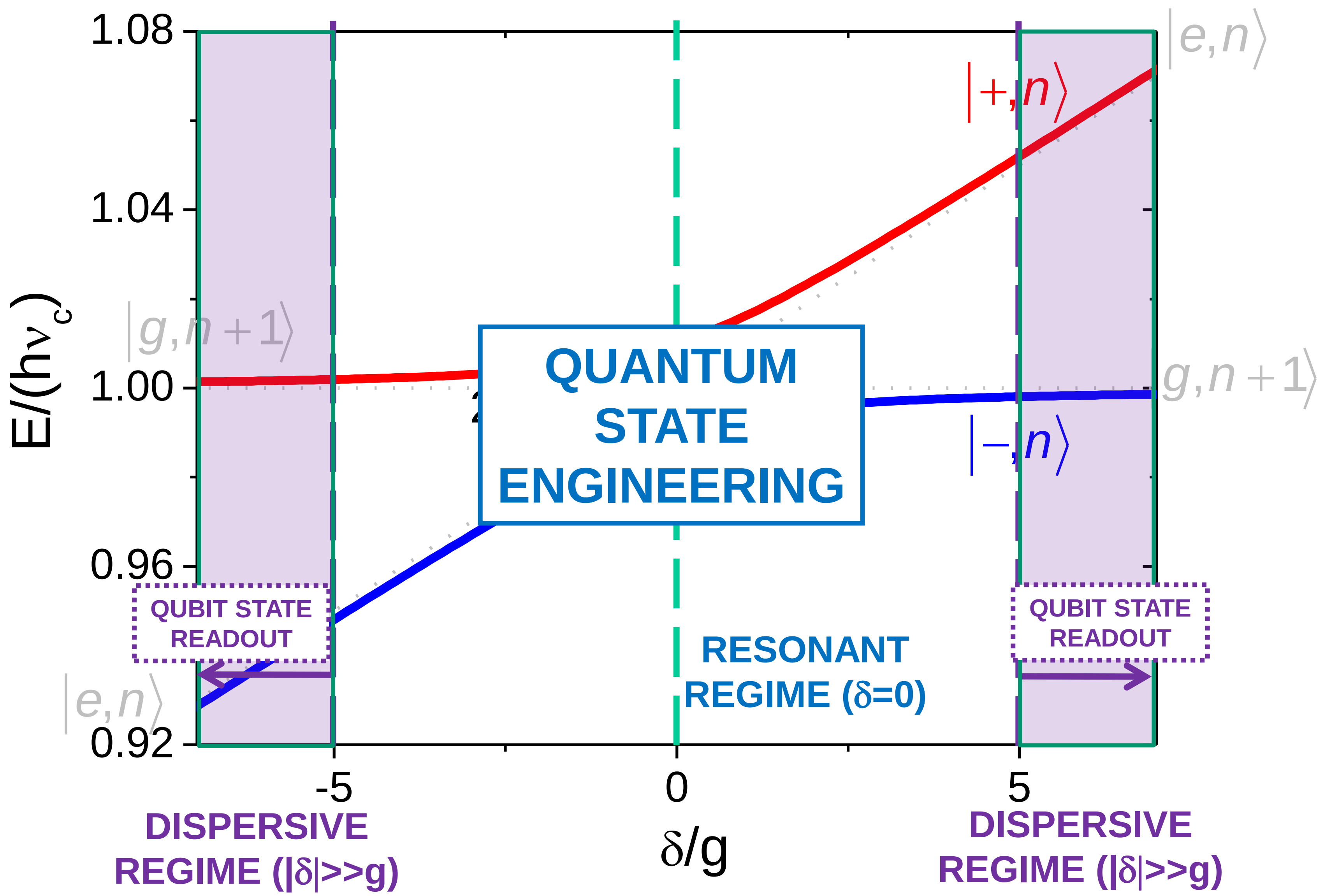
- The ultimate quantum feedback machine




Fowler, PRA (2012)

# PART III: QUANTUM STATE ENGINEERING

# Two Interesting Limits



# Resonant JC: Vacuum Rabi Osc.

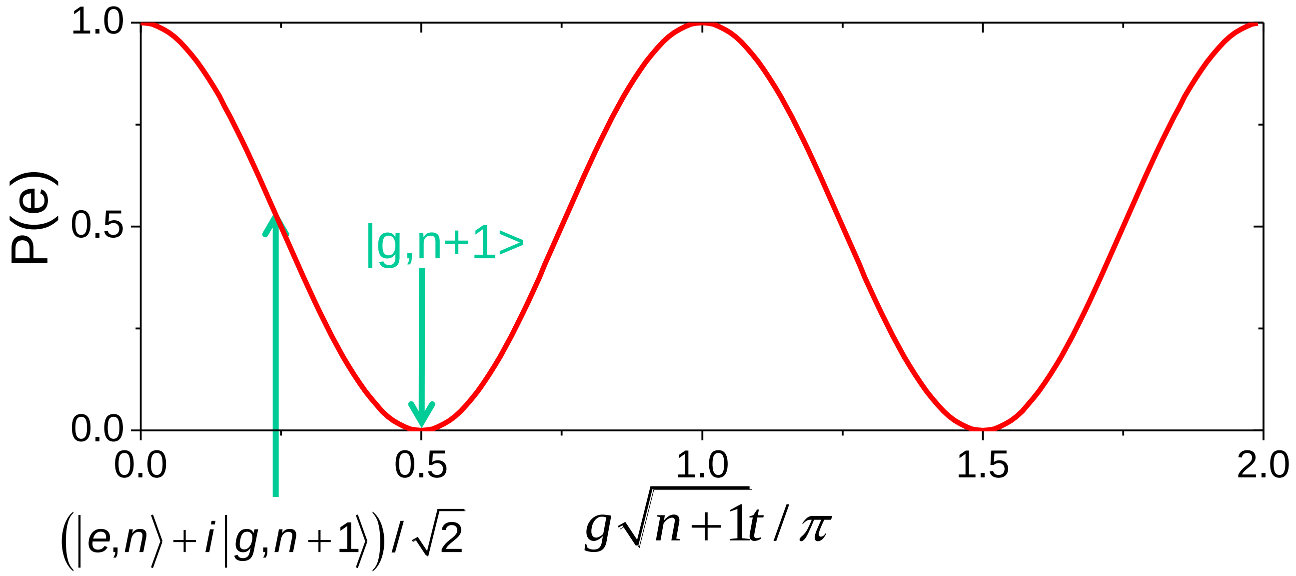
$$|+,n\rangle = \left(|e,n\rangle + |g,n+1\rangle\right)/\sqrt{2}$$
$$|-,n\rangle = \left(-|e,n\rangle + |g,n+1\rangle\right)/\sqrt{2}$$

COMPLETE ATOM-PHOTON MIXING

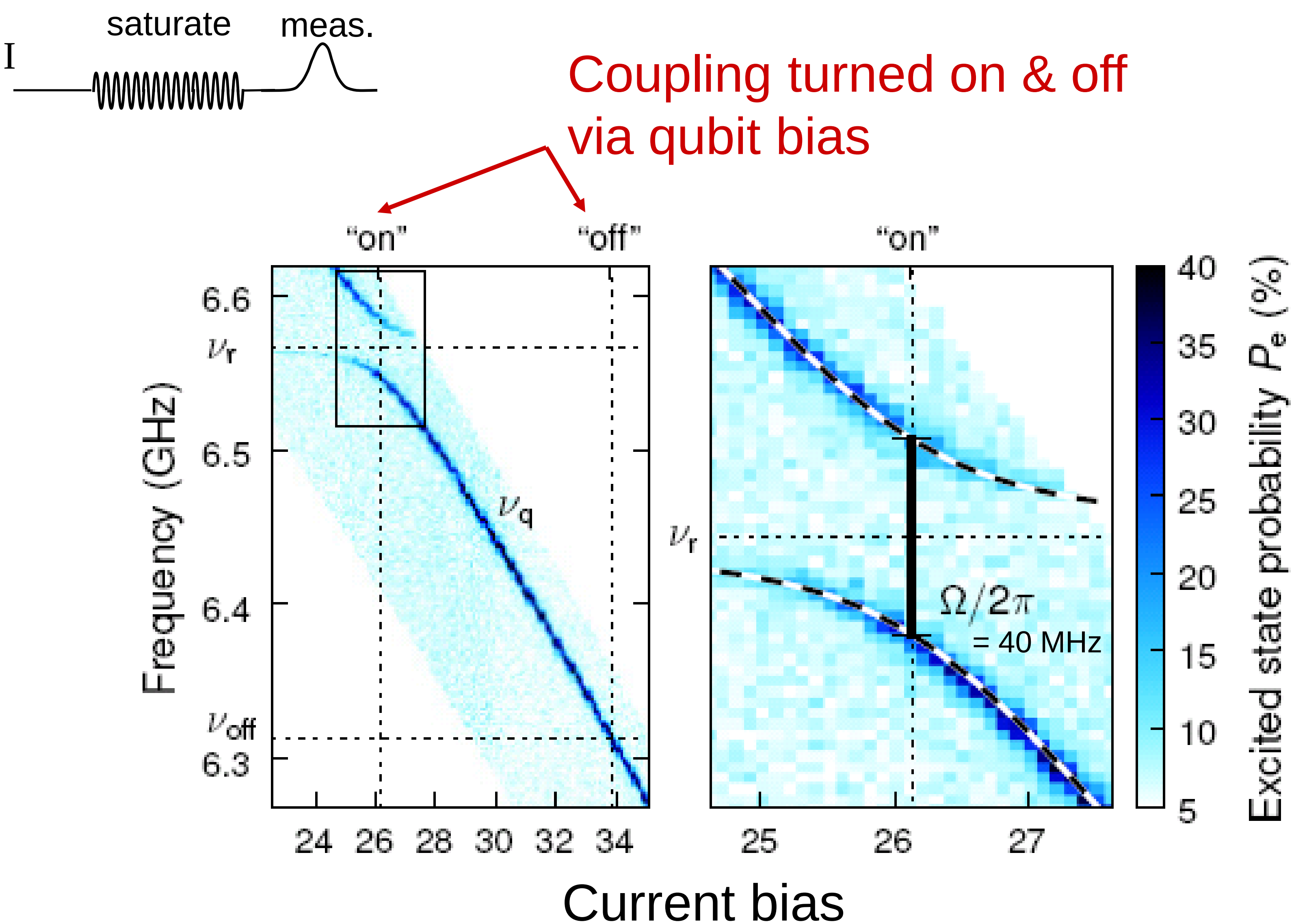
COUPLING SUDDENLY ON at t=0

$$P(e) = 1/2\left(1+\cos 2g\sqrt{n+1}t\right)$$

Uncoupled sytem prepared in |e,n>



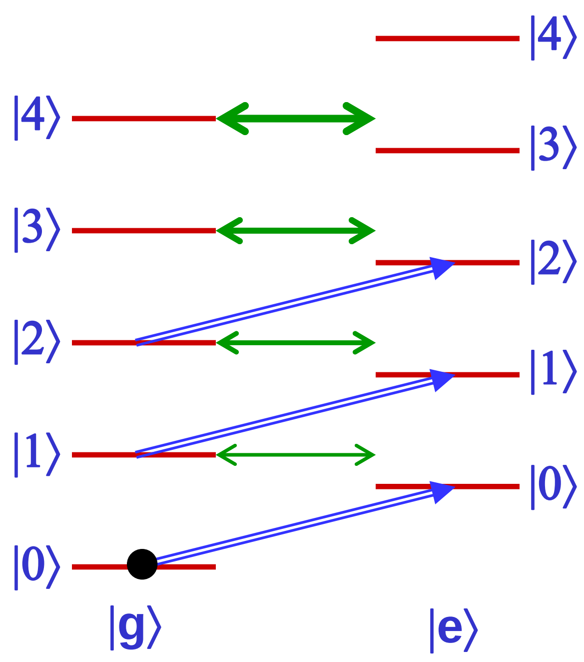
# Phase Qubit Coupled to Resonator

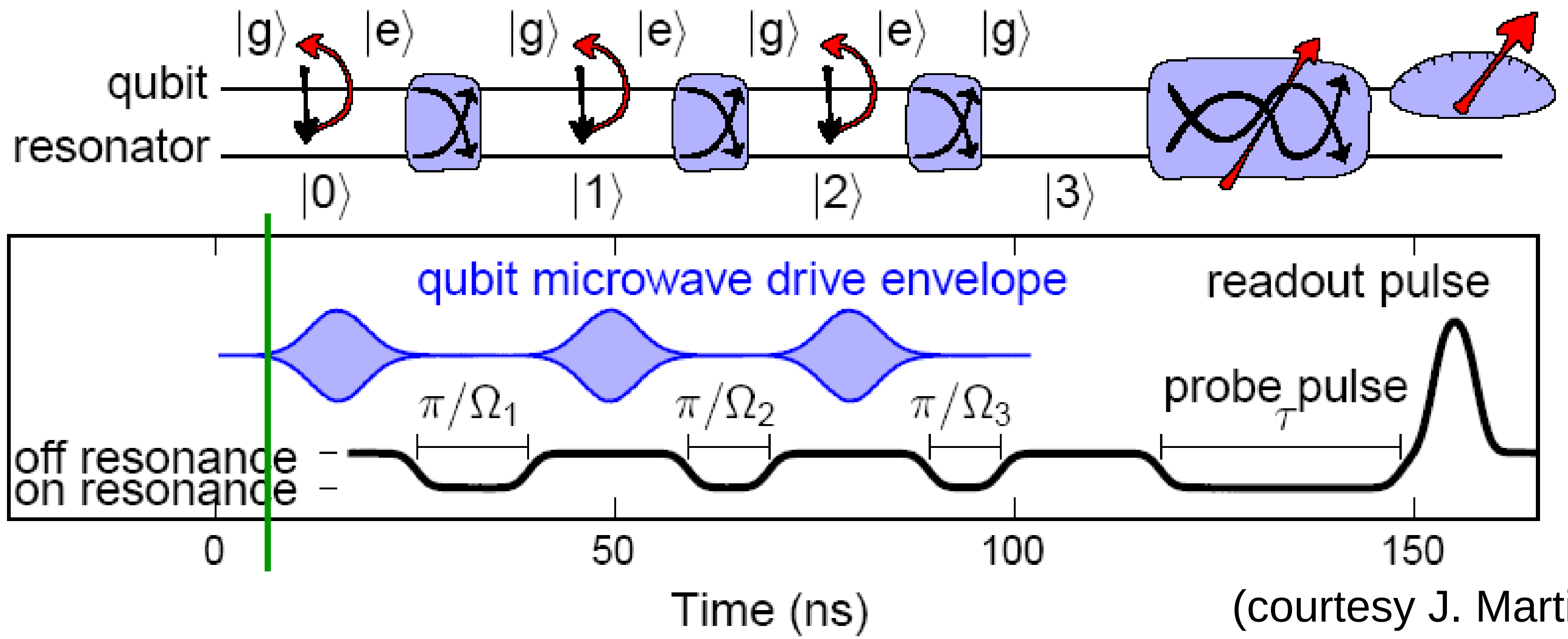



(courtesy J. Martinis)

# Generating Fock States: Pumping Photons One by One

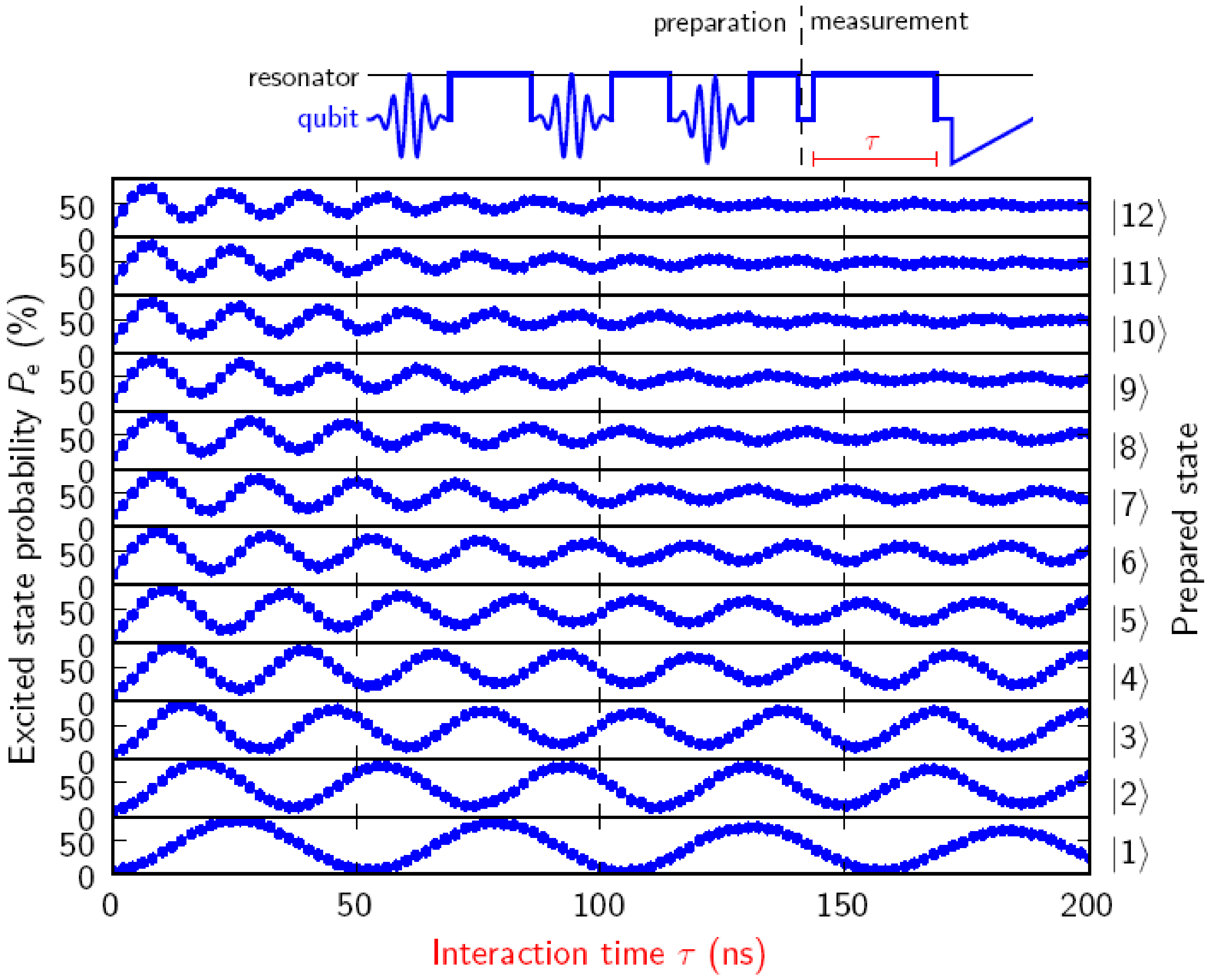
(As done for ion traps)

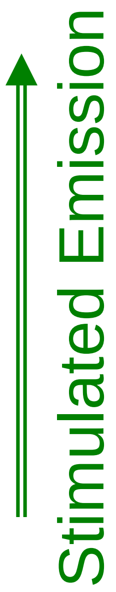



(courtesy J. Martinis)

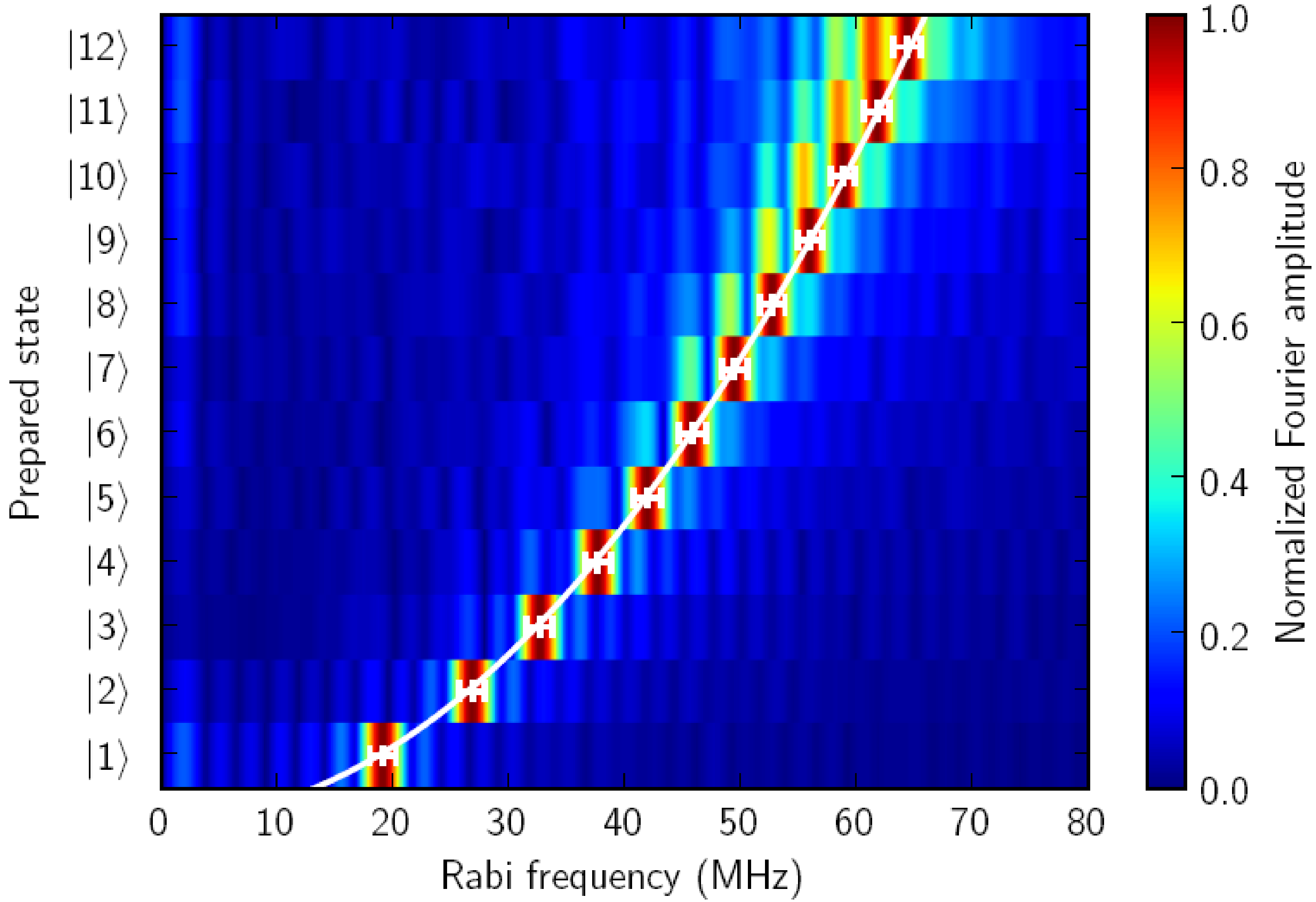
# Swap Oscillations Depend on |n⟩




(courtesy J. Martinis)

# Swap Oscillations Depend on |n⟩

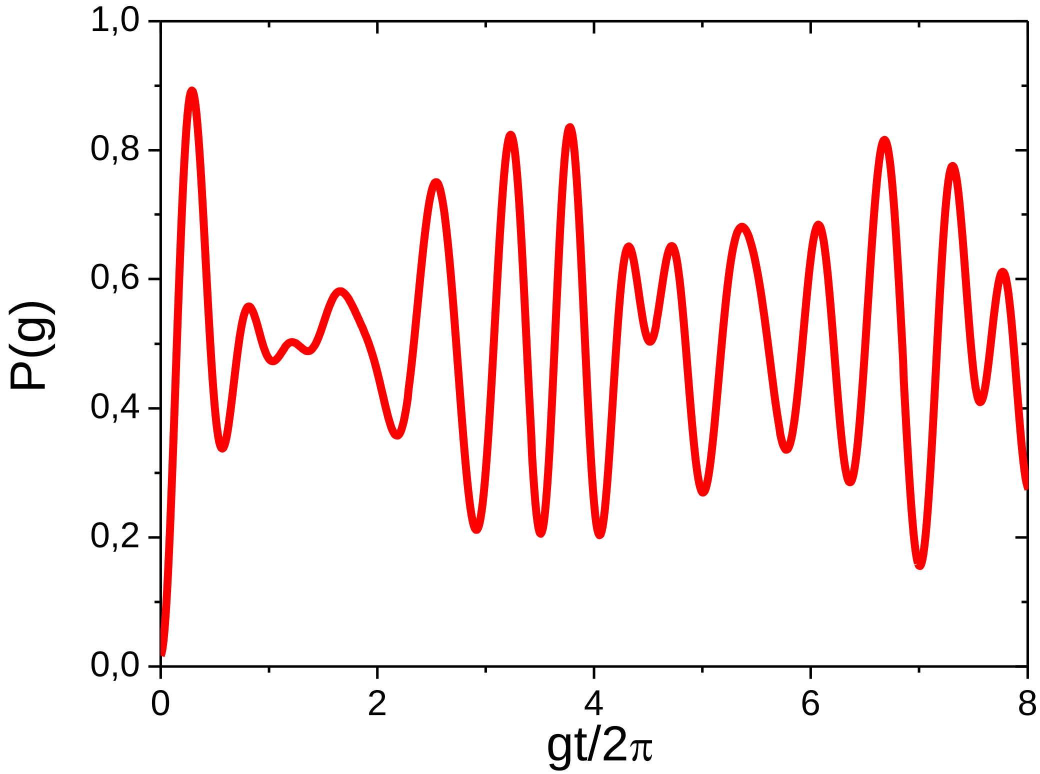



(courtesy J. Martinis)

# Swap Oscillations Depend on |n⟩

COUPLING
SUDDENLY ON at t=0

- Atom prepared in |g>
- Oscillator in unknown state




$$P(g) = 1/2\left[1 + \sum_n p(n)\cos\left(2g\sqrt{n}t\right)\right]$$

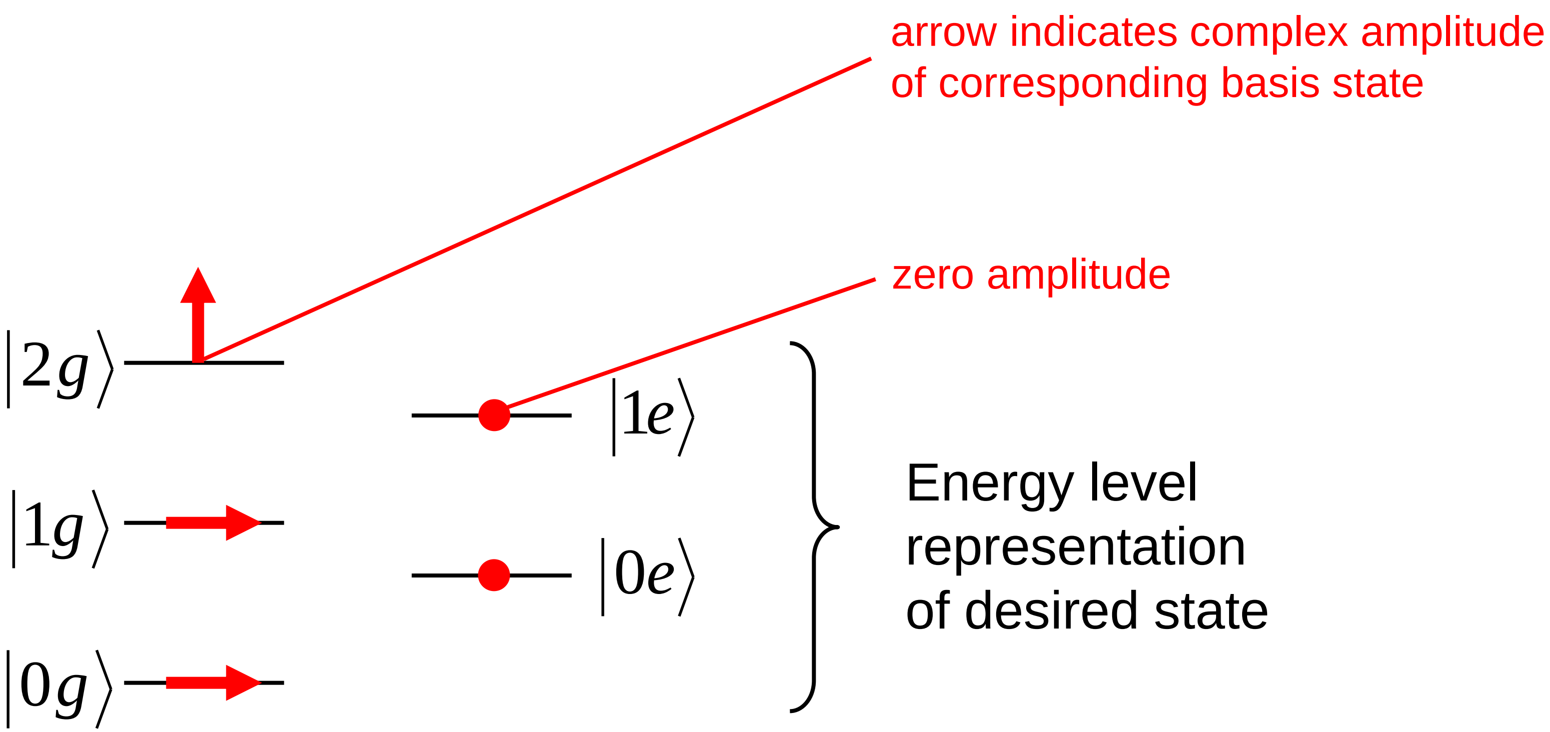
→ Measurement of p(n)

# Generating Arbitrary States

Desired final state ➡ $$|\psi\rangle = 0.577\big(|0\rangle + |1\rangle + i|2\rangle\big) \otimes |g\rangle$$

**Law and Eberly, PRL (1996)**

Reverse-engineer final state by building pulse sequence backwards



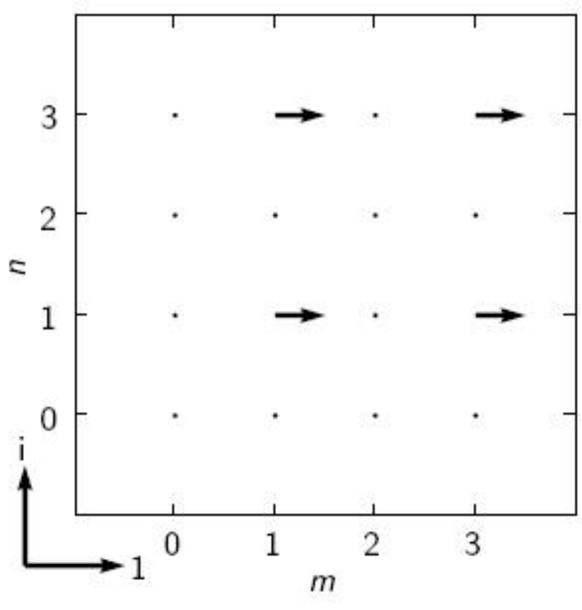

(courtesy J. Martinis)

# State Tomo. of Harmonic Oscillator

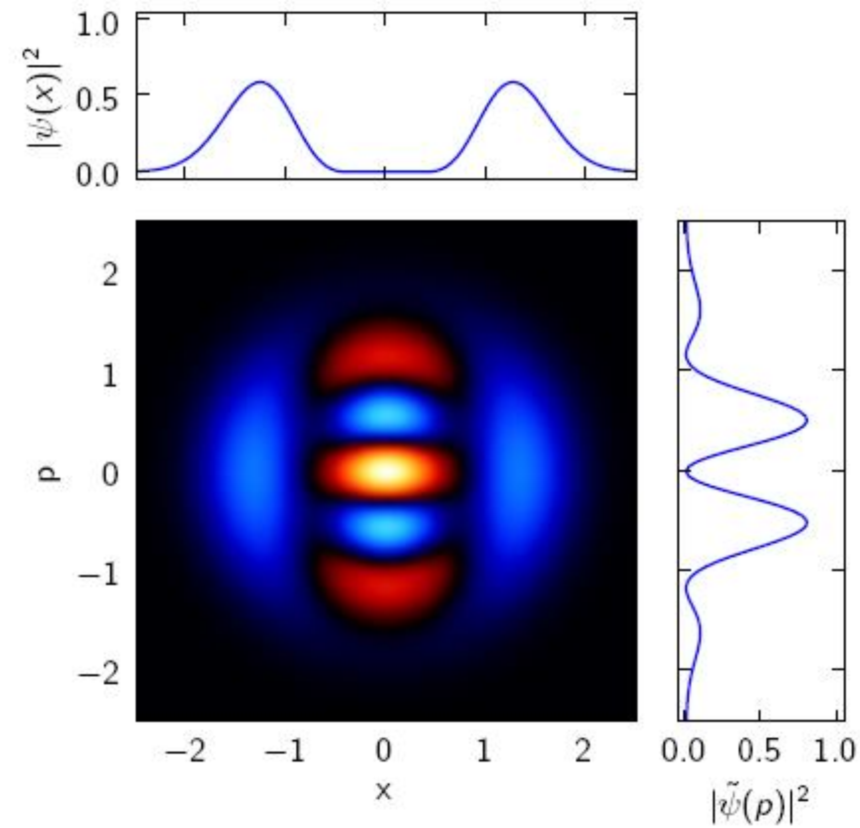
Example: $|\psi\rangle = |1\rangle + \ |3\rangle$

Density matrix:

$$\rho = |\psi\rangle\langle\psi|$$

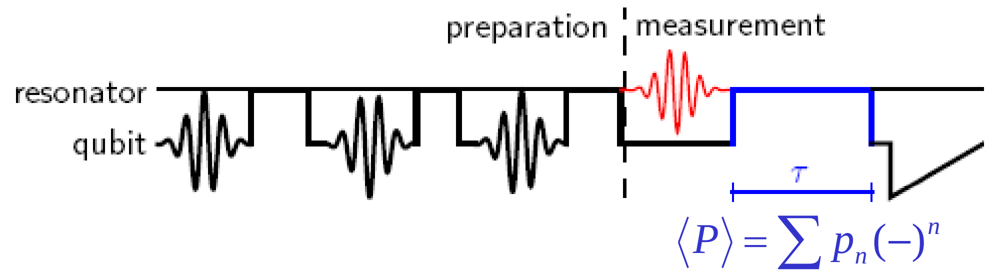



Wigner function:




(courtesy J. Martinis)

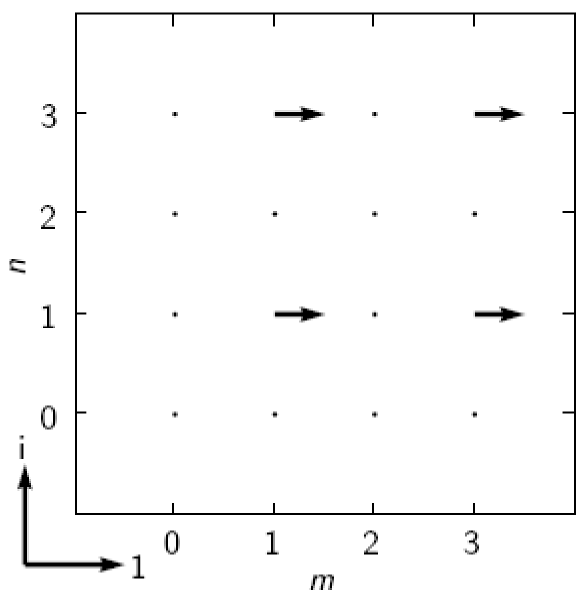
# State Tomo. of Harmonic Oscillator




Example: $|\psi\rangle = |1\rangle + \ |3\rangle$

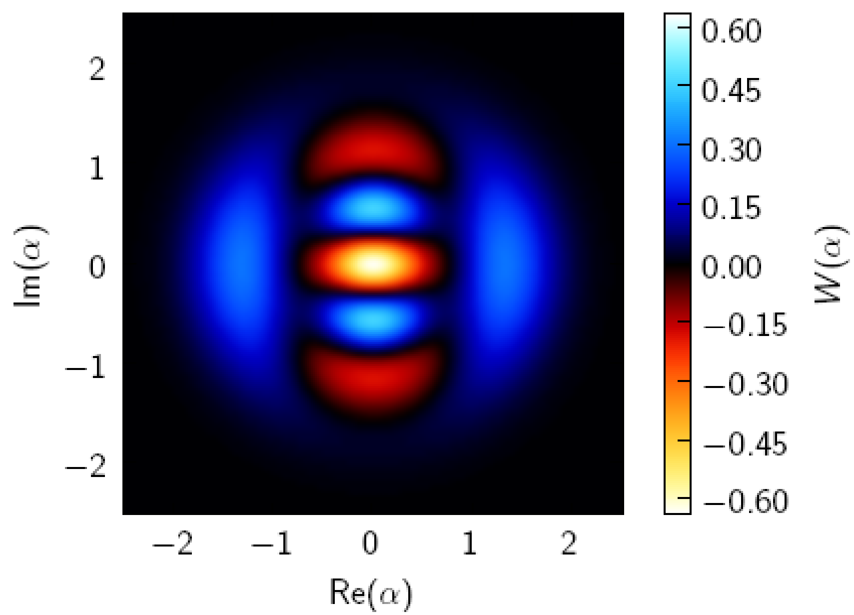
Density matrix:

$$\rho = |\psi\rangle\langle\psi|$$



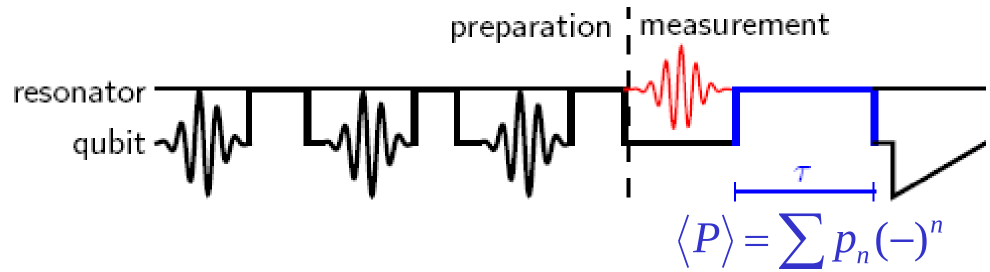

Wigner function:

$$W(\alpha) = \frac{2}{\pi}\langle\psi|D(\alpha)PD(-\alpha)|\psi\rangle$$




(courtesy J. Martinis)

# State Tomo. of Harmonic Oscillator



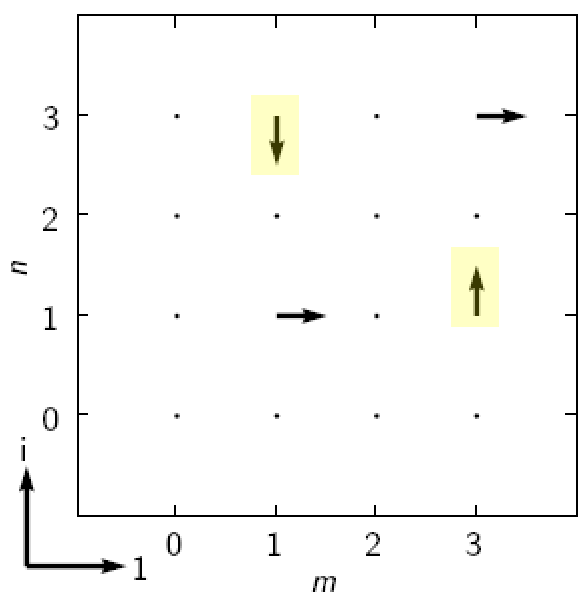

Example: $|\psi\rangle = |1\rangle + i|3\rangle$

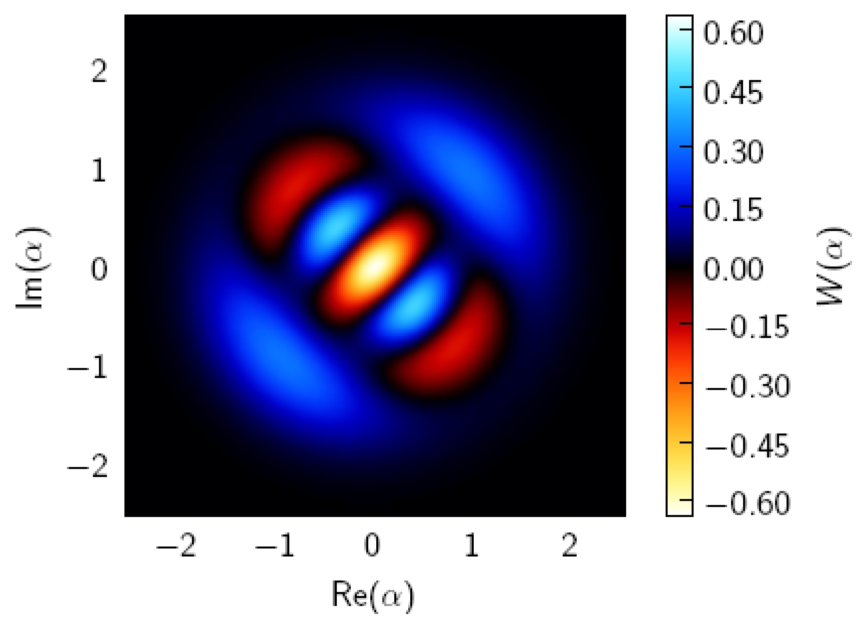
Density matrix:

$$\rho = |\psi\rangle\langle\psi|$$

Wigner function:

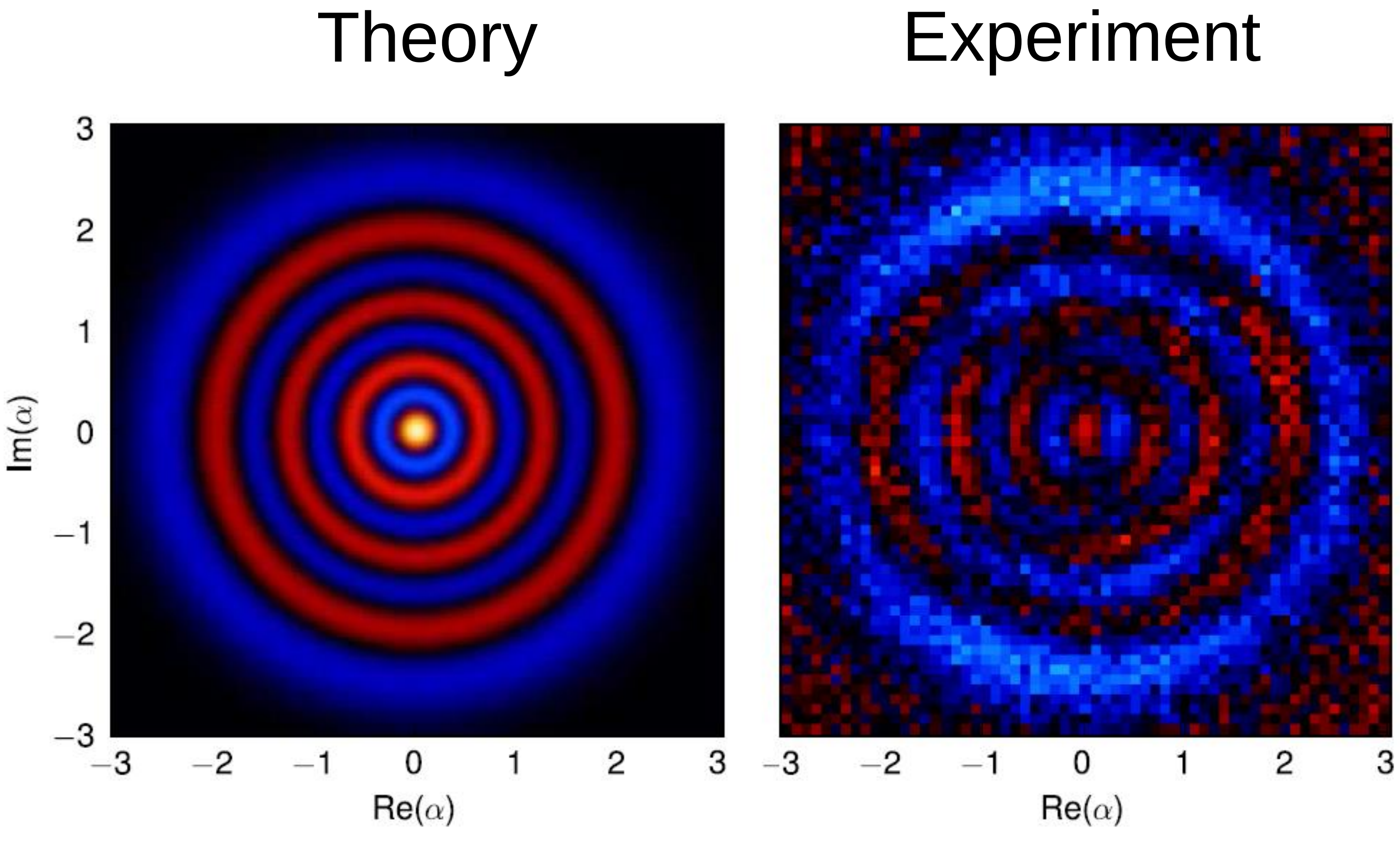
$$W(\alpha) = \frac{2}{\pi}\langle\psi|D(\alpha)PD(-\alpha)|\psi\rangle$$



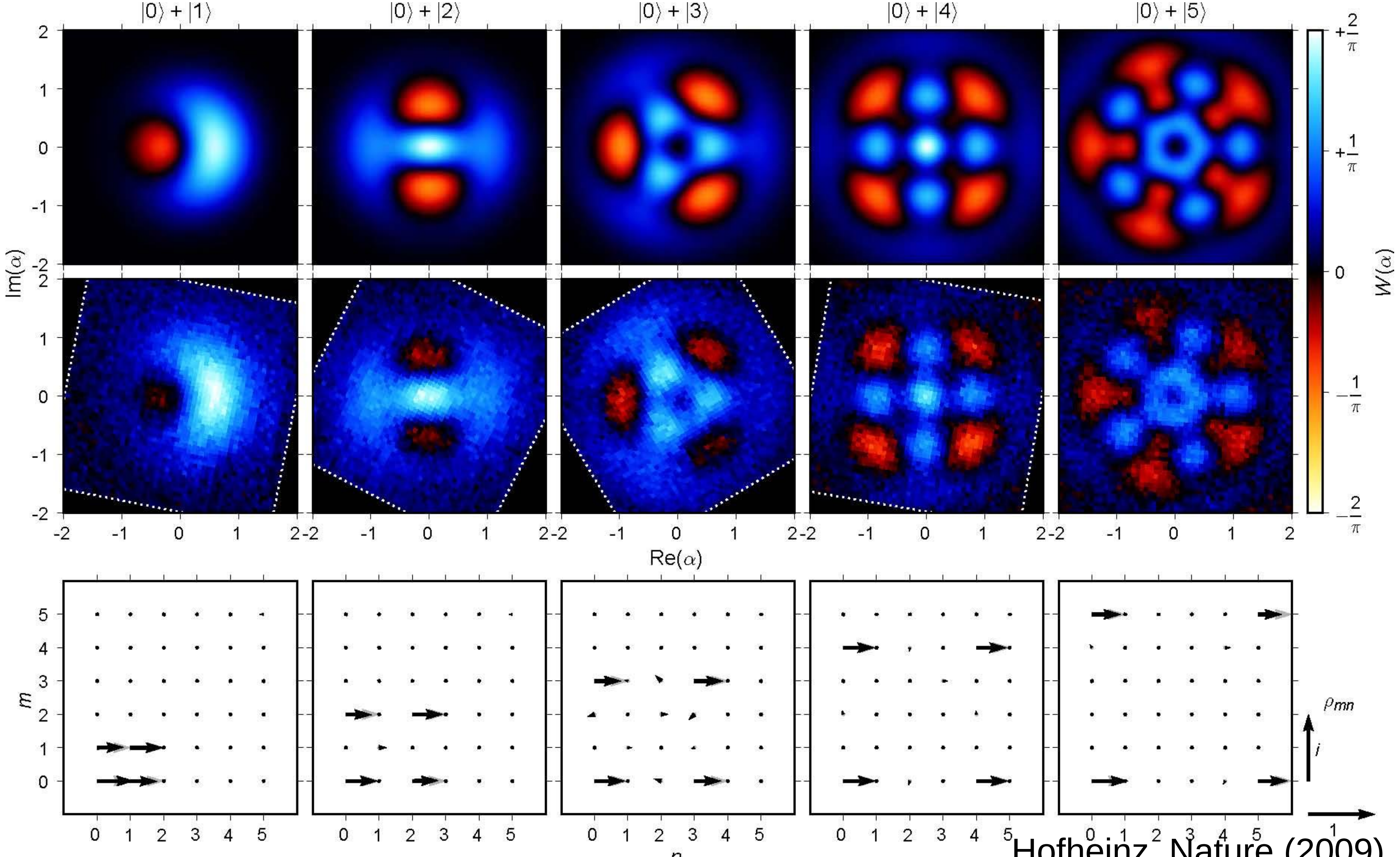

(courtesy J. Martinis)

# Wigner Tomography of Fock State $|7\rangle$



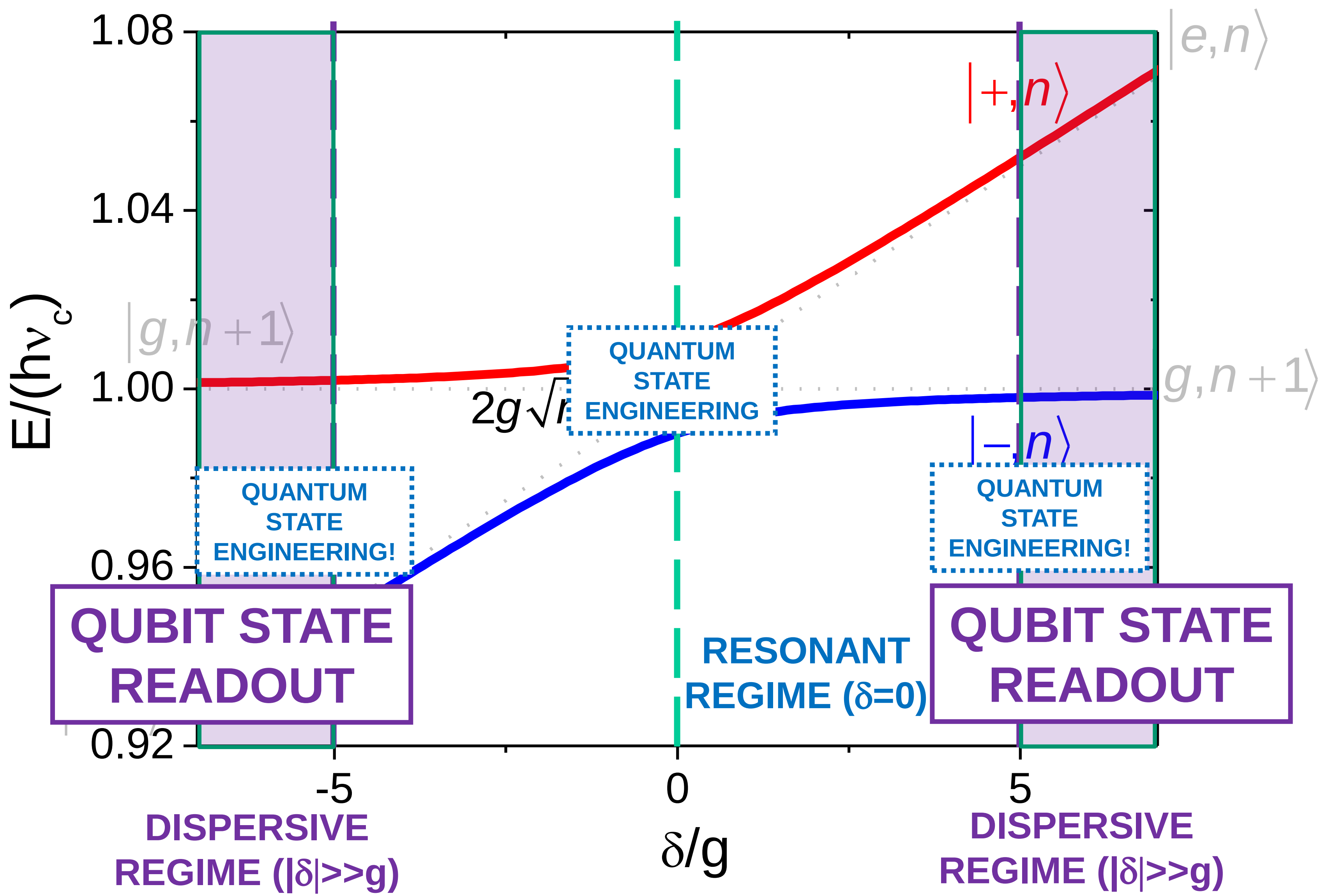

(courtesy J. Martinis)

# Wigner Tomography, $|0\rangle + |N\rangle$ states:



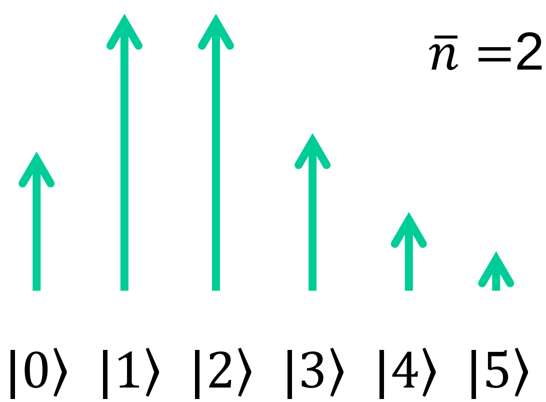

Hofheinz, Nature (2009)

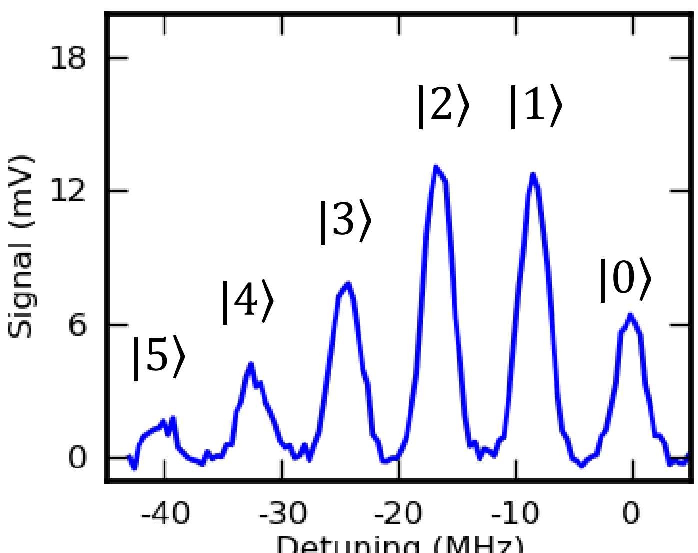
# Two Interesting Limits



# Dispersive State Manipulation

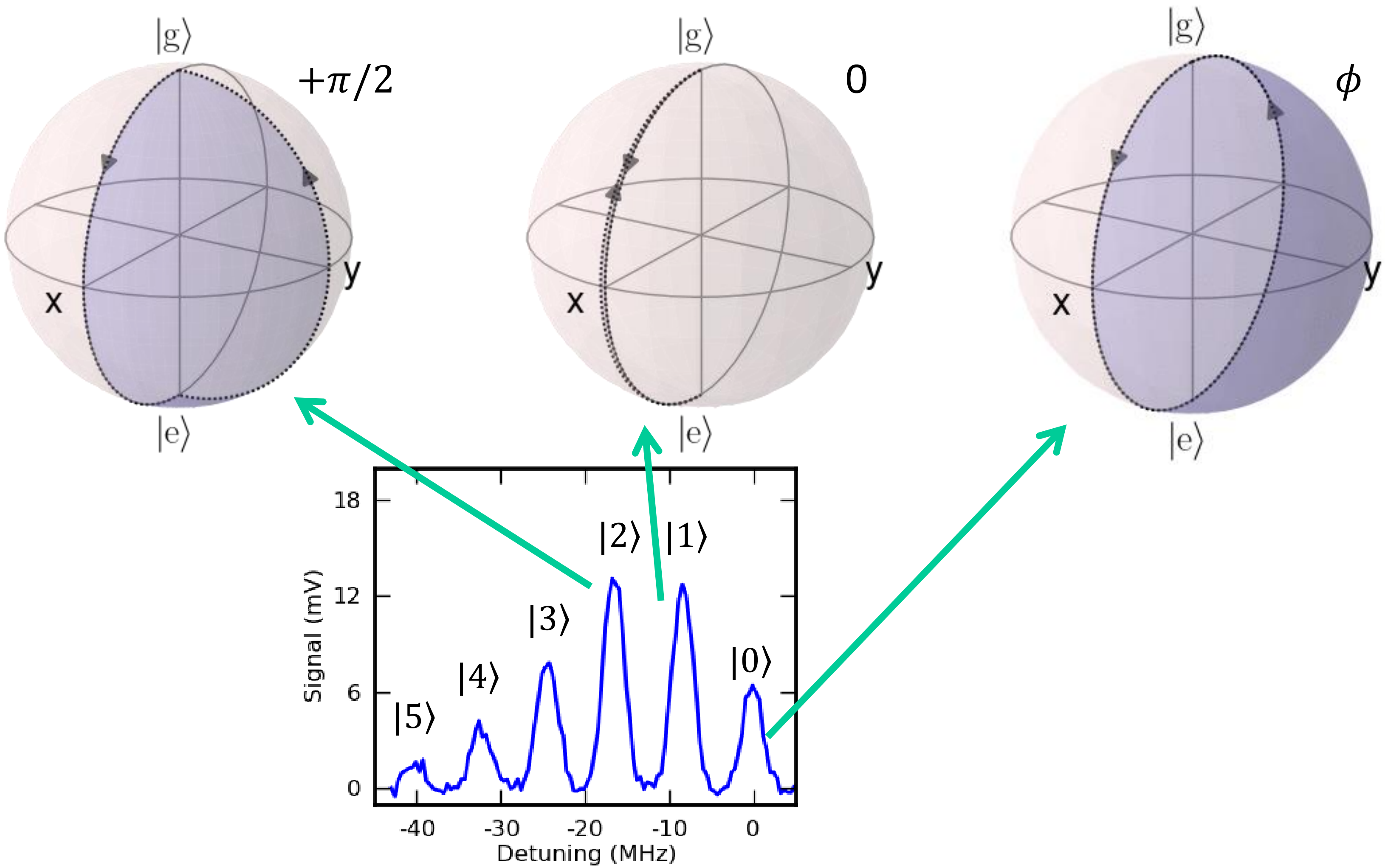
- We can "displace" the oscillator state
  - Creates coherent state from vacuum
  - Closest analog to a classical state
  - Poisson photon number distribution
  - Average photon number $\bar{n} = |\alpha|^2$
  - $|\alpha\rangle = \sum c_n |n\rangle$



# Dispersive State Manipulation

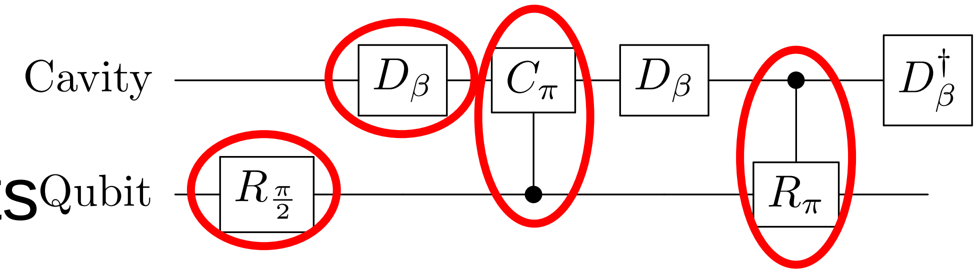
- The dispersive Hamiltonian allows *selective qubit rotations*



# Dispersive State Manipulation

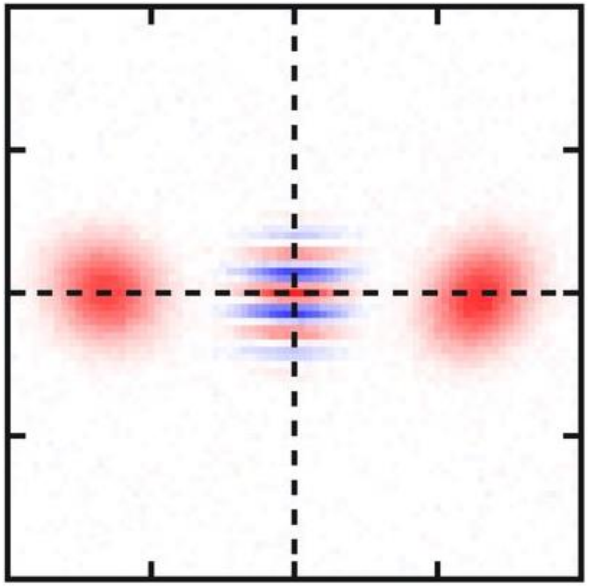
$$H = \omega_q \sigma_z + \omega_c a^\dagger a + \chi a^\dagger a \sigma_z$$

- Operation library:
  - Qubit rotations
  - Cavity displacements
  - Conditional qubit rotations
  - Conditional cavity phase




- Entangling gates: $\tau \approx \frac{1}{\chi}$
- *Theoretically* universal (using SNAP gate)

“Cat state”



Krastanov et al., PRA 2015, Heeres et al., PRL 2016

Vlastakis et al., Science 2013

# Dispersive State Manipulation

- How to account for additional terms?

$$H = \omega_q \sigma_z + \omega_c a^\dagger a + \chi a^\dagger a \sigma_z + \frac{K}{2}(a^\dagger)^2 a^2 + \frac{\chi'}{2}(a^\dagger)^2 a^2 \sigma_z$$

- How to do non-coherent state operations?
- Faster by making control-knobs more flexible?

$$H(t) = H_0 + \sum_k \varepsilon_k(t) H_k$$

$$H_{C,I} = \hat{a} + \hat{a}^\dagger$$

$$H_{C,Q} = i(\hat{a} - \hat{a}^\dagger)$$

$$H_{T,I} = \hat{b} + \hat{b}^\dagger$$

$$H_{T,Q} = i(\hat{b} - \hat{b}^\dagger)$$

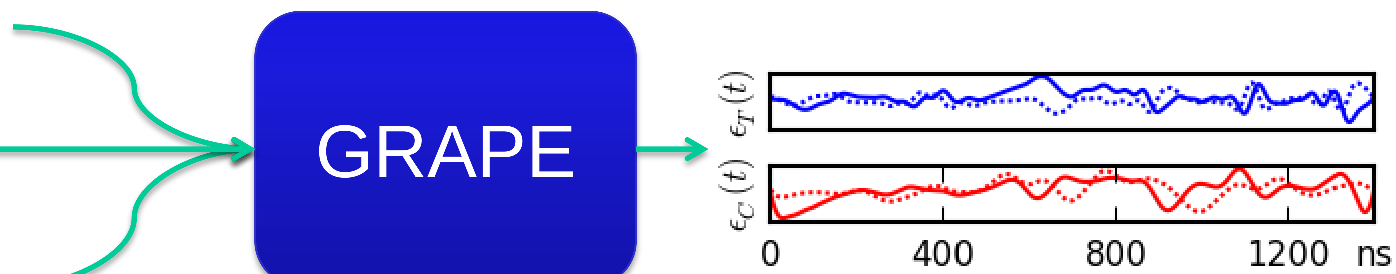
# GRadient Ascent Pulse Engineering

- Numerical search for pulse implementing desired operation

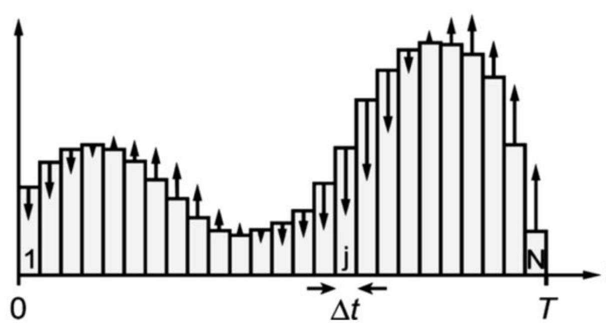
$H_0, \{H_k\}$

$t_{\mathrm{final}}, \delta t$

$F(U) = \mathrm{Tr}[U^\dagger U_{\mathrm{target}}]$




$$\frac{\partial F}{\partial \epsilon_k(t_j)}$$

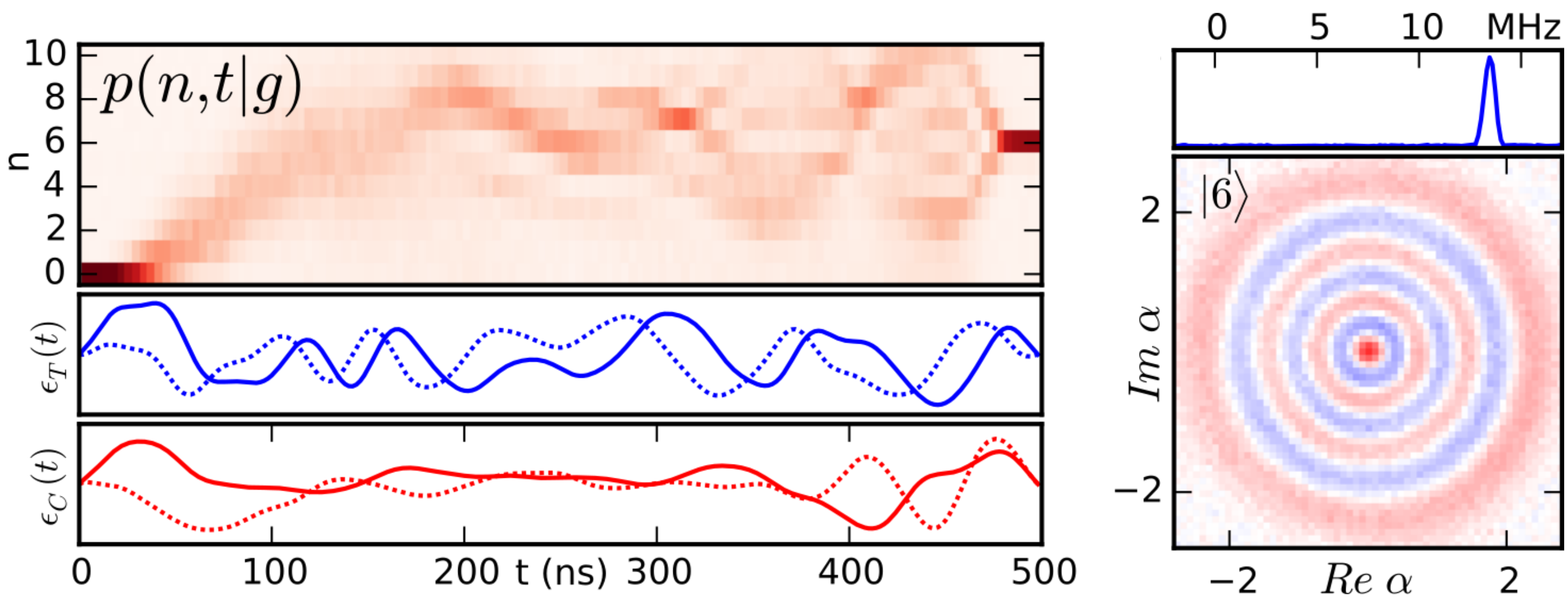



Khaneja et al., J. Mag. Res. (2005)

# Example Application

- Realize $|6\rangle\langle 0|$ to make Fock state $|6\rangle$?
  - The effect of the pulse is a unitary operation
  - We have only specified what happens to $|0\rangle$!

    (many unitaries satisfy our “request”)




Heeres et al., arXiv

# Conclusions

- Circuit Quantum Electrodynamics:
  - Allows QND readout of qubit state in the dispersive regime
  - Using parametric amplifiers high-fidelity single-shot readout is possible
  - In both the resonant and the dispersive regime oscillator control and tomography is possible