University Paris-Saclay - IQUPS

Optical Quantum Engineering: From fundamentals to applications

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- Lecture 1 (7 March, 9:15-10:45) :
  Qubits, entanglement and Bell's inequalities.

- Lecture 2 (14 March,11:00-12:30) :
  From QND measurements to quantum gates.

- Lecture 3 (21 March, 9:15-10:45) :
  Quantum optics with discrete and continuous variables.

- Lecture 4 (28 March, 11:10-12:30) :
  Quantum cryptography and optical quantum networks.

Lecture 1 :
Qubits, entanglement, Bell's inequalities.

1. Dirac's notations, two-state systems, Bloch sphere (1 qubit...)

2. Tensor product of Hilbert spaces, entanglement (2 qubits...)

3. From the EPR argument to Bell's inequalities.

4. Loophole-free Bell tests and some perspectives.

1. Quantum states, Dirac's notations, qubits.

Hilbert spaces, Dirac's notations.

The state of a quantum system is described by a vector $|\psi\rangle$ belonging to an Hilbert space $\mathcal{E}$ (complex vector space, complete and separable).

The physical quantities are described by linear and Hermitian operators $\hat{A}$ (observables) acting in $\mathcal{E}$. The measurement results correspond to (real !) eigenvalues of $\hat{A}$.

In the space $\mathcal{E}$ one defines a scalar product : $(|\phi\rangle, |\psi\rangle) = \langle\phi|\psi\rangle$ which is linear on the right side ($|\psi\rangle$), and antilinear on the left side ($|\phi\rangle$).

Dirac's notations : ket : $\lambda A|\psi\rangle \rightarrow$ bra : $\langle\psi|A^\dagger\,\lambda^* = \lambda^*\,\langle\psi|A^\dagger$

$$\text{Scalar product} = \langle \text{bra} \mid \text{ket} \rangle$$

Home exercise : Using Dirac's notations show that an Hermitian operator has real eigenvalues, and that its eigenvectors corresponding to different eigenvalues are orthogonal.

Quantum states, eigenvalues and measurement results.

Physically, giving a quantum state is equivalent to giving the values of a set of physical properties, which can be predicted with certainty, and measured repeatedly without changing the state.

Mathematically, this corresponds to the fact that a state $|\psi\rangle$ is a joint eigen-vector of a set of commuting observables $\hat{A}$, $\hat{B}$, $\hat{C}$ ... :

$$\hat{A}|\psi\rangle = a|\psi\rangle, \ \hat{B}|\psi\rangle = b|\psi\rangle, \ \hat{C}|\psi\rangle = c|\psi\rangle ...$$

The state $|\psi\rangle$ is thus equivalent to the set of eigenvalues $(a, \ b, \ , c ... \ )$.

If this set of eigenvalues specifies the state $|\psi\rangle$ in a unique way, the operators $\hat{A}$, $\hat{B}$, $\hat{C}$ ... form a Complete Set of Commuting Observables ("CoSCO"). The set of eigenvalues is often used instead of the state, by denoting :

$$|\psi\rangle = |a, \ b, \ , c ...\rangle.$$

Exemple of Hilbert spaces with dimension equal to two.

* Spin $1/2$ particle

Spin : angular momentum taking values that are half-integer multiple of $\hbar$, purely quantum origin. The Hilbert space for a spin $1/2$ has dimension two. In this space the operators for the components of the spin (angular momentum) are $\vec{S} = (S_x, S_y, S_z)$ where :

$\vec{S} = \frac{\hbar}{2}\,\vec{\sigma}$ with $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$, où $\sigma_x, \sigma_y, \sigma_z$ are Pauli's matrices :

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The usual "computational" basis is usually taken as the eigenvectors $|\pm\rangle_z$ of σ_z, so that :

$$\sigma_z|\pm\rangle_z = \pm|\pm\rangle_z.$$

Exemples of spin $1/2$ particles: electron, proton, neutron...

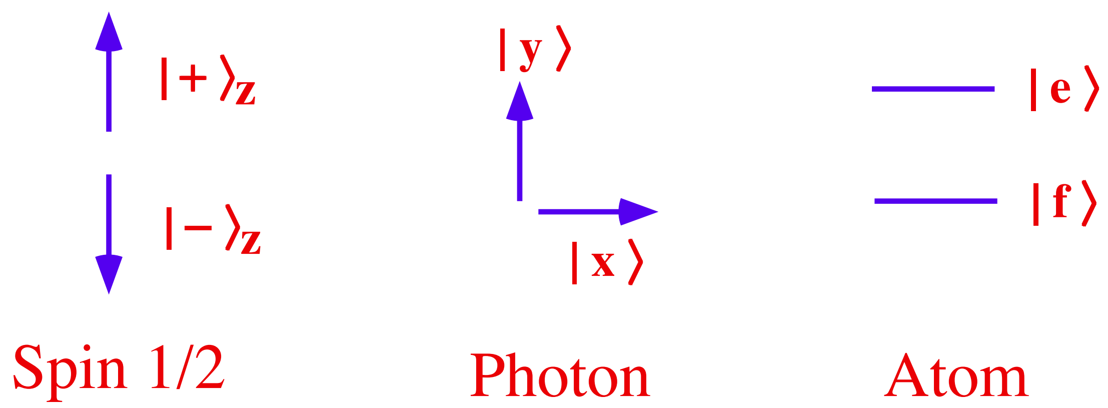
A spin is associated with a magnetic momentum : the quantum state of a spin $1/2$ particle can be controlled easily by having it rotating ("precessing") in a prescribed magnetic field.

Exemple of Hilbert spaces with dimension equal to two.

* Spin 1/2 particle

* Polarized photon : states with linear or circular polarisation; mathematical structure very close to a spin 1/2 (factor 2 on angles, see below).

* "Two-level atom" (attention ! spontaneous emission).



These systems are various implementations of a "quantum bit" (qubit).

Exemple of Hilbert spaces with dimension equal to two.

Home exercise : Let us consider a qubit with the quantum state :

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$

which is a quantum superposition of the two logical states $|0\rangle$ et $|1\rangle$.

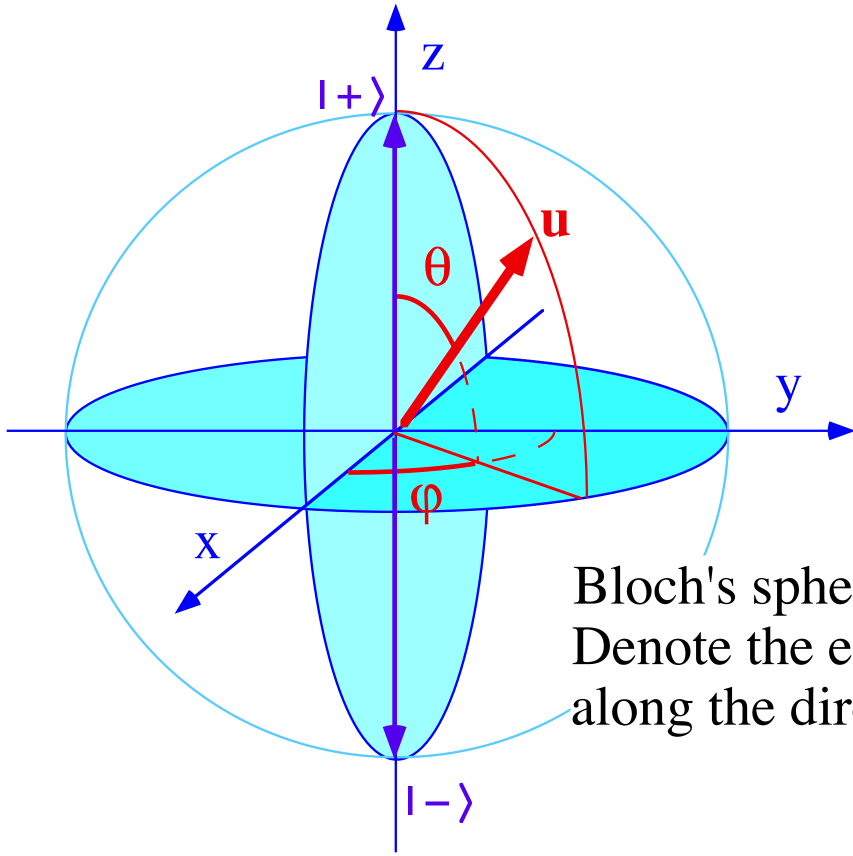
Depending on the implementation (spin 1/2, photon, atom...) one will get :

$$|1\rangle = |+\rangle_z = |x\rangle = |e\rangle...$$

$$|0\rangle = |-\rangle_z = |y\rangle = |g\rangle...$$

- Which results can be found when measuring the qubit's logical state ?
- With which probabilities?
- What is the difference between the qubit and a "random classical bit" which would have the same probabilities of finding either 0 or 1 ?

Bloch's sphere.



Bloch's sphere (spin 1/2)
Denote the eigenstate along the direction **u**(θ, φ)

Normalized vector $\vec{u}$

$u_x = \cos(\phi)\sin(\theta)$,
$u_y = \sin(\phi)\sin(\theta)$,
$u_z = \cos(\theta)$.

$$\vec{S}.\vec{u} = \frac{\hbar}{2}\,\vec{\sigma}.\vec{u}$$

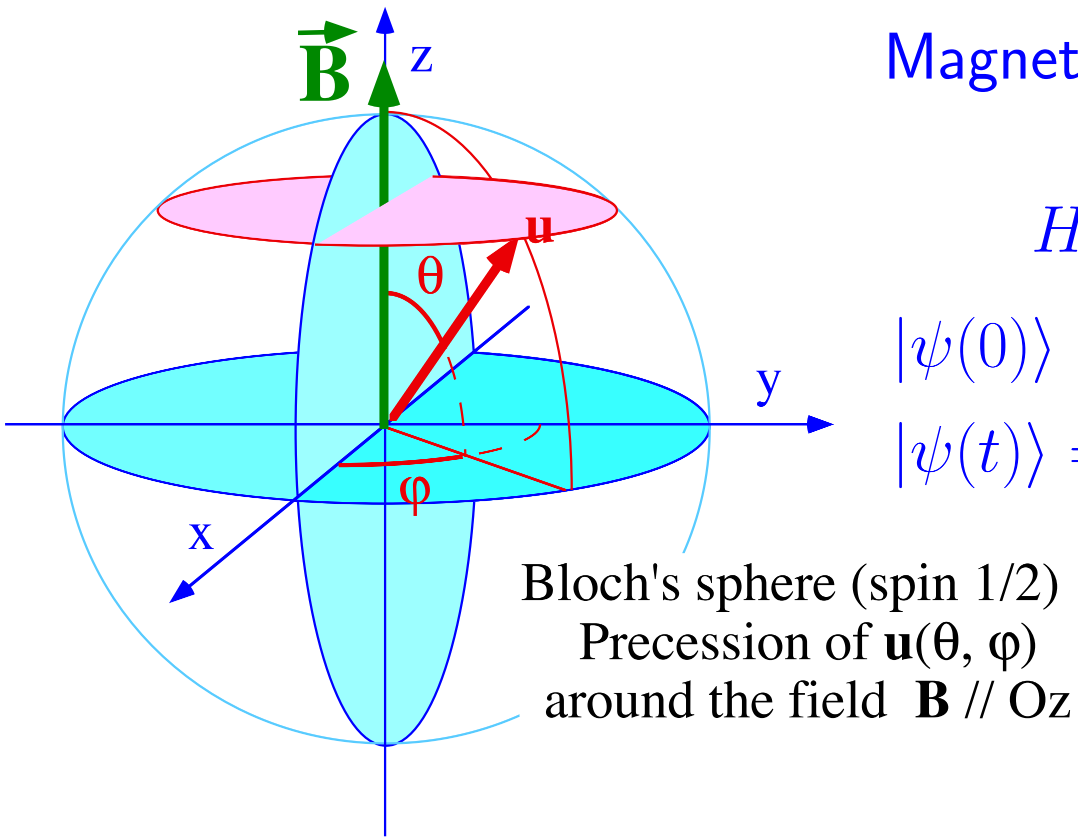
$$\vec{\sigma}.\vec{u} = \begin{pmatrix} \cos(\theta) & \sin(\theta)e^{-i\phi} \\ \sin(\theta)e^{i\phi} & -\cos(\theta) \end{pmatrix}$$

Eigenvalues of $\vec{\sigma}.\vec{u}$: ± 1, eigenstates of $\vec{S}.\vec{u}$ = eigenstates of $\vec{\sigma}.\vec{u}$:

$$|+_{\vec{u}}\rangle = \cos(\theta/2)e^{-i\phi/2}\,|+_z\rangle + \sin(\theta/2)e^{i\phi/2}\,|-_z\rangle$$

$$|-_{\vec{u}}\rangle = -\sin(\theta/2)e^{-i\phi/2}\,|+_z\rangle + \cos(\theta/2)e^{i\phi/2}\,|-_z\rangle$$

Bloch's sphere : magnetic field along Oz



Magnetic field along Oz

$$H_z = \frac{\hbar g}{2}\,\sigma_z$$

$$|\psi(0)\rangle = \alpha|+_z\rangle + \beta|-_z\rangle$$

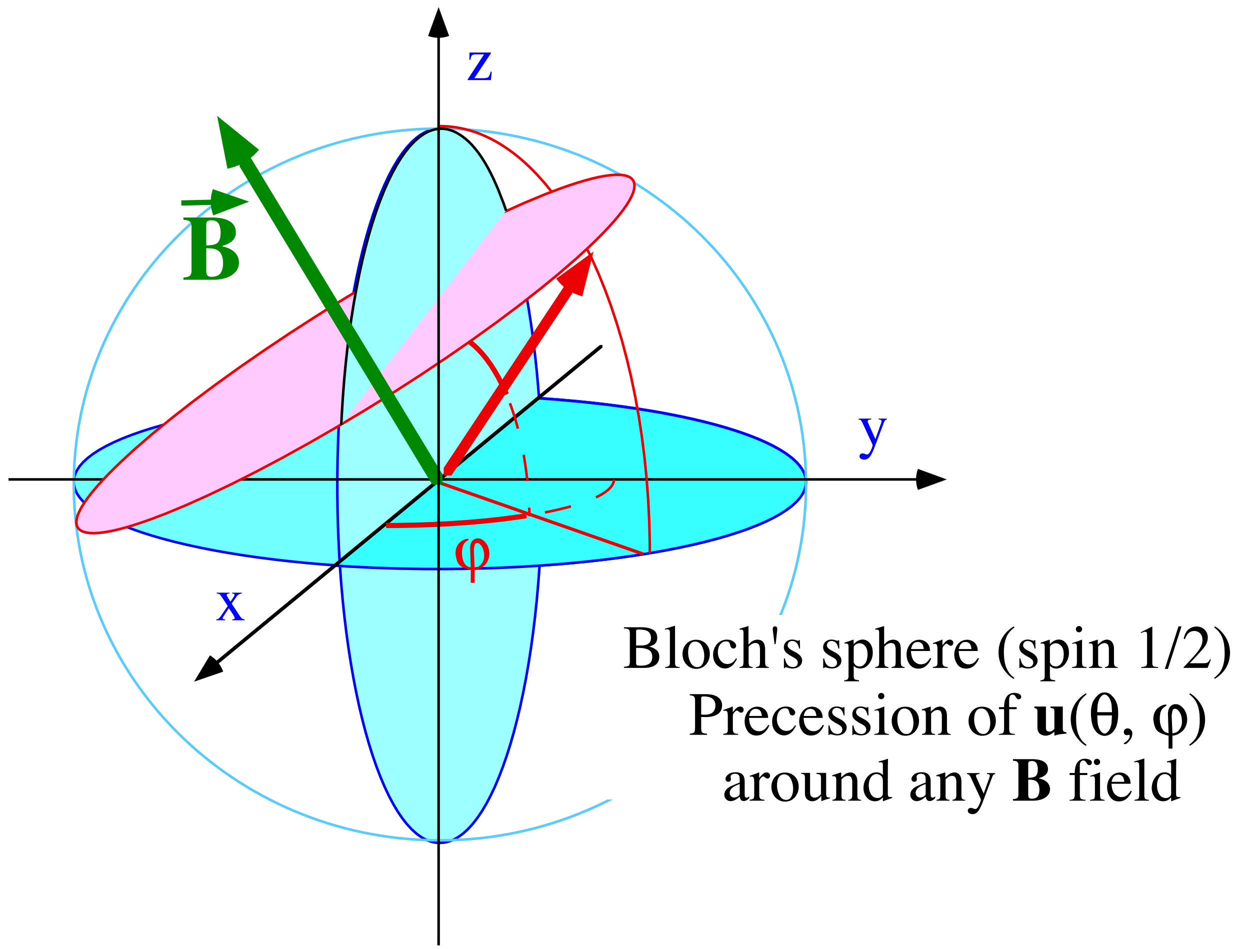
$$|\psi(t)\rangle = \alpha e^{-igt/2}|+_z\rangle + \beta e^{igt/2}|-_z\rangle$$

Evolution with time : "precession" around $\vec{B}$

$$|\psi(0)\rangle = |+_{\vec{u}}\rangle = \cos(\theta/2)e^{-i\phi/2}\,|+_z\rangle + \sin(\theta/2)e^{i\phi/2}\,|-_z\rangle$$

$$|\psi(t)\rangle = \cos(\theta/2)e^{-i(\phi+gt)/2}\,|+_z\rangle + \sin(\theta/2)e^{i(\phi+gt)/2}\,|-_z\rangle$$

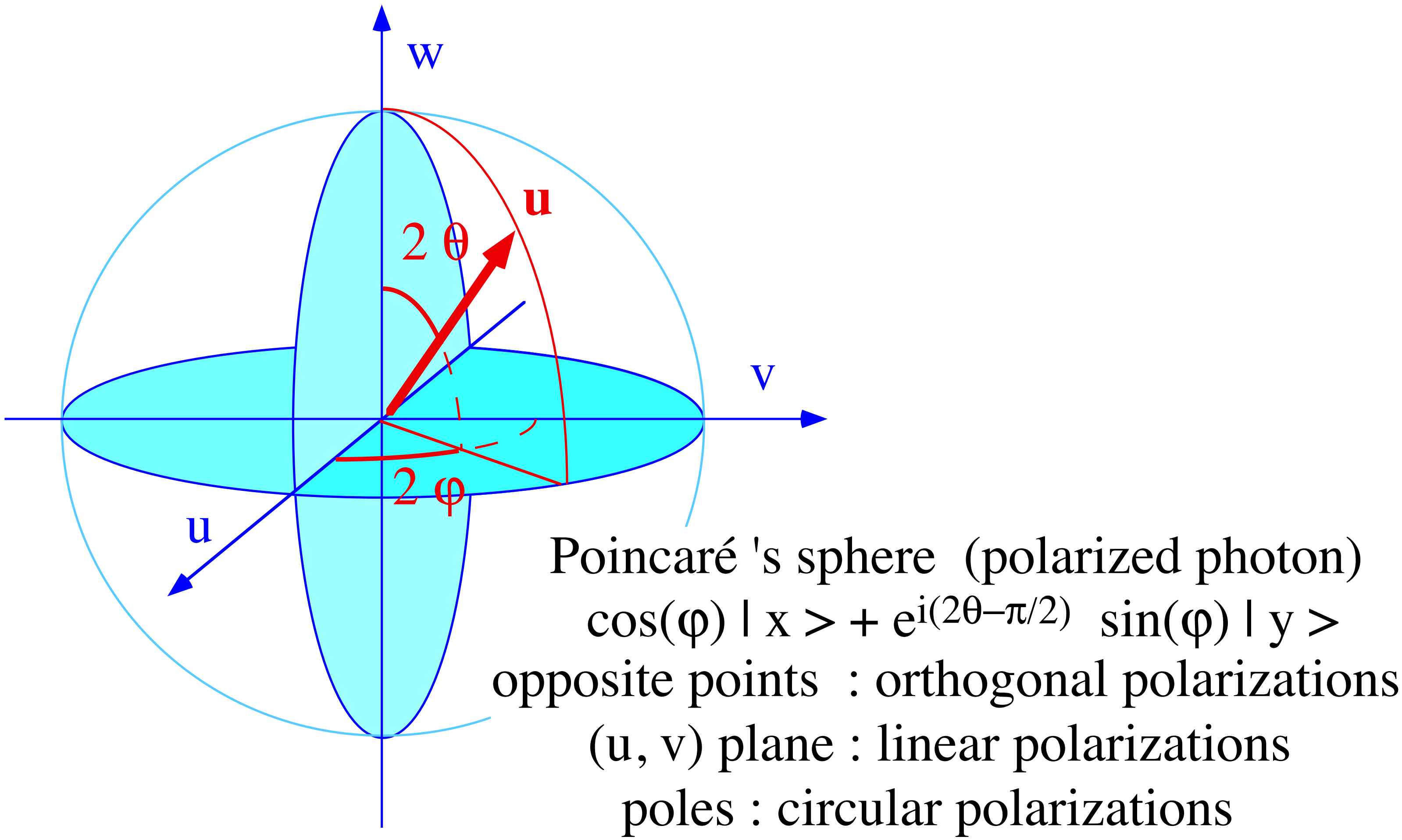
Bloch's sphere : magnetic field with arbitrary orientation



Magnetic field with arbitrary orientation : $H = \frac{\hbar g}{2}\,\vec{\sigma}.\vec{B}/B$.

Always a precession around $\vec{B}$

Poincaré's sphere



With polarized photons instead of spins 1/2, Bloch's sphere is replaced by Poincaré's sphere, with a very similar behaviour (qubits !).

2. Tensor products of Hilbert spaces.

What is the problem ?

Question : Which Hilbert space must be used
- to describe a system with several particles
- to describe a particle with several degrees of freedom (motion + spin...)
- in a general way, to "combine" 2 Hilbert spaces $\mathcal{E}_1$ and $\mathcal{E}_2$, with basis $\{|\phi_n^{(1)}\rangle,\ n = 1... \dim(\mathcal{E}_1)\}$ and $\{|\phi_p^{(2)}\rangle,\ p = 1... \dim(\mathcal{E}_2)\}$?

One may try to define the joint state, for instance for two particles, by specifying the state of each one :

$$|\psi\rangle = (\ 1\ :\ |\phi_n^{(1)}\rangle,\ 2\ :\ |\phi_p^{(2)}\rangle\ )$$

Answer : One defines the tensor product of two Hilbert spaces $\mathcal{E}_1$ and $\mathcal{E}_2$ as the space $\mathcal{E} = \mathcal{E}_1 \otimes \mathcal{E}_2$ generated by the basis obtained by combining vectors from the two initial basis: $|\phi_n^{(1)}\rangle$ et $|\phi_p^{(2)}\rangle$:

$$|\phi_{n,p}\rangle = |\phi_n^{(1)}\rangle\ \otimes\ |\phi_p^{(2)}\rangle$$

Properties of the tensor product space.

* Dimension of $\mathcal{E}$: dim($\mathcal{E}$) = dim($\mathcal{E}_1$) $\times$ dim($\mathcal{E}_2$).

* Scalar product in $\mathcal{E}$:

If $|\mu\rangle = |\mu_1\rangle \otimes |\mu_2\rangle$, and $|\phi\rangle = |\phi_1\rangle \otimes |\phi_2\rangle$ then : $\langle\mu|\phi\rangle = \langle\mu_1|\phi_1\rangle\langle\mu_2|\phi_2\rangle$.

* Operators in $\mathcal{E}$:

One defines $A = A_1 \otimes A_2$ by : $A(|\phi_{n,1}\rangle \otimes |\phi_{p,2}\rangle) = (A_1|\phi_{n,1}\rangle) \otimes (A_2|\phi_{p,2}\rangle)$

* Semi-sloppy notation, very often used :

- $A = A_1 \otimes I_2$ (very useful) is most often denoted as A_1

- $|\phi_1\rangle \otimes |\phi_2\rangle$ is often denoted as $|\phi_1\rangle|\phi_2\rangle$ or $|\phi_1,\ \phi_2\rangle$

Entanglement

* Let us consider two vectors in the spaces $\mathcal{E}_1$ and $\mathcal{E}_2$: $|\psi_1\rangle = \sum_n a_n |\phi_n^{(1)}\rangle$ and $|\psi_2\rangle = \sum_p b_p |\phi_p^{(2)}\rangle$. Then :

$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle = \sum_{n,p} a_n \, b_p \, |\phi_n^{(1)}\rangle \otimes |\phi_p^{(2)}\rangle$$

is obviously a vector in $\mathcal{E}$.

* **The reciprocal is wrong :** there are "non factorisable" vectors in $\mathcal{E}$, which cannot be written as a tensor product $|\psi_1\rangle \otimes |\psi_2\rangle$.

Demonstration :

$$|\psi\rangle = \sum_{n,p} c_{n,p} \, |\phi_n^{(1)}\rangle \otimes |\phi_p^{(2)}\rangle \quad \neq \quad |\psi_1\rangle \otimes |\psi_2\rangle = \sum_{n,p} a_n \, b_p \, |\phi_n^{(1)}\rangle \otimes |\phi_p^{(2)}\rangle$$

It is generally impossible to find N_1 coefficients a_n and N_2 coefficients b_p so that the $N_1 \times N_2$ equations $c_{n,p} = a_n b_p$ are all obeyed, because there are $(N_1 + N_2)$ unknown and $N_1 \times N_2$ equations.

In that case one says that the state $|\psi\rangle$ is "entangled".

Exemples of entangled states

* Spin 1/2 particle moving in space : $\mathcal{E} = \mathcal{E}_{\mathbf{r}} \otimes \mathcal{E}_{\mathbf{s}}$

Basis $\{|\mathbf{r}\rangle \otimes |\epsilon\rangle, \mathbf{r} = (x, y, z), \epsilon = \pm 1\}$ with $S_z|\epsilon\rangle = \epsilon\, \hbar/2\, |\epsilon\rangle$.
Wave function $\psi_\epsilon(\mathbf{r}) = (\ \psi_+(\mathbf{r}), \psi_-(\mathbf{r})\ )$ (often called a “spinor”).

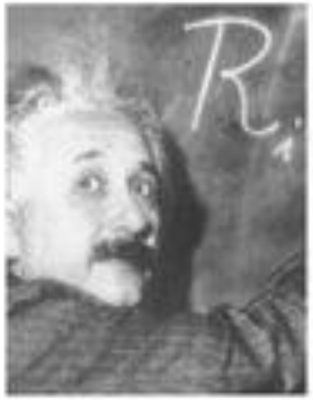
* Two spins 1/2 particles (= two qubits)

The states are denoted in the basis $\{\ |+\rangle,\ |-\rangle\}$ of eigenstates along z.

- Give a basis of the tensor product space.
- Choose a new basis where each vector is either symmetrical or antisymmetrical by exchanging the two spins.
- Show that there are three symmetrical states (“triplet” states) and one antisymmetrical state (“singlet” state).
- Show that the singlet state is entangled.

3. From the Einstein-Podolsky-Rosen argument to Bell's inequalities.

What was their worry ?



A. Einstein

B. Podolsky

N. Rosen

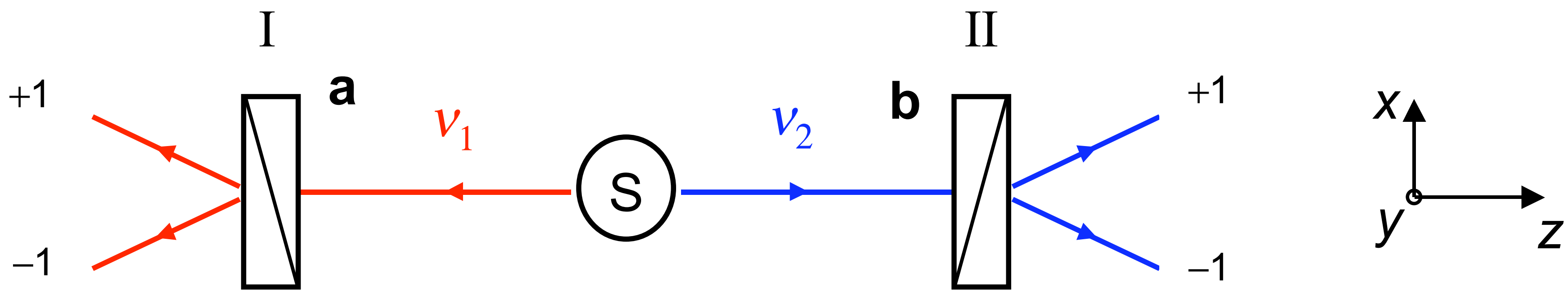
* Quantum physics has a non-deterministic character !

* But is it possible to explain the probabilistic character of quantum predictions by invoking a supplementary underlying level of description (supplementary parameters, hidden variables...) ?

* It was the conclusion of the Einstein-Podolsky-Rosen reasoning (1935), but Bohr strongly opposed, saying that quantum mechanics tells everything, and there is "nothing more" to be looked for.

* Bell's theorem (1964) allowed experiments to enter the debate.

The Einstein-Podolsky-Rosen GedankenExperiment with photons correlated in polarization



Measurement of the polarization de ν_1 along orientation **a** and of the polarization de ν_2 along orientation **b** : results +1 or –1

Probabilities to find +1 or –1 for ν_1 (measured along **a**) and +1 or –1 for ν_2 (measured along **b**).

Single probabilities	Joint probabilities
$P_+(\mathbf{a})$, $P_-(\mathbf{a})$	$P_{++}(\mathbf{a},\mathbf{b})$, $P_{+-}(\mathbf{a},\mathbf{b})$
$P_+(\mathbf{b})$, $P_-(\mathbf{b})$	$P_{-+}(\mathbf{a},\mathbf{b})$, $P_{--}(\mathbf{a},\mathbf{b})$

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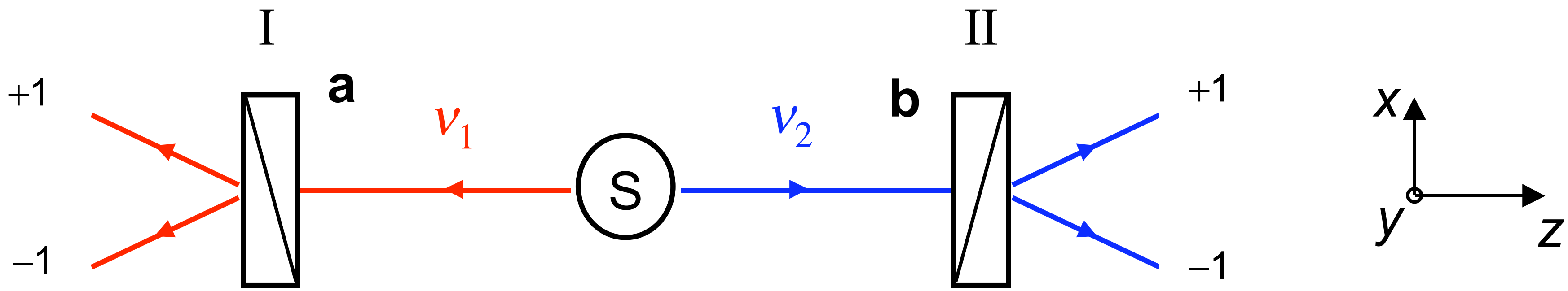


For the entangled EPR state… $\left|\Psi(\nu_1,\nu_2)\right\rangle = \frac{1}{\sqrt{2}}\left\{\left|x,x\right\rangle + \left|y,y\right\rangle\right\}$

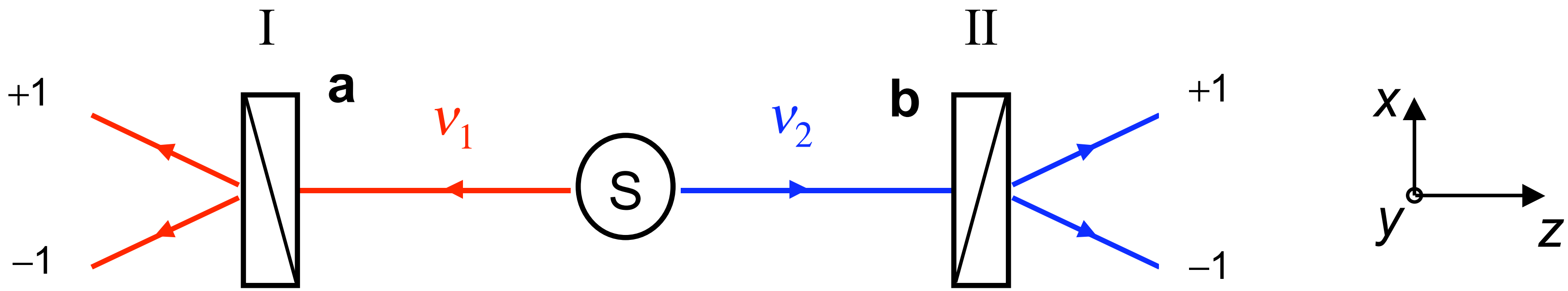
Quantum mechanical predicted results are separately random... $P_+(\mathbf{a}) = P_-(\mathbf{a}) = \frac{1}{2}$; $P_+(\mathbf{b}) = P_-(\mathbf{b}) = \frac{1}{2}$

... but strongly correlated :

$$P_{++}(\mathbf{a},\mathbf{b}) = P_{--}(\mathbf{a},\mathbf{b}) = \frac{1}{2}\cos^2(\mathbf{a},\mathbf{b})$$

$$P_{+-}(\mathbf{a},\mathbf{b}) = P_{-+}(\mathbf{a},\mathbf{b}) = \frac{1}{2}\sin^2(\mathbf{a},\mathbf{b})$$

The Einstein-Podolsky-Rosen GedankenExperiment with photons correlated in polarization



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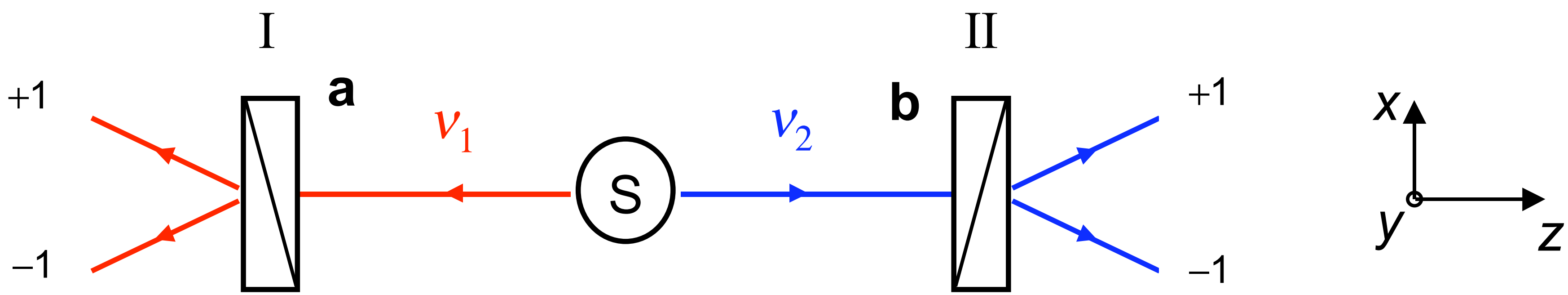
if $\mathbf{a}=\mathbf{b}$, and if one gets $+1$ for ν_1, then one gets with certainty $+1$ also for ν_2 (i.e. -1 for ν_2 never happens)

$$\mathbf{a}=\mathbf{b} \quad \Rightarrow \quad P_{++} = P_{--} = \frac{1}{2}$$

$$P_{+-} = P_{-+} = 0$$

Correlation coefficient for polarization results



The correlation coefficient E measures the correlations between the results obtained from the measurements I and II :

$$E = P_{++} + P_{--} - P_{+-} - P_{-+} = P(\text{same results}) - P(\text{different results})$$

QM predictions for parallel polarizers

$$P_{++} = P_{--} = \frac{1}{2} \qquad \Rightarrow \quad E_{\text{MQ}} = 1$$

$$P_{+-} = P_{-+} = 0$$

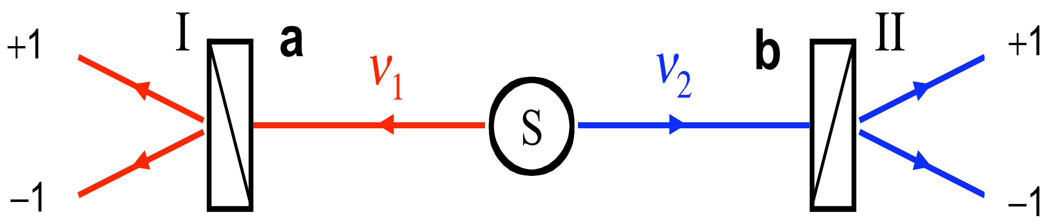
Full correlation

More generally, for an arbitrary angle (a,b) between the polarizers

$$E_{\text{MQ}}(\mathbf{a},\mathbf{b}) = \cos 2(\mathbf{a},\mathbf{b})$$

NB: $E_{\text{MQ}}(\mathbf{a},\mathbf{a}) = 1$

1964: Bell's theorem



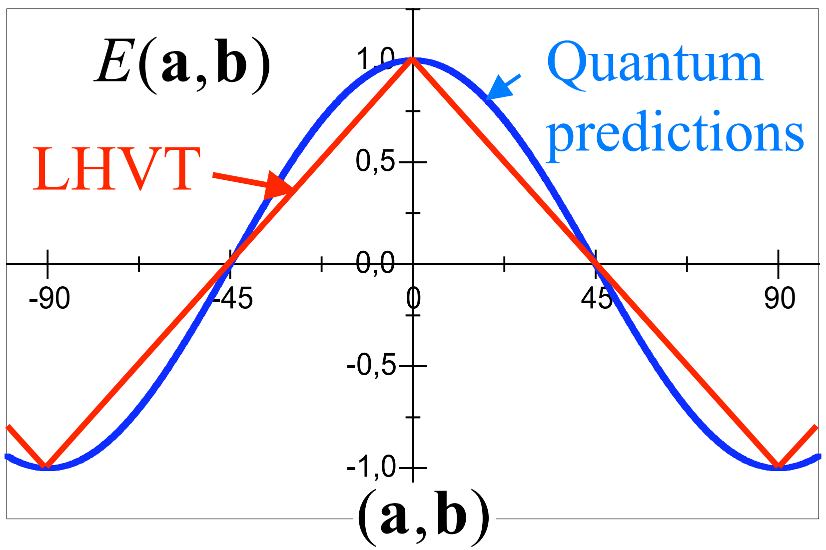
Example of LHVT

- Common direction of polarisation λ , different for each pair : $\rho(\lambda) = 1/2\pi$
- Result (± 1) depends on the angle between λ and polarizer orientation (**a** or **b**)

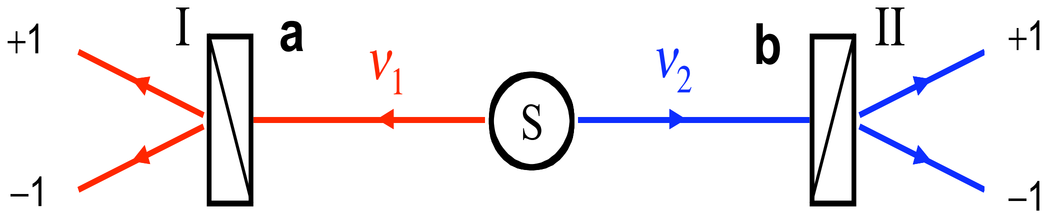
$$A(\lambda, \mathbf{a}) = \operatorname{sign}\left\{\cos 2(\theta_{\mathrm{a}} - \lambda)\right\}$$

$$B(\lambda, \mathbf{b}) = \operatorname{sign}\left\{\cos 2(\theta_{\mathrm{b}} - \lambda)\right\}$$

Not bad for a first try, but…



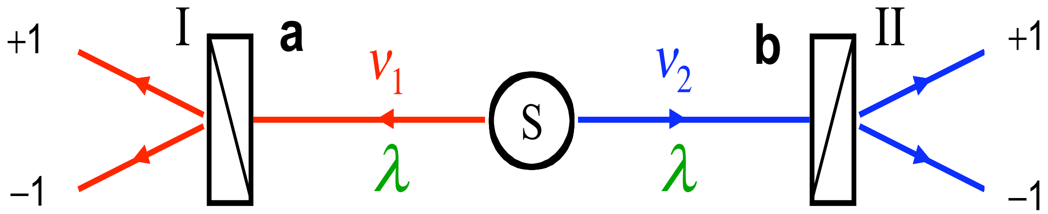
1964: Bell's theorem



No local hidden variable theory (in the spirit of Einstein's ideas) can reproduce quantum mechanical predictions for EPR correlations for **all** orientations of the polarizers.

So let us consider two measurements on each side, and evaluate the four correlation functions E(a,b), E(a,b'), E(a',b) , E(a',b') ...

1964: Bell's theorem



Consider local supplementary parameters theories (in the spirit of Einstein's ideas on EPR correlations):

- The two photons of a same pair have a common property λ

- The value of λ determines the measurements results at I and II $\Leftrightarrow$ $A(\lambda, \mathbf{a}) = +1 \text{ or } -1$ at polarizer I; $B(\lambda, \mathbf{b}) = +1 \text{ or } -1$ at polarizer II

- The values of λ are randomly distributed at the source $\Leftrightarrow$ $\rho(\lambda) \geq 0$ and $\int \rho(\lambda)\, d\lambda = 1$

Then look at the coefficient E, which measures the correlations between the results obtained from the measurements I and II :

$$E = P_{++} + P_{--} - P_{+-} - P_{-+} = P(\text{same results}) - P(\text{different results})$$

Bell-CHSH inequalities

Any local hidden variables theory $\Rightarrow$ Bell's inequalities

$$-2 \leq S \leq 2 \quad \text{with} \quad S = E(\mathbf{a},\mathbf{b}) - E(\mathbf{a},\mathbf{b}') + E(\mathbf{a}',\mathbf{b}) + E(\mathbf{a}',\mathbf{b}')$$

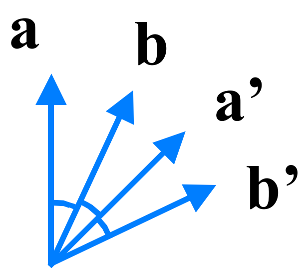
CHSH inequalities (Clauser, Horne, Shimony, Holt, 1969)

Dem : $A(\lambda,\mathbf{a})\, B(\lambda,\mathbf{b}) - A(\lambda,\mathbf{a})\, B(\lambda,\mathbf{b}') + A(\lambda,\mathbf{a}')\, B(\lambda,\mathbf{b}) + A(\lambda,\mathbf{a}')\, B(\lambda,\mathbf{b}') = \pm 2$

Quantum mechanics $\quad E_{\text{MQ}}(\mathbf{a},\mathbf{b}) = \cos 2(\mathbf{a},\mathbf{b})$

for the orientations $\quad (\mathbf{a},\mathbf{b}) = (\mathbf{b},\mathbf{a}') = (\mathbf{a}',\mathbf{b}) = \dfrac{\pi}{8}$

$$S_{\text{QM}} = 2\sqrt{2} = 2.828\ldots > 2$$



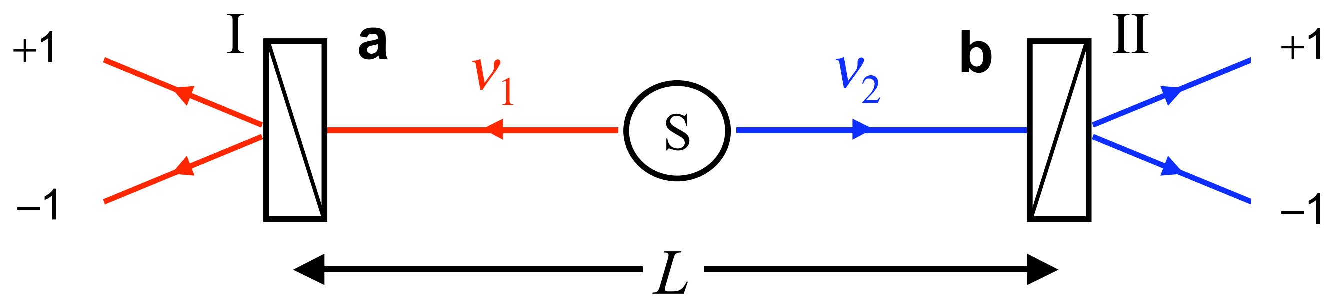
CONFLICT ! The possibility to complete quantum mechanics is no longer a matter of taste (of interpretation). It has turned into an experimental question.

Bell's locality condition

$$\boxed{A(\lambda,\mathbf{a},\cancel{\mathbf{b}}) \qquad B(\lambda,\cancel{\mathbf{a}},\mathbf{b}) \qquad \rho(\lambda,\cancel{\mathbf{a}},\cancel{\mathbf{b}})}$$

can be stated as a reasonable hypothesis, but…

…in an experiment with variable polarizers (orientations modified faster than the propagation time L/c of light between polarizers) Bell's locality condition becomes a consequence of Einstein's relativistic causality (no faster than light influence)



Conflict between quantum mechanics and
Einstein's world view (local realism based on relativity)
... but one must carry out specifically designed experiments

Four generations of experiments

Pioneers (1972-76): Berkeley, Harvard, Texas A&M

- Convenient inequalities: CHSH (Clauser Horne Shimony Holt)
- First results contradictory (Clauser = QM; Pipkin ≠ QM), but clear trend in favour of Quantum mechanics (Clauser, Fry)
- Significantly different from the ideal scheme

Experiments at Institut d'Optique (1977-82)

- A source of entangled photons of unprecedented efficiency
- Schemes closer and closer to the ideal GedankenExperiment
- Test of quantum non locality (relativistic separation)

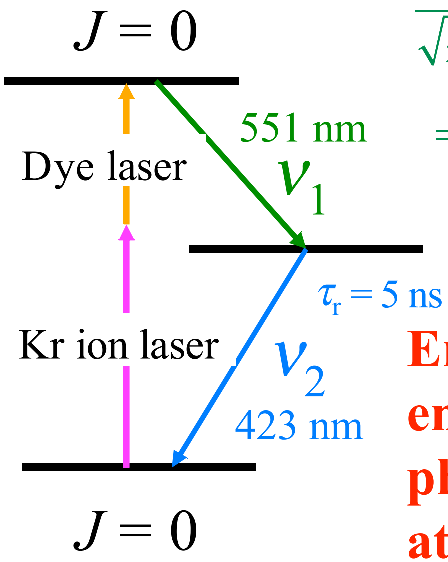
Third generation experiments (1988-2014, many places...)

- New sources of entangled pairs
- Separate closure of loopholes (improved locality, detection..)
- Entanglement at a large distance... towards Q. communications

Fourth generation experiments (2015 - ...)

- Simultaneous closure of all loopholes (nearly ideal expts)

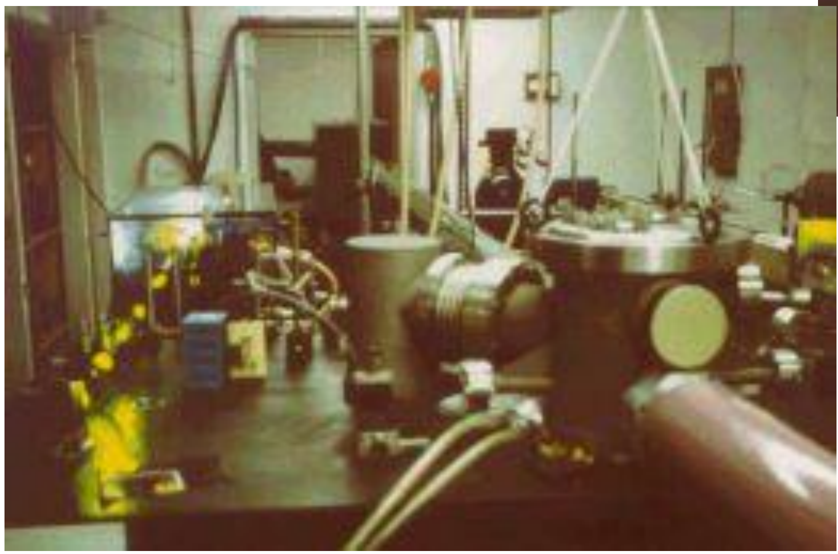
Orsay's source of pairs of entangled photons (1980-82)



$$\frac{1}{\sqrt{2}}\left\{\left|\sigma_+,\sigma_-\right\rangle + \left|\sigma_-,\sigma_+\right\rangle\right\}$$

$$= \frac{1}{\sqrt{2}}\left\{\left|x,x\right\rangle + \left|y,y\right\rangle\right\}$$

Emission of two entangled photons by an atomic cascade.



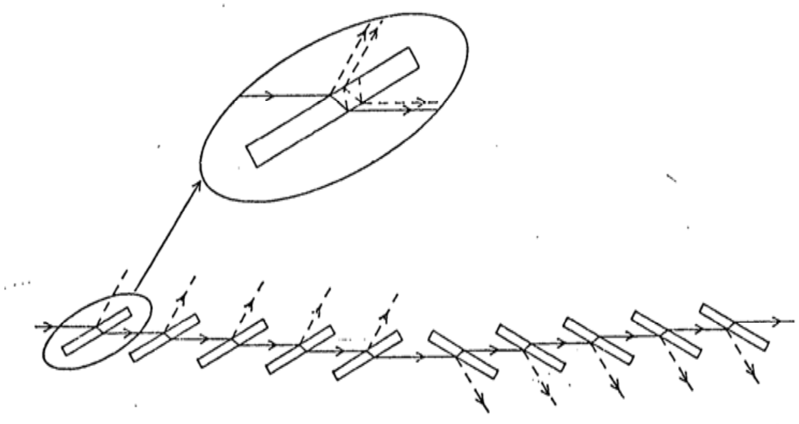
* Laser-induced two-photon excitation of a cascade in a Calcium 40 atomic beam.

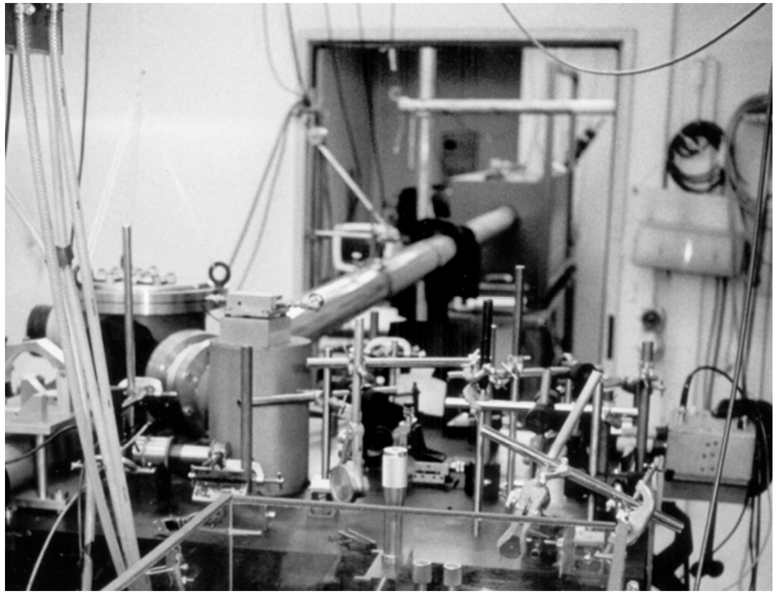
☺ 100 detected pairs per second

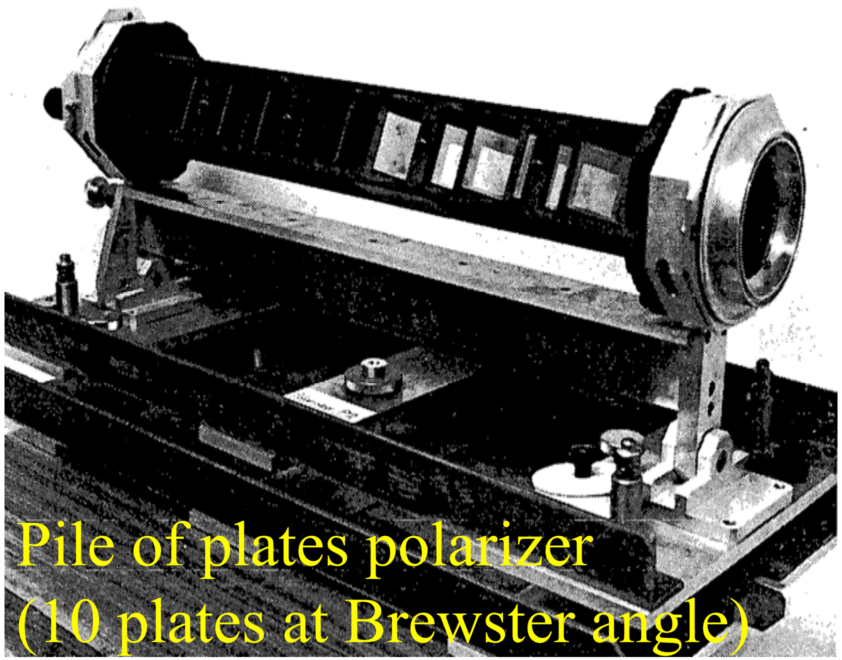
1% precision for 100 s counting

Experiment with one-way polarizers

(A. Aspect, P. Grangier, G. Roger, 1981)



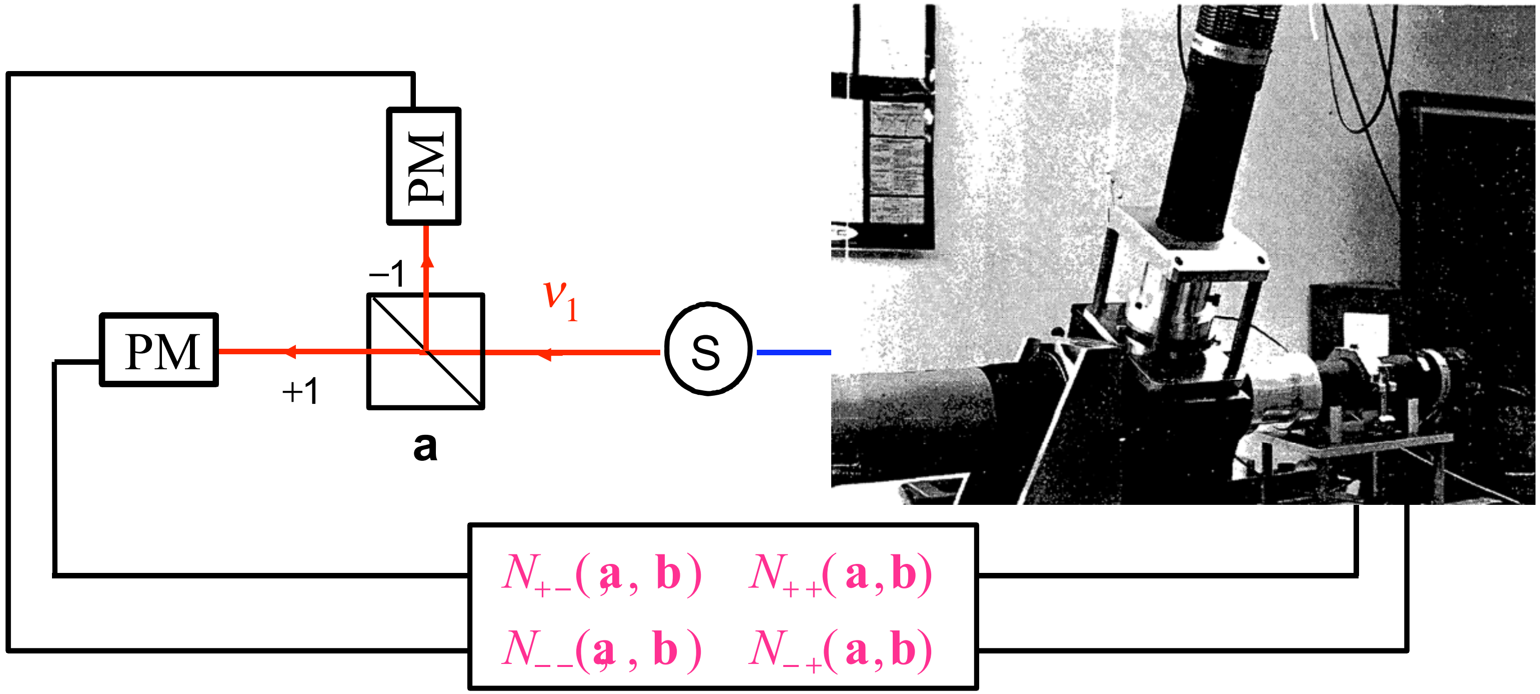




* Same idea as J.F. Clauser's 1972 experiment, but laser excitation : much shorter integration time.
* Violation of Bell's inequalities with polarizers 6 m away from the source : entanglement survives at large distance.

Experiment with 2-channel polarizers

(A. Aspect, P. Grangier, G. Roger, 1982)



Direct measurement of the polarization correlation coefficient:
simultaneous measurement of the 4 coincidence rates

$$E(\mathbf{a},\mathbf{b}) = \frac{N_{++}(\mathbf{a},\mathbf{b}) - N_{+-}(\mathbf{a},\mathbf{b}) - N_{-+}(\mathbf{a},\mathbf{b}) + N_{--}(\mathbf{a},\mathbf{b})}{N_{++}(\mathbf{a},\mathbf{b}) + N_{+-}(\mathbf{a},\mathbf{b}) + N_{-+}(\mathbf{a},\mathbf{b}) + N_{--}(\mathbf{a},\mathbf{b})}$$

Smuggling polarizers from Amsterdam...

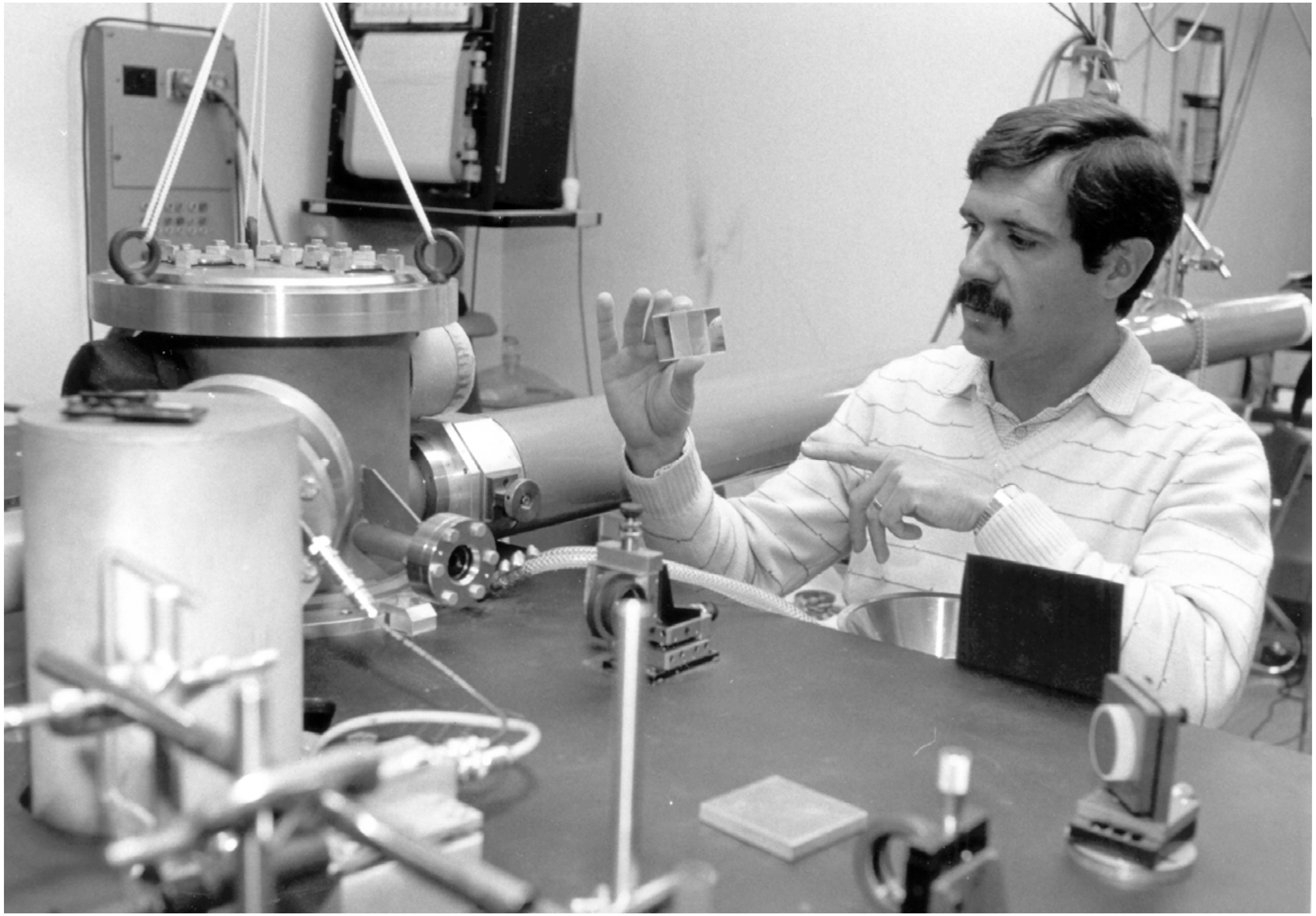
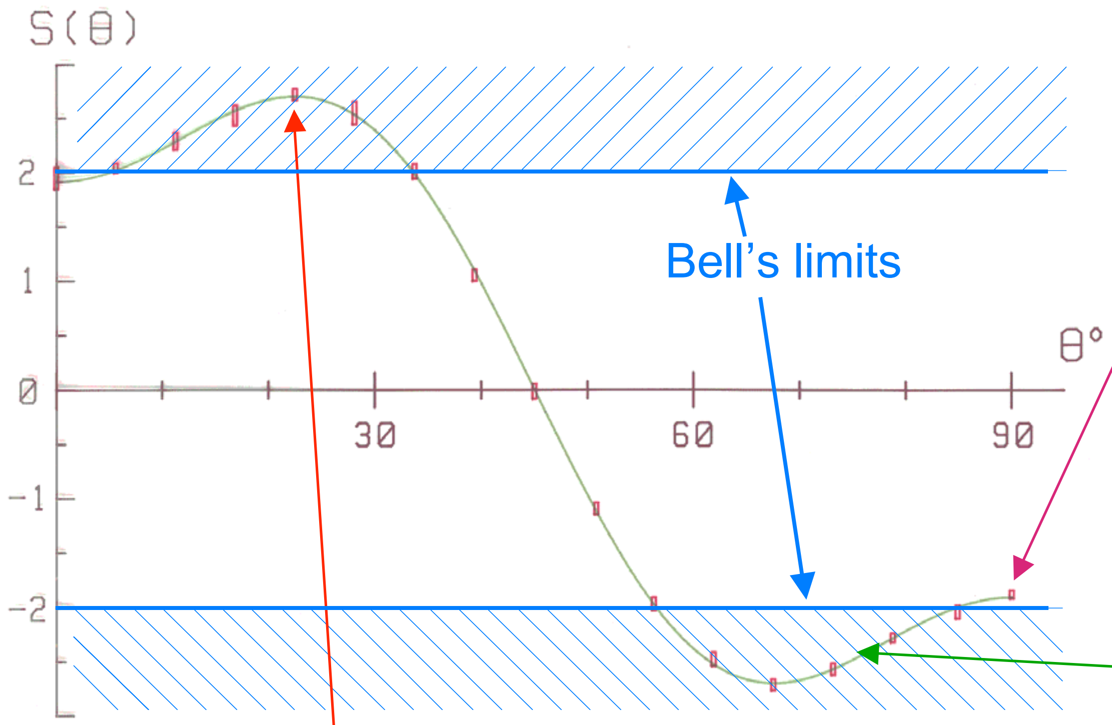


Photo by Lars Becker-Larsen

Experiment with 2-channel polarizers

(A. Aspect, P. Grangier, G. Roger, 1981)



▯ Measured value ± 2 standard dev.

— Quantum mechanical prediction (including imperfections of real experiment)

For $\theta = (\mathbf{a},\mathbf{b}) = (\mathbf{b},\mathbf{a}') = (\mathbf{a}',\mathbf{b}) = 22.5°$ $\quad S_{\text{exp}}(\theta) = 2.697 \pm 0.015$

Violation of Bell's inequalities $S \leq 2$ by more than $40\ \sigma$

Excellent agreement with quantum predictions $S_{MQ} = 2.70$

Experiment with variable polarizers

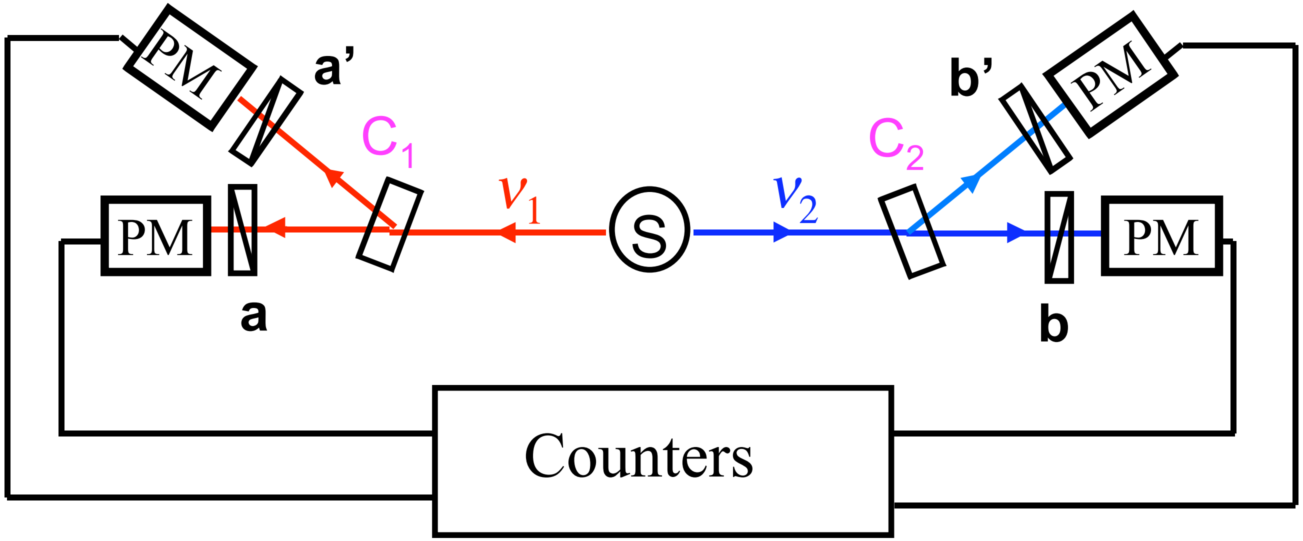
(A.A. 1976; A. Aspect, J. Dalibard, G. Roger, 1982)

Bell's locality hypothesis : $\boxed{A(\lambda,\mathbf{a},\cancel{\mathbf{b}}) \quad B(\lambda,\cancel{\mathbf{a}},\mathbf{b}) \quad \rho(\lambda,\cancel{\mathbf{a}},\cancel{\mathbf{b}})}$

In an experiment with variable polarizers (switch faster than L / c), results from relativistic causality (no faster than light influence) !

☹ Not possible with massive polarizer

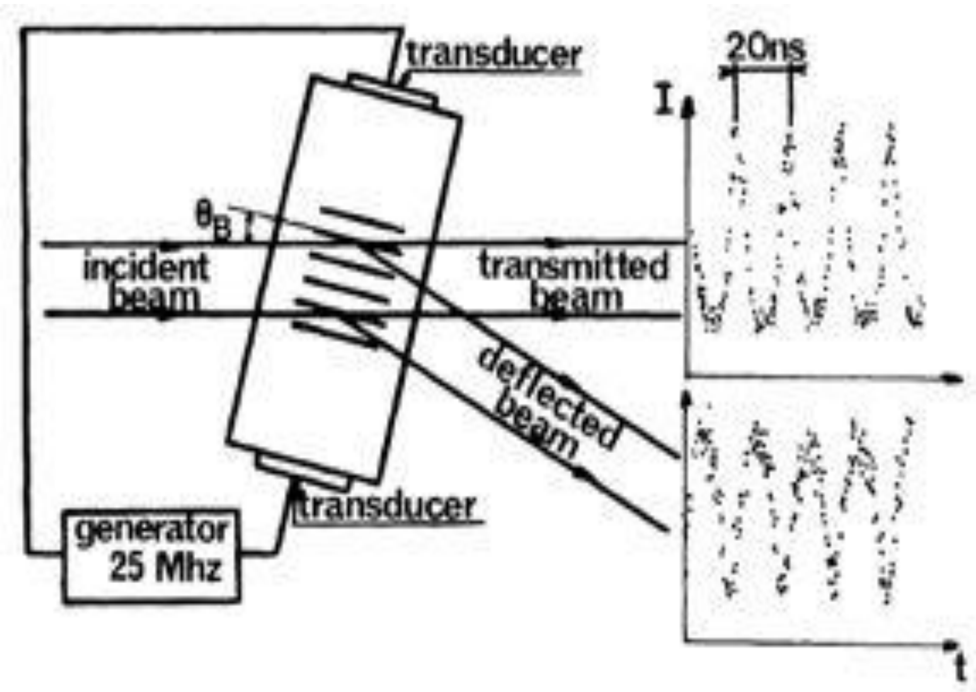
☺ Possible with optical switches



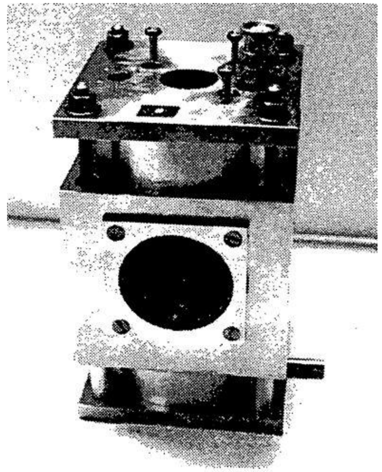
Switches C_1 and C_2 redirects light
- either towards polarizer **a** or **a'**
- either towards polarizer **b** or **b'**

Equivalent to polarizers switching on both sides !

Experiment with variable polarizers

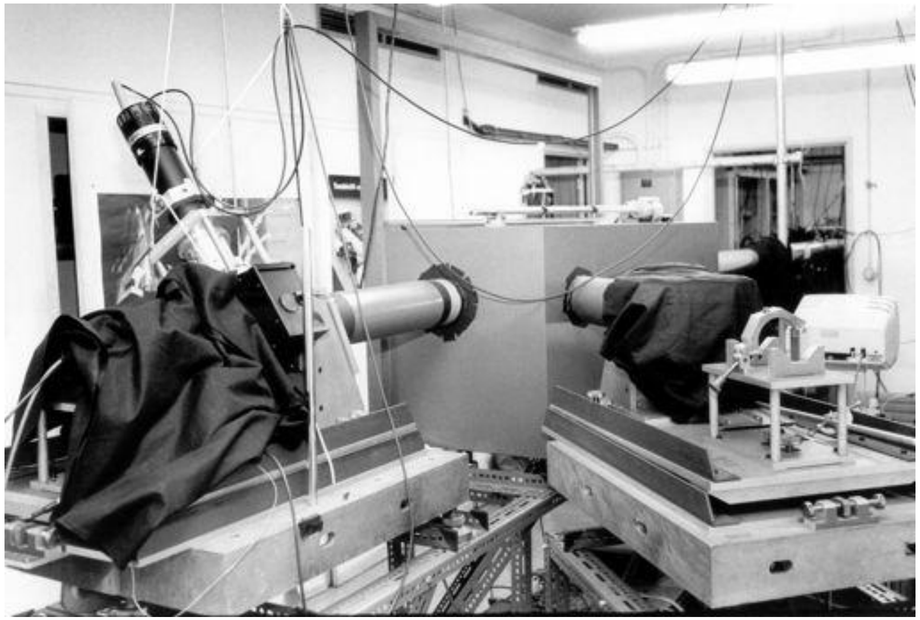


Optical switches



in action....
from the front...

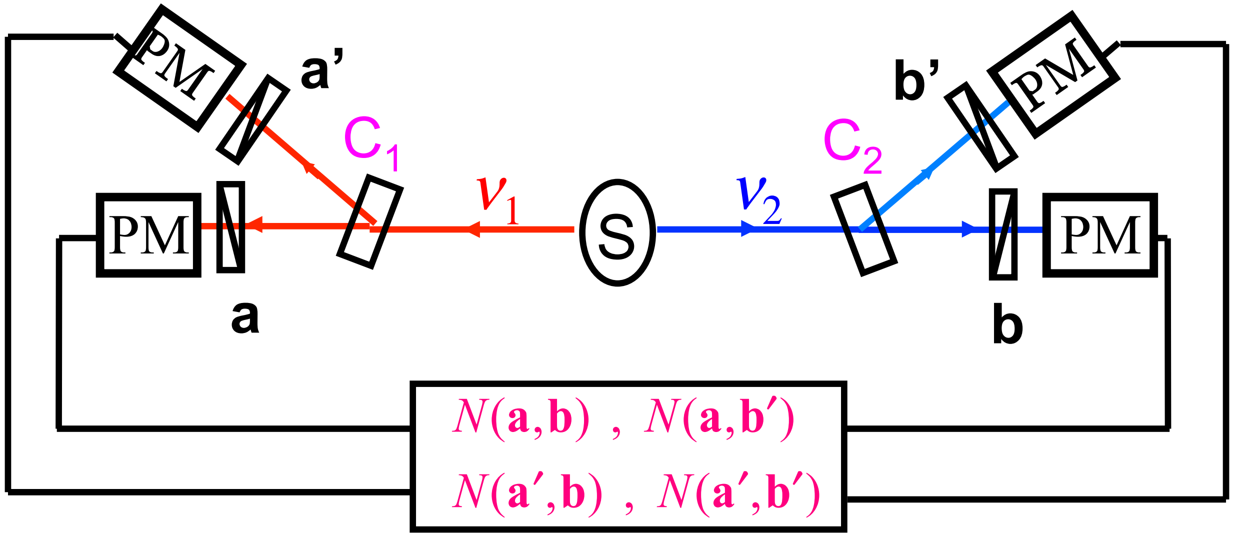
from the back...



* Acousto optical switch :
change every 10 ns.
* Faster than propagation of light between polarizers (40 ns) and even than time of flight of photons between the source S and each switch (20 ns).

Experiment with variable polarizers

(A. Aspect, J. Dalibard, G. Roger, 1982)



Difficult experiment...

* reduced signal

* data accumulation for several hours (enough for tasting foie gras and Sauternes...)

* switching not fully random

... but convincing results :

* Bell's inequalities violated by par 6 standard deviations.
* Each measurement space-like separated from setting of distant polarizer -> Einstein's causality enforced

What are the "loopholes" in BI tests ?

They are various "defects" which make that the practical implementations of the BI tests do not follow Bell's hypothesis : logical gap in the result.

Usually they are corrected by adding "supplementary assumptions", which are "ad hoc" hypothesis, allowing to exploit the experimental data.

The most famous loopholes are :

- **the locality loophole** (can be eliminated in principle)

- **the detection loophole** (can be eliminated in principle)

- **the universal conspiration loophole** (telling for instance that there is no "true" randomness, and that everything is determined in advance... harder to test, but also hardly compatible with physics in general).

The locality loophole

Ideally, the choice of the analyzer orientation **a** must be space-like separated from the choice of **b**, and from the emission of the correlated pairs.

Then relativistic causality enforces Bell's locality condition, as shown by Alain Aspect, Phys. Rev. D 14, 1944 (1976) :

$$\boxed{A(\lambda, \mathbf{a}, \cancel{\mathbf{b}}) \qquad B(\lambda, \cancel{\mathbf{a}}, \mathbf{b}) \qquad \rho(\lambda, \cancel{\mathbf{a}}, \cancel{\mathbf{b}})}$$

Until recently two experiments (Aspect et al 1982, Zeilinger et al 1998) have fulfilled this condition. For other ones, Bell's locality condition is added as a "reasonable hypothesis", but not experimentally demonstrated.

Remark : in principle, nothing forbids that QM might be true as long as detection events are not space-like separated, and then become wrong when they are (= locality loophole). This invalidates most "algebraic" proofs of the impossibility of hidden variables (e.g. Fine, Malley...).

The detection loophole

Ideally, all pairs of particles emitted by the source must be detected and must be taken into account when evaluating the correlation functions.

Alternatively, one must have an "event ready detector" at the source, which should be space-like separated from the choice of measurements, and which tells that a given emitted pair is valid and must be detected.

Without that, one may imagine that the polarizer orientations select different "sub-ensembles" among the emitted pairs, and BI can be easily violated.

Until recently all optical test of BI were subject to the detection loophole.

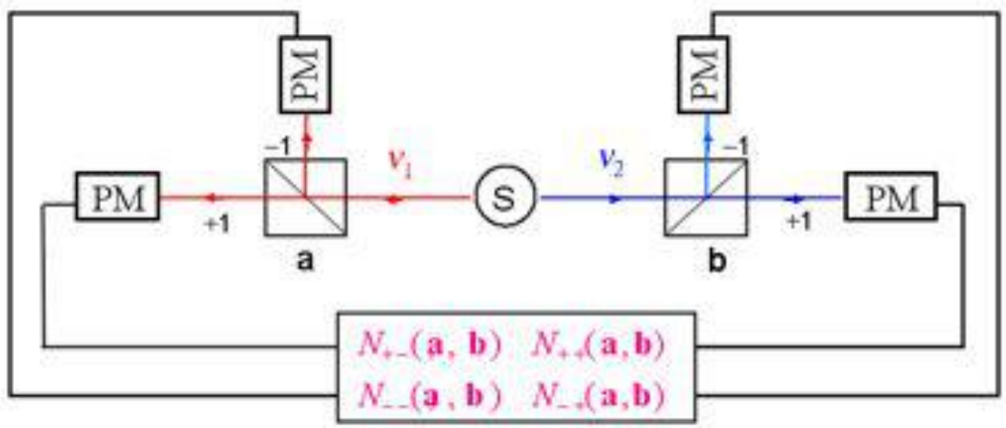
Several "auxiliary assumptions" may be used to solve the problem, e.g. :

- the "no-enhancement hypothesis" (Clauser, Shimony, Horne ...)
- the "fair sampling" hypothesis (Aspect, Grangier..)

The basic goal of these hypothesis is to allow to evaluate correlation functions over the ensemble of detected pairs, rather than on all emitted pairs.

Experiment with 2-channel polarizers

(A. Aspect, P. Grangier, G. Roger, 1982)



Direct measurement of the polarization correlation coefficient : simultaneous measurement of the 4 coincidence rates

$$\boxed{E(\mathbf{a},\mathbf{b}) = \frac{N_{++}(\mathbf{a},\mathbf{b}) - N_{+-}(\mathbf{a},\mathbf{b}) - N_{-+}(\mathbf{a},\mathbf{b}) + N_{--}(\mathbf{a},\mathbf{b})}{N_{++}(\mathbf{a},\mathbf{b}) + N_{+-}(\mathbf{a},\mathbf{b}) + N_{-+}(\mathbf{a},\mathbf{b}) + N_{--}(\mathbf{a},\mathbf{b})}}$$

However note that : $N_{++}(\mathbf{a,b}) + N_{+-}(\mathbf{a,b}) + N_{-+}(\mathbf{a,b}) + N_{--}(\mathbf{a,b}) = N_{detected}(\mathbf{a,b})$

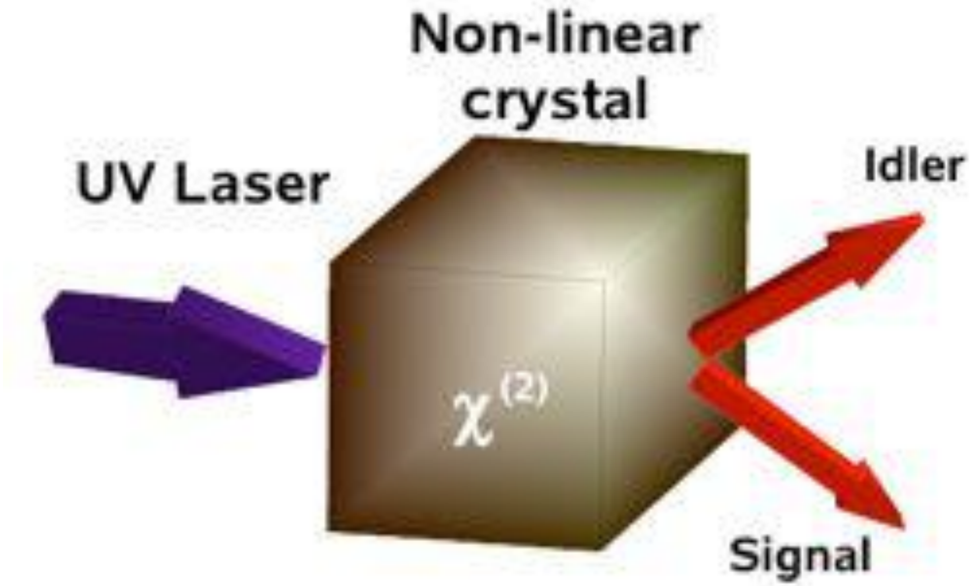
$$\mathbf{N_{detected}(a,b) = \varepsilon_1 \, \varepsilon_2 \, \xi \, N_{emitted} \approx 10^{-6} \, N_{emitted}}$$

ε_1, ε_2 overall detection efficiencies, ξ correlation (angle and time)

A lot of pairs are missing...

The situation is much better with parametric photons, but still the condition to eliminate this loophole are extremely stringent : typically the overall detection efficiency should be larger than 80% (see below)

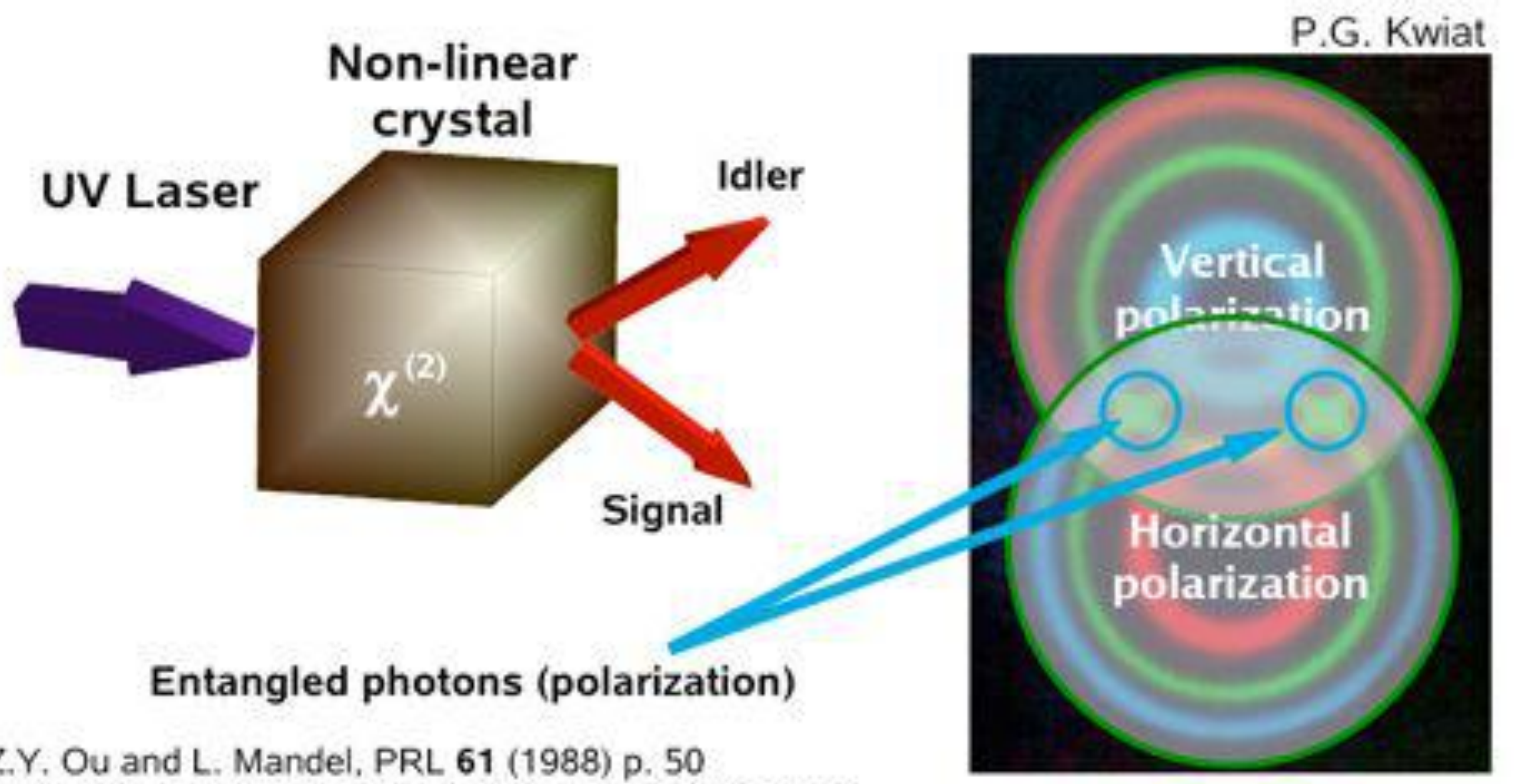
Entangled photon pairs by parametric down conversion



Z.Y. Ou and L. Mandel, PRL **61** (1988) p. 50
J. G. Rarity and P.R. Tapster, PRL

P.G. Kwiat

Entangled photon pairs by parametric down conversion



Z.Y. Ou and L. Mandel, PRL **61** (1988) p. 50
J. G. Rarity and P.R. Tapster, PRL **64** (1990) p. 2495
P.G. Kwiat, K. Mattle, H. Weinfurter, A. Zeilinger, A.V. Sergienko, and Y. Shih, PRL **75**, (1995) 4337

Well defined directions: can be injected into optical fibers.

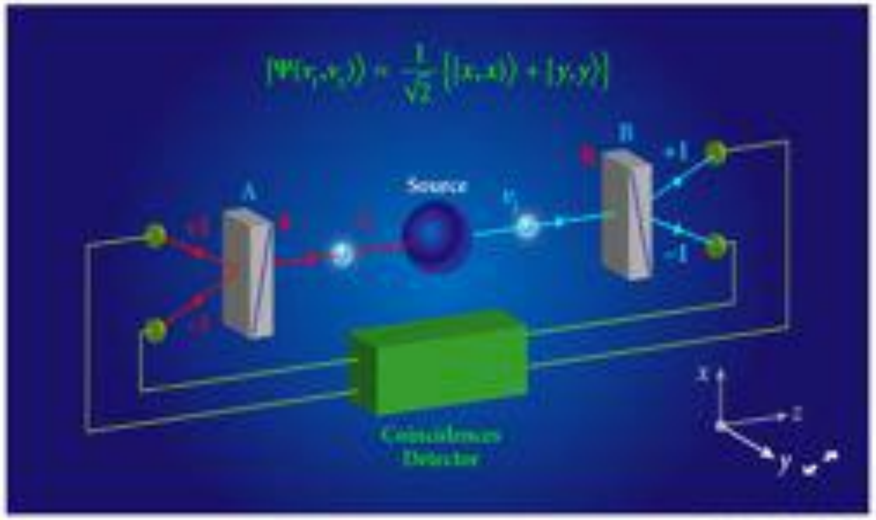
Physics ABOUT BROWSE PRESS

Viewpoint: Closing the Door on Einstein and Bohr's Quantum Debate

Alain Aspect, Laboratoire Charles Fabry, Institut d'Optique Graduate School, CNRS, Université Paris-Saclay, Palaiseau, France

December 16, 2015 • *Physics* 8, 123

By closing two loopholes at once, three experimental tests of Bell's inequalities remove the last doubts that we should renounce local realism. They also open the door to new quantum information technologies.



APS/Alan Stonebraker

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"Loophole-free Bell Inequality Violation Using Electron Spins Separated by 1.3 Kilometres,"

M. Giustina et al., Phys. Rev. Lett. 115, 250401 (2015).
"Significant-Loophole-Free Test of Bell's Theorem with Entangled Photons,"

L. K. Shalm et al., Phys. Rev. Lett. 115, 250402 (2015).
"Strong Loophole-Free Test of Local Realism,"

Loophole-free Bell tests in 2015

B. Hensen et al., Nature 526, 682 (2015).
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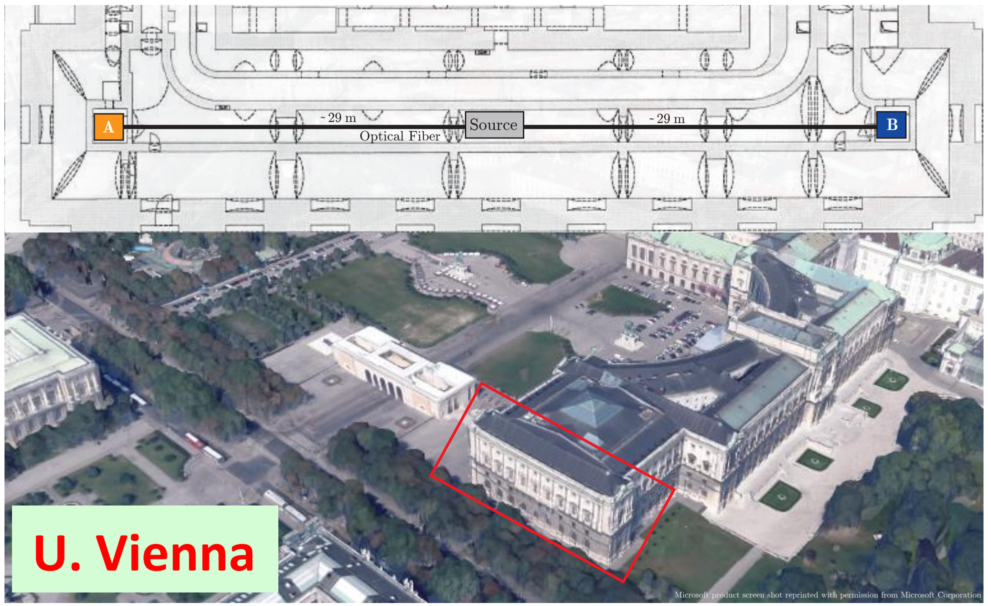

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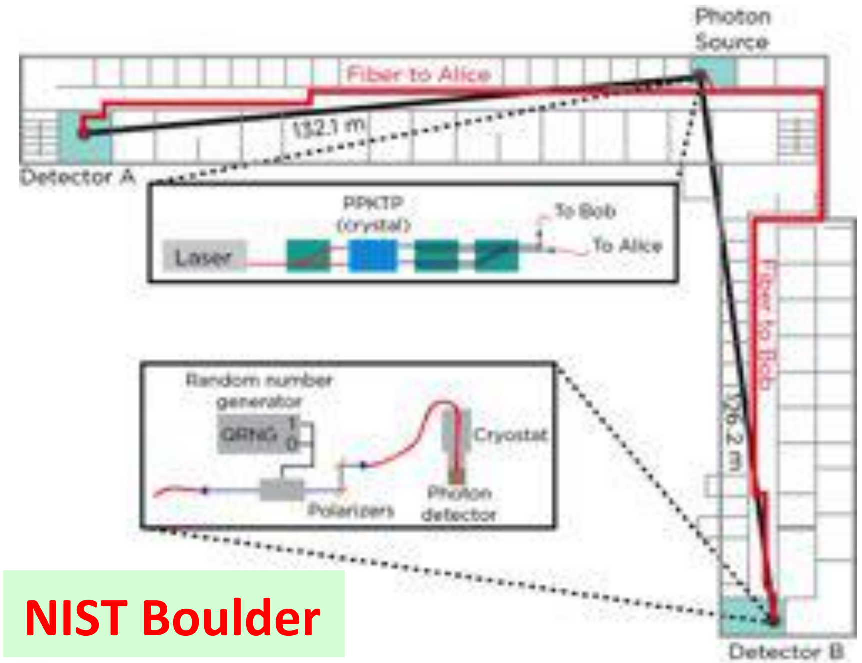

L. K. Shalm et al., PRL 115, 250402 (2015).
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A few considerations on entangled systems.

* Within classical physics, correlations between measurements carried out on separated subsystems are explained by attributing to each subsystems some properties which are correlated to properties of the other subsystem.

* If one tries to reproduce quantum correlations using such a model, Bell's inequalities show that these properties must be non-local, i.e. must contradict relativistic causality $\rightarrow$ **inacceptable**.

* Quantum mechanics remains in perfect agreement with relativistic causality, but there is a price : it is impossible to attribute a "local physical reality" to the state of each subsystem.

"EPR Paradox" (Einstein Podolsky Rosen, 1935)
"Quantum non-separability"

* We will see now that entanglement plays an essential role in quantum mechanics in general, and especially in quantum information...