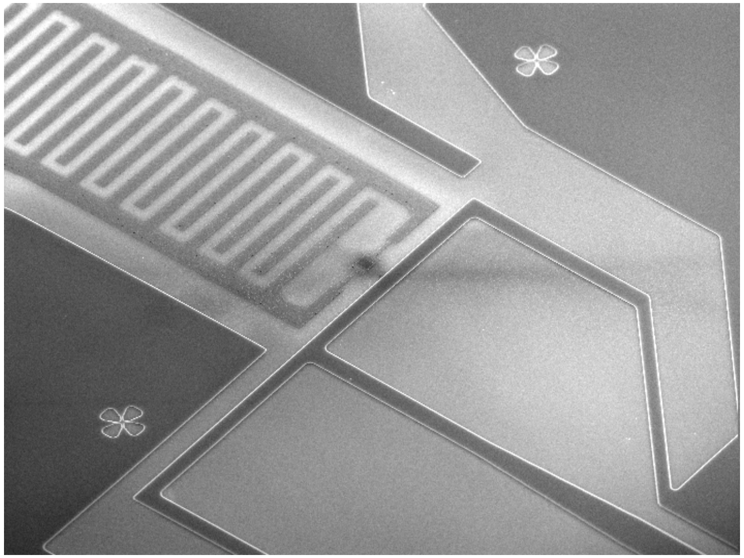
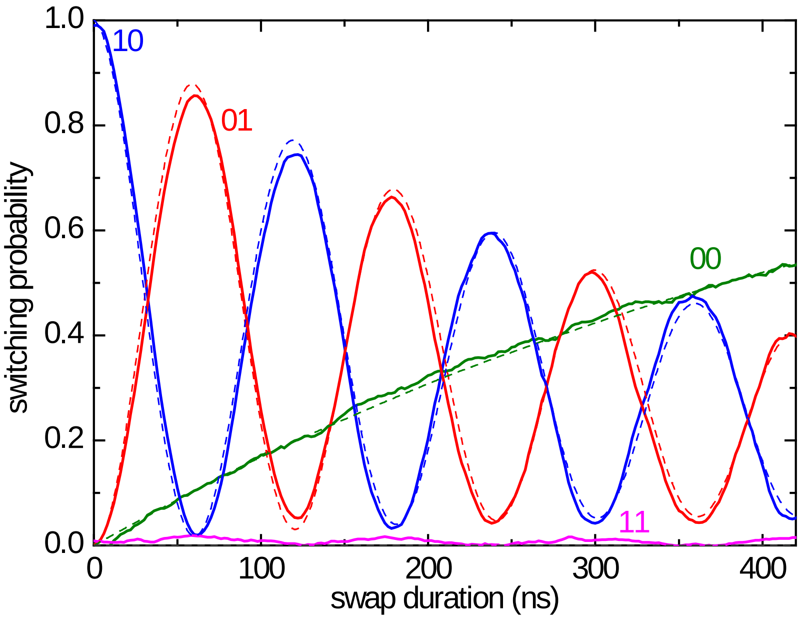


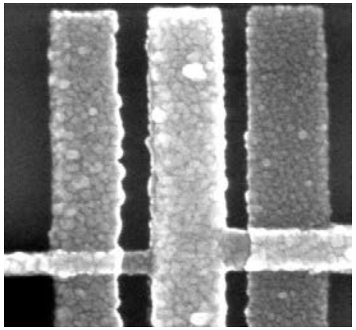


Electrical quantum engineering with superconducting circuits

P. Bertet & R. Heeres

SPEC, CEA Saclay (France),
Quantronics group

Introduction : Josephson circuits for quantum physics



From a ***fundamental question*** (25 years ago) ….

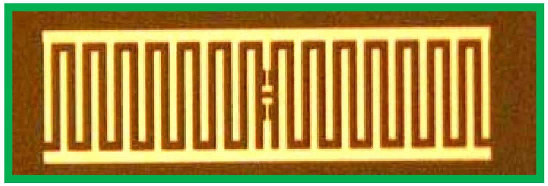
CAN MACROSCOPIC « MAN-MADE » ELECTRICAL CIRCUITS BEHAVE QUANTUM-MECHANICALLY ????

M.H. Devoret, J.M. Martinis and J. Clarke, *PRL* **85**, 1908 (1985)

M.H. Devoret, J.M. Martinis and J. Clarke, *PRL* **85**, 1543 (1985)

YES THEY CAN
Discrete energy levels

… to genuine artificial atoms



$$\alpha|0\rangle + \beta|1\rangle$$

Introduction : Josephson circuits for quantum physics

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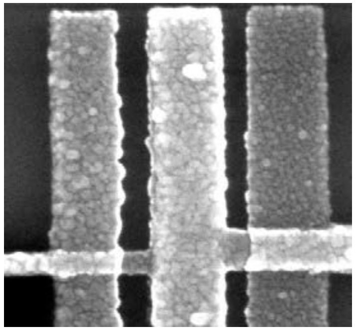

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… to genuine artificial atoms for quantum information and quantum optics ***on a chip***

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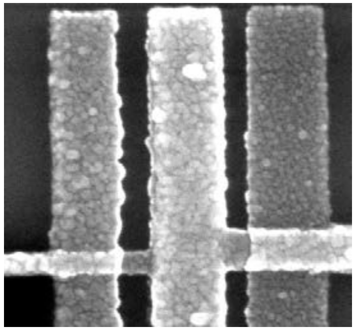

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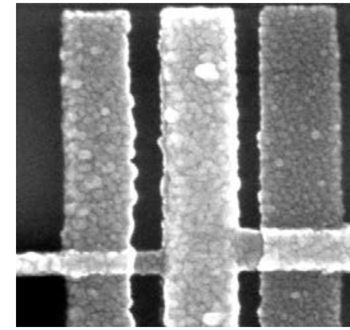
… to genuine artificial atoms for quantum information and quantum optics ***on a chip***

$$|\psi\rangle = \sum_{i,j,k=0,1} c_{ijk} |i_1 j_2 k_3\rangle$$

Introduction : Josephson circuits for quantum physics



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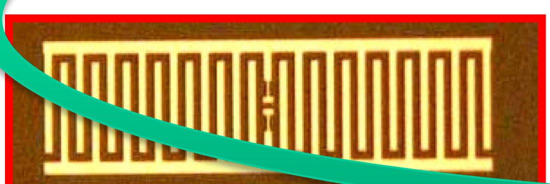


$$|\psi\rangle = \sum_{i,j,k=0,1} c_{ijk} |i_1 j_2 k_3\rangle$$

QUANTUM PHYSICS

QUANTUM ALGORITHMS
QUANTUM SIMULATORS

Outline

Lecture 1: Basics of superconducting qubits

Lecture 2: Qubit readout and circuit quantum electrodynamics

Lecture 3: Multi-qubit gates and algorithms

Lecture 4: Introduction to Hybrid Quantum Devices

Outline

Lecture 1: Basics of superconducting qubits

1) Introduction: Hamiltonian of an electrical circuit

2) The Cooper-pair box

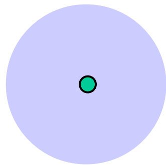
3) Decoherence of superconducting qubits

Lecture 2: Qubit readout and circuit quantum electrodynamics

Lecture 3: Multi-qubit gates and algorithms

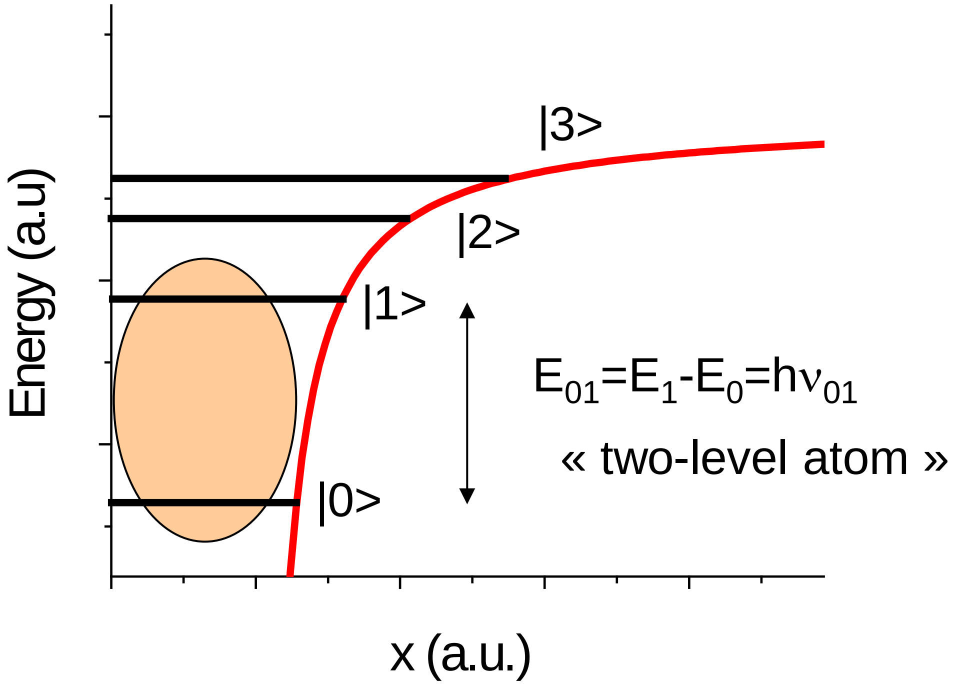
Lecture 4: Introduction to Hybrid Quantum Devices

Real atoms

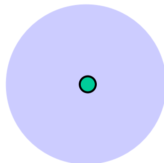


Hydrogen atom

$$V(r) = -\frac{e^2}{4\pi\varepsilon_0 r} \xrightarrow{\text{quantization}} \begin{array}{l} [\hat{r}, \hat{p}] = i\hbar \\ \hat{H} = \frac{\hat{p}^2}{2m} + V(\hat{r}) \end{array}$$



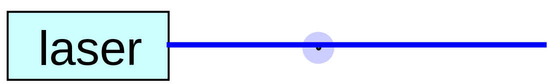
Real atoms



Hydrogen atom

$$V(r) = -\frac{e^2}{4\pi\varepsilon_0 r} \xrightarrow{\text{quantization}} \left[\hat{r}, \hat{p}\right] = i\hbar, \quad \hat{H} = \frac{\hat{p}^2}{2m} + V(\hat{r})$$

$$E(t) = E_0 \cos 2\pi\nu t$$

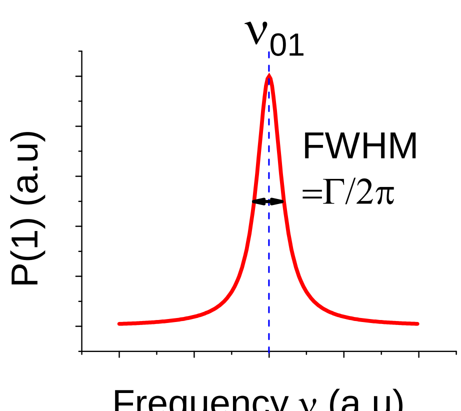


$$H_I = e\vec{r} \cdot \vec{E}(t)$$

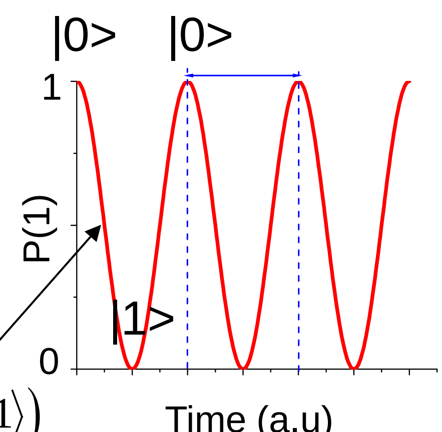
+ spontaneous emission Γ

} Optical Bloch equations

Spectroscopy (weak field)



Rabi oscillations (short pulses, strong field at $\nu=\nu_{01}$)



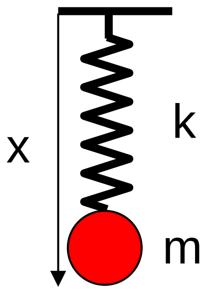
$$\frac{1}{\sqrt{2}}\left(\left|0\right\rangle + \left|1\right\rangle\right)$$

$$\Omega_R = e\langle 0|\vec{r}|1\rangle \cdot \vec{E}_0/\hbar$$

$$\Phi(t) = \int_{-\infty}^{t} V(t')dt'$$

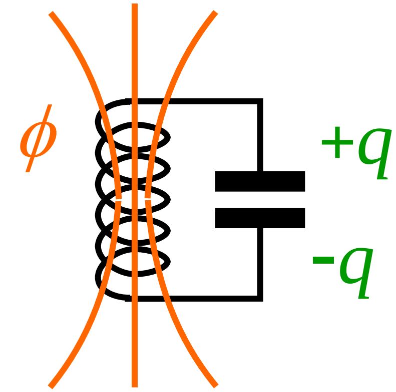
$$Q(t) = \int_{-\infty}^{t} i(t')dt'$$

Electrical harmonic oscillator



$$\omega_0 = \sqrt{\frac{k}{m}}$$

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

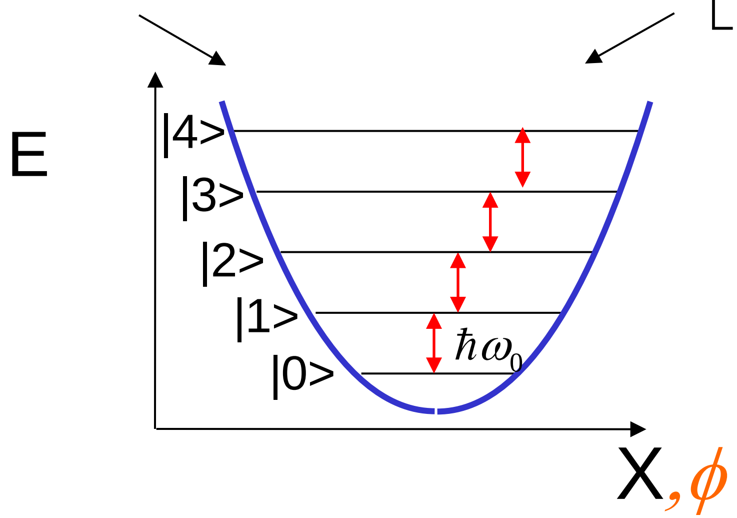


$$H(x,p) = \frac{kx^2}{2} + \frac{p^2}{2m}$$

$$H(\Phi,Q) = \frac{\Phi^2}{2L} + \frac{Q^2}{2C}$$

$$[\hat{x},\hat{p}] = i\hbar$$

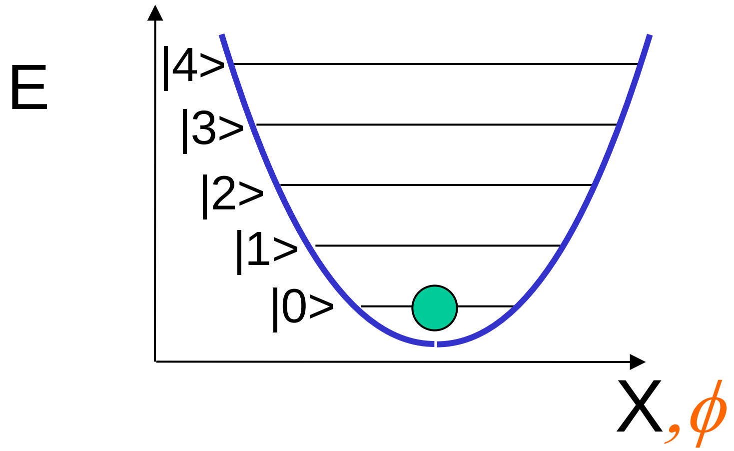
$$[\hat{\Phi},\hat{Q}] = i\hbar$$



Quantum regime ??

LC oscillator in the quantum regime ?

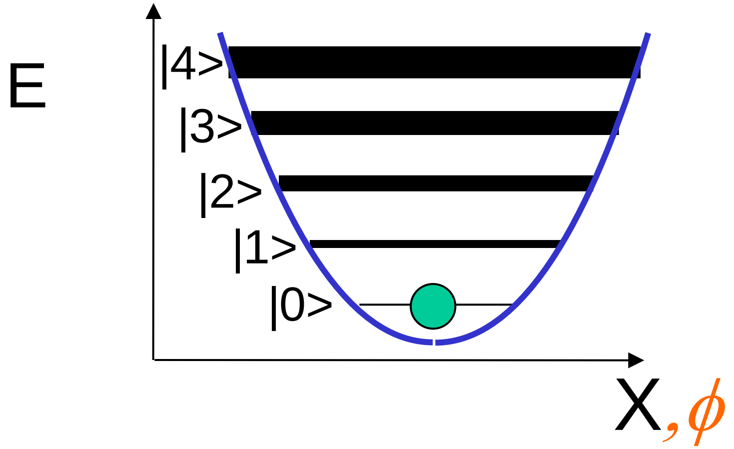
2 conditions :



$$kT << \hbar\omega_0$$

Typic : $L \approx nH \quad C \approx pF \quad \nu_0 \approx 5GHz$

At ***T=30mK*** : $\frac{h\nu_0}{kT} \approx 8$

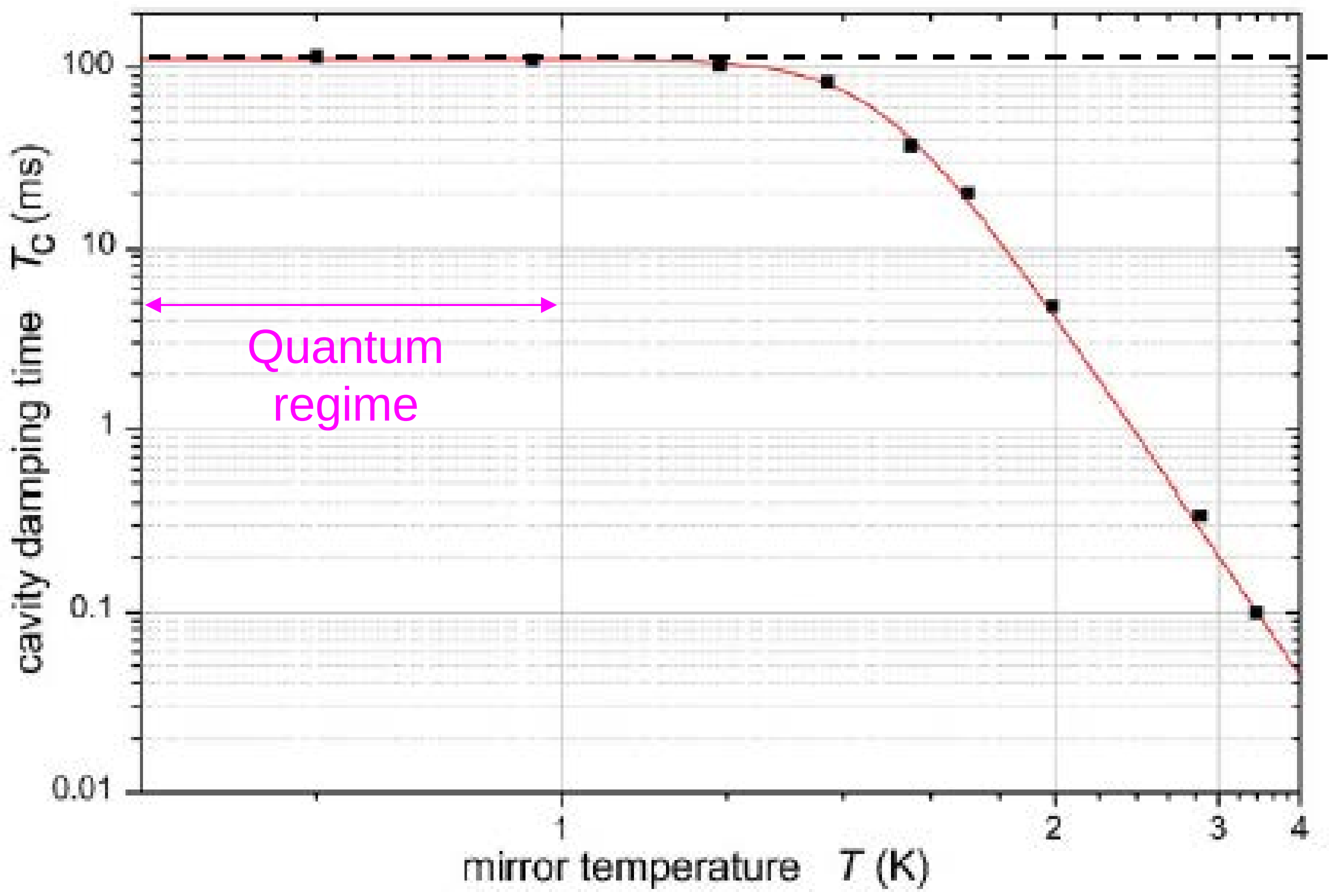


$$Q >> 1$$

OK if dissipation negligible

→ ***Superconductors*** at T<<Tc

Microwave superconducting resonators



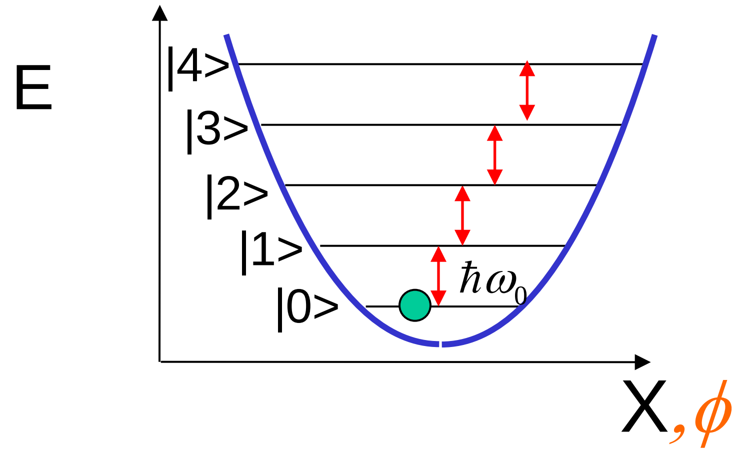
Q=4 10^{10}
at 51GHz
and at 1K

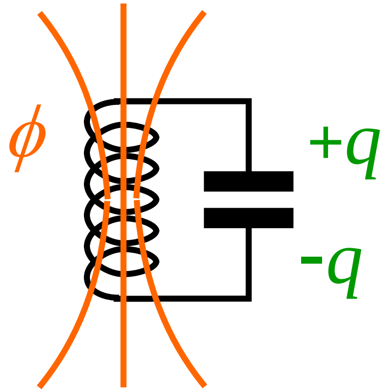
T_c(Nb)=9.2K

S. Kuhr et al., *APL* **90**, 164101 (2006)

T<<Tc : dissipation negligble at GHz frequencies

Necessity of anharmonicity





How to prepare |1> ?

→ **Need non-linear** and **non dissipative** element : Josephson junction

Basics of the Josephson junction

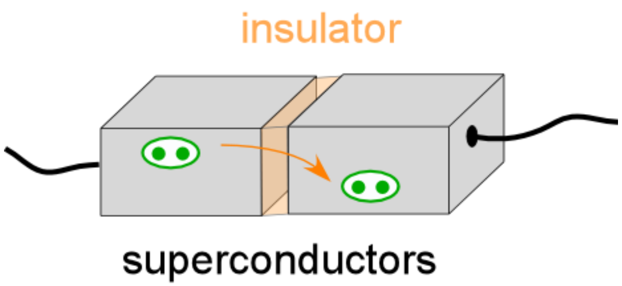
$$\Phi(t) = \int_{-\infty}^{t} V(t')dt'$$

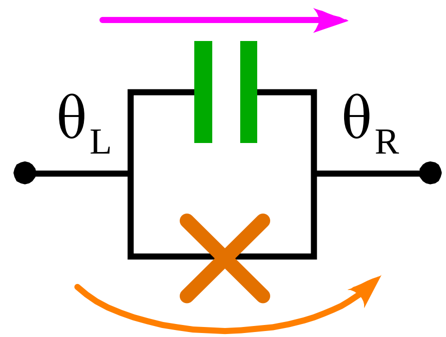
$$Q(t) = \int_{-\infty}^{t} i(t')dt'$$

The building block of superconducting qubits

$$\theta = \frac{\Phi}{\varphi_0} \mathrm{mod}(2\pi) = \theta_R - \theta_L \in [0, 2\pi]$$

$$\varphi_0 = \hbar/(2e)$$





$N = Q/2e \in \mathbb{N}$

Josephson DC relation : $I = I_C \sin\theta$

Josephson AC relation : $V = \varphi_0 \frac{d\theta}{dt} = \frac{Q}{C}$

B. Josephson, *Phys. Lett.* **1**, 251 (1962)

P.W. Anderson & J.M. Rowell, *Phys. Rev.* **10**, 230 (1963)

S. Shapiro, *Phys. Rev.* **11**, 80 (1963)

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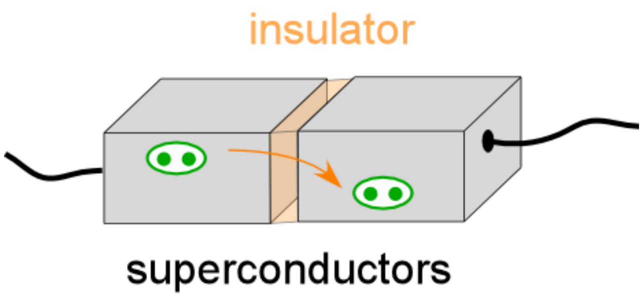
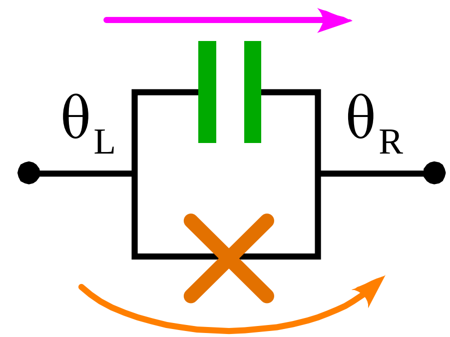




N=Q/2e∈ ℕ

Josephson DC relation : $I = I_C \sin\theta$

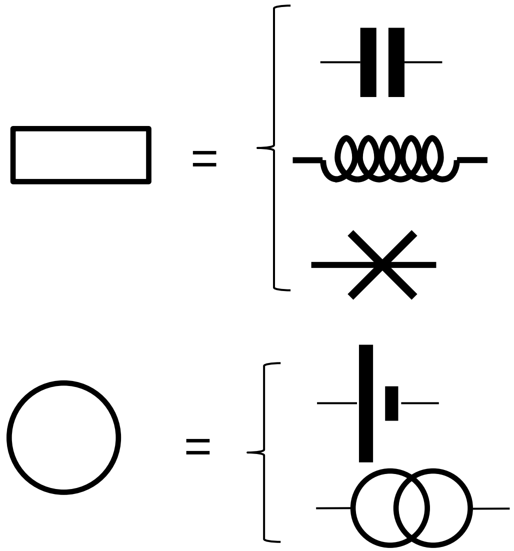
Josephson AC relation : $V = \varphi_0 \frac{d\theta}{dt} = \frac{Q}{C}$

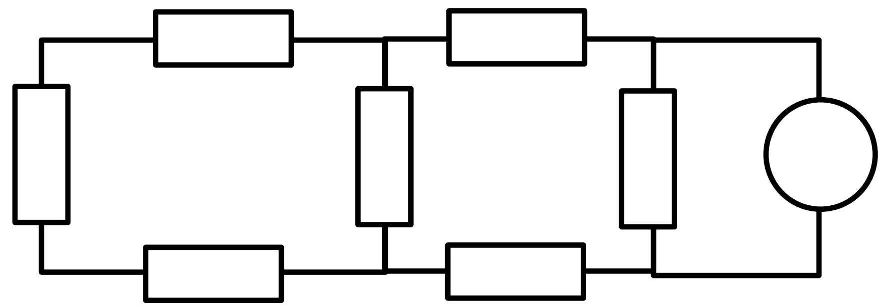
Classical variables ??

→ NON-LINEAR INDUCTANCE $$L_J(I) = \frac{\varphi_0}{I_C\sqrt{1-(I/I_C)^2}}$$

→ POTENTIAL ENERGY $$E_J(\theta) = -\varphi_0 I_C \cos\theta = -E_J \cos\theta$$

Hamiltonian of an arbitrary circuit





HAMILTONIAN ???

Correct procedure described in :

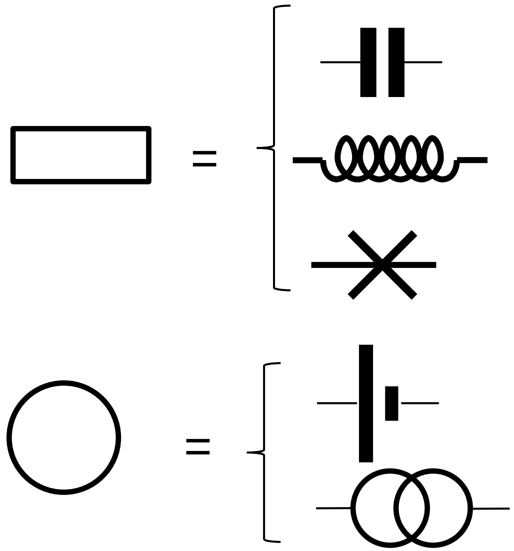
M. H. Devoret, p. 351 in *Quantum fluctuations* (Les Houches 1995)

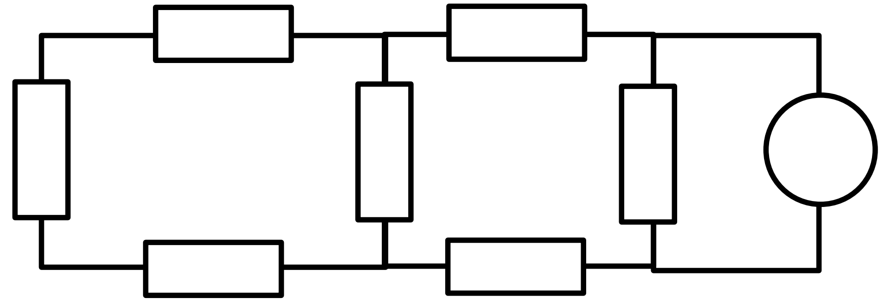
G. Burkard et al., Phys. Rev. B **69**, 064503 (2004)

G. Wendin and V. Shumeiko, cond-mat/0508729

M.H. Devoret, lectures at Collège de France (2008) accessible online

Hamiltonian of an arbitrary circuit





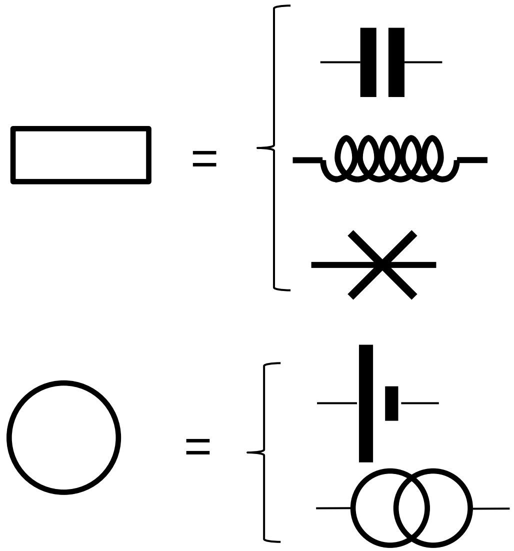
HAMILTONIAN ???

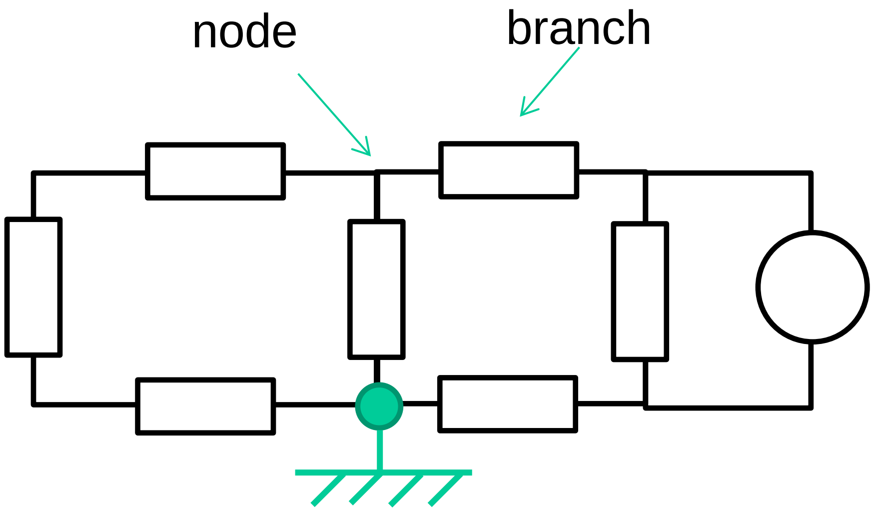
1) Identify the relevant independent circuit variables

2) Write the circuit Lagrangian

3) Determine the canonical conjugate variables and the Hamiltonian

Hamiltonian of an arbitrary circuit

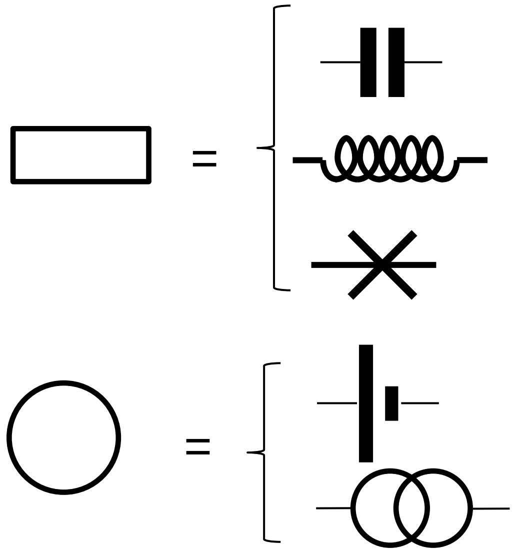


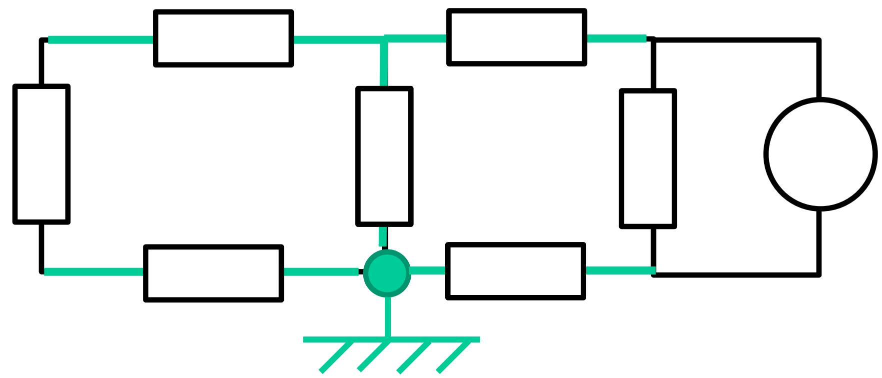


Identifying the relevant independent circuit variables

1) Choose reference node (ground)

Hamiltonian of an arbitrary circuit



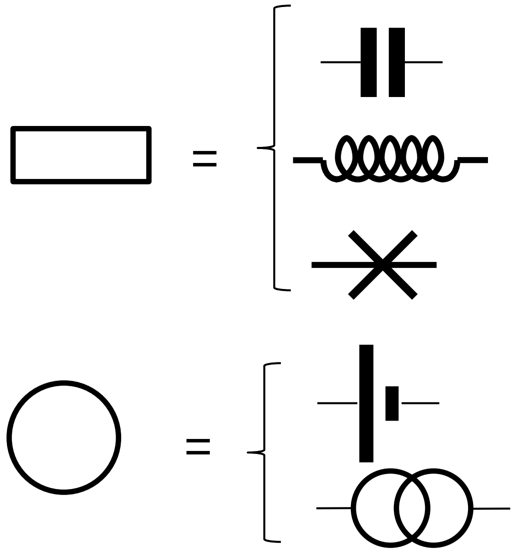


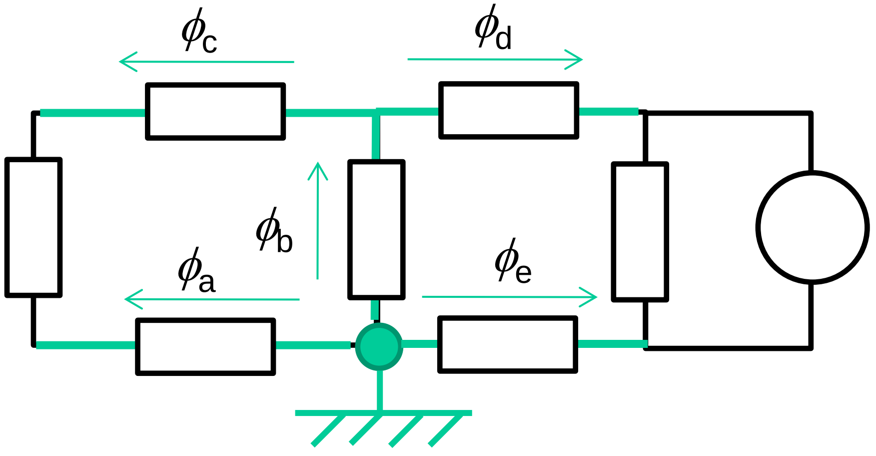
Identifying the relevant independent circuit variables

1) Choose reference node (ground)

2) Choose « spanning tree » (no loop)

Hamiltonian of an arbitrary circuit





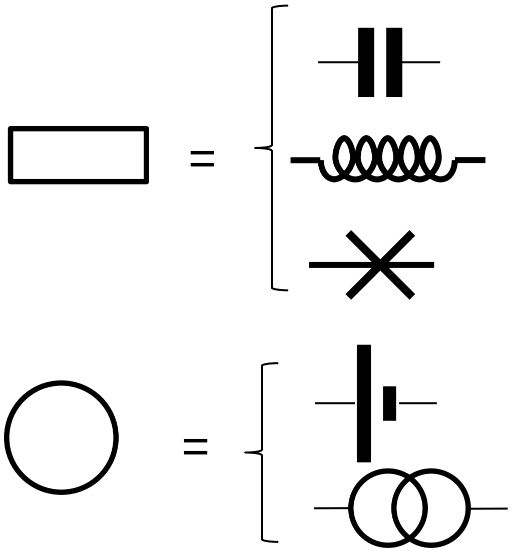
Identifying the relevant independent circuit variables

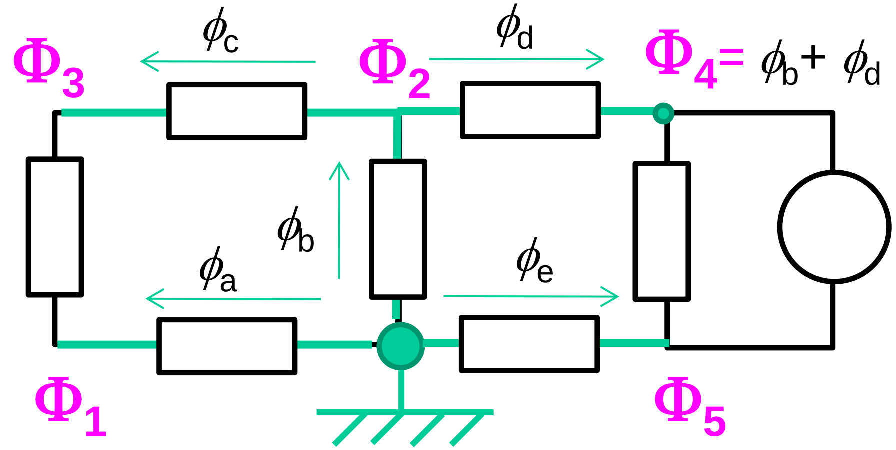
1) Choose reference node (ground)

2) Choose « spanning tree » (no loop)

3) Define « tree branch fluxes » $\phi_i(t) = \int_{-\infty}^{t} V(t')dt'$

Hamiltonian of an arbitrary circuit





Identifying the relevant independent circuit variables

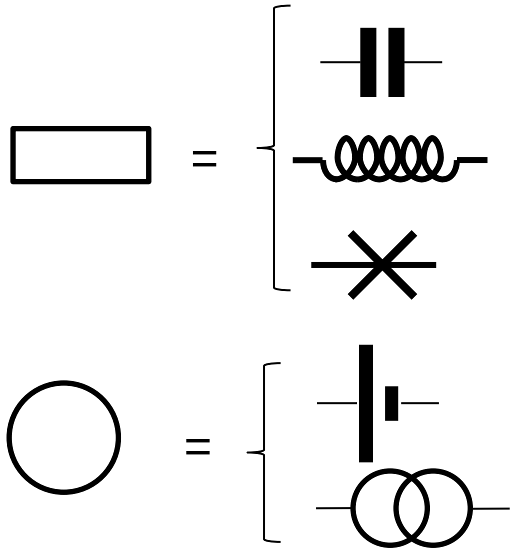
1) Choose reference node (ground)

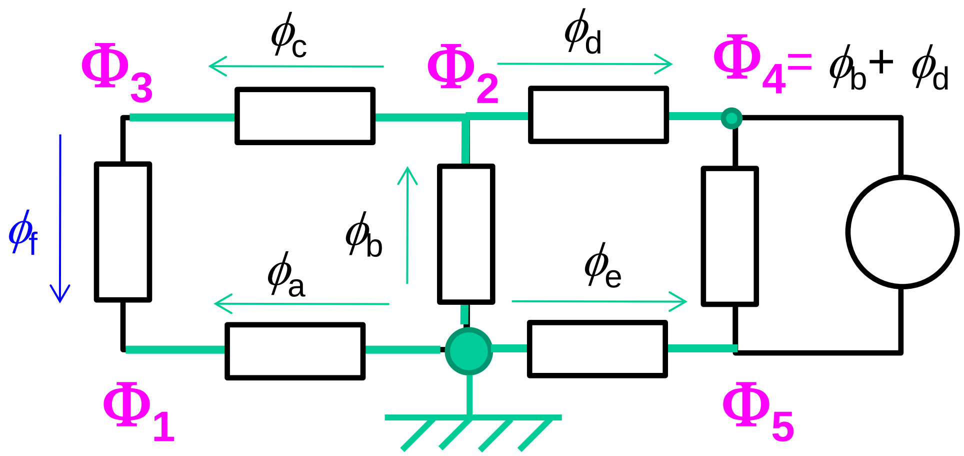
2) Choose « spanning tree » (no loop)

3) Define « tree branch fluxes » $\phi_i(t) = \int_{-\infty}^{t} V(t')dt'$

4) Define **node fluxes** $\Phi_n = \sum_{branches\, \beta\, leading to\, n} \phi_\beta$

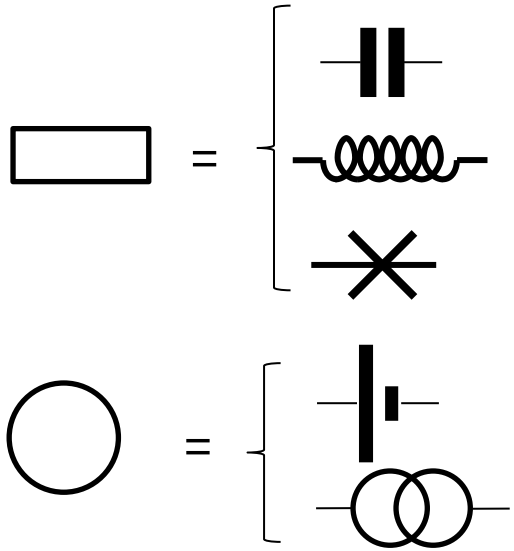
Hamiltonian of an arbitrary circuit

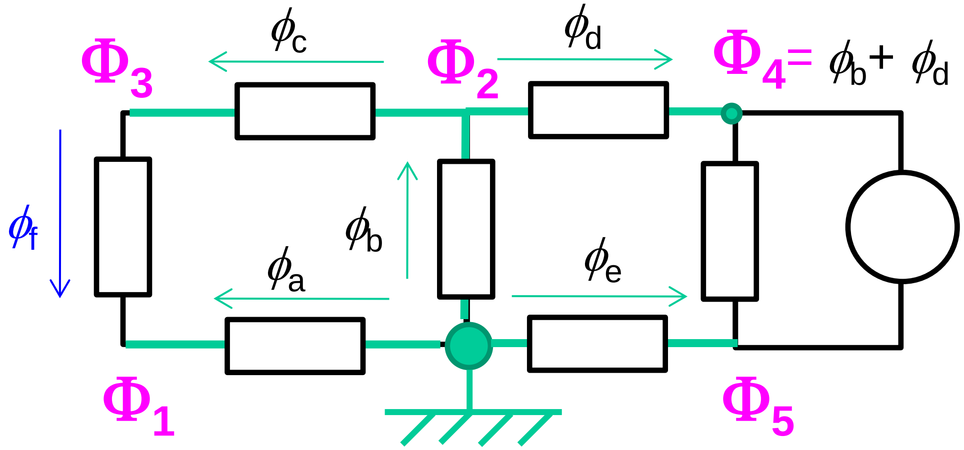




5) What about « closure branches » ??

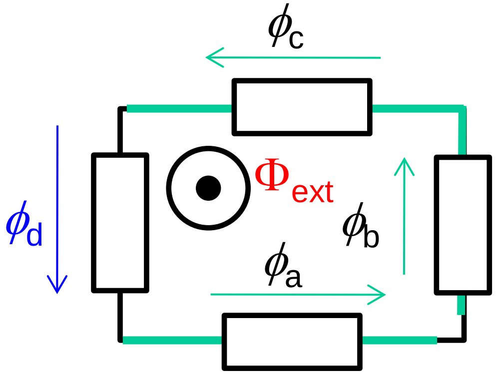
Hamiltonian of an arbitrary circuit





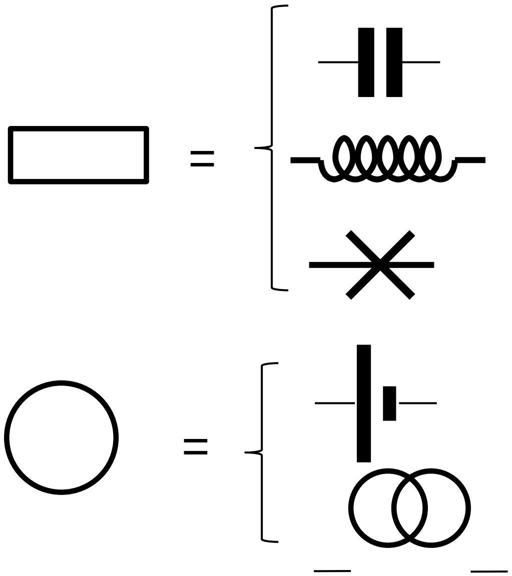
5) What about « closure branches » ??

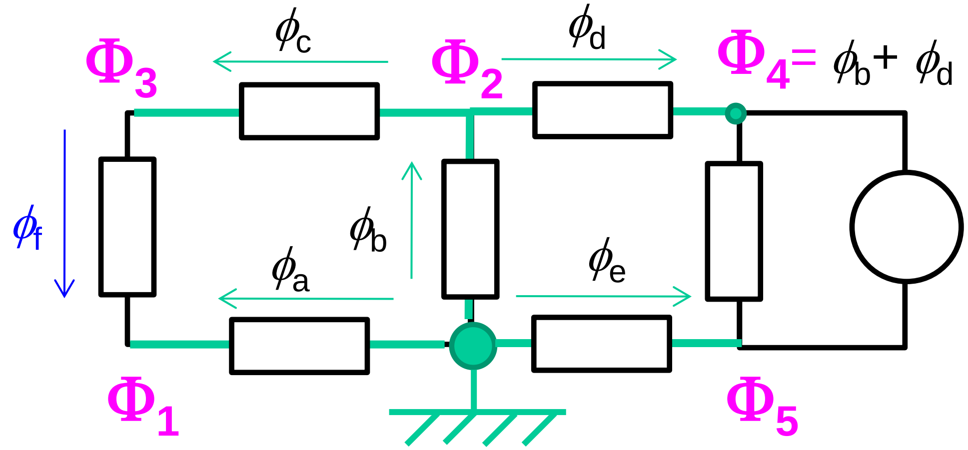
PHASE QUANTIZATION CONDITION (superconducting loop)



$$\phi_a+\phi_b+\phi_c+\phi_d=\Phi_{ext}$$

Hamiltonian of an arbitrary circuit

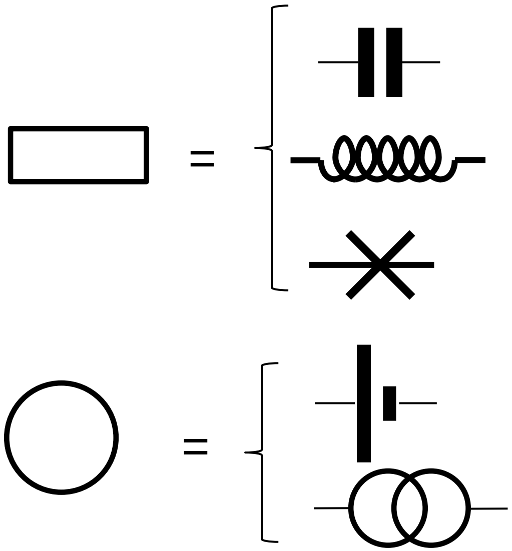


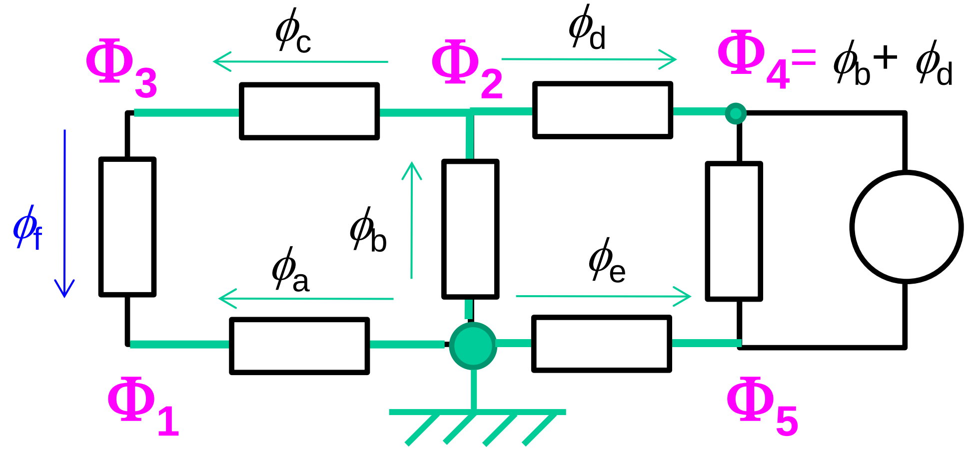


6) Classical Lagrangian $L(\Phi_i, \dot{\Phi}_i) = E_{electrostatic}(\dot{\Phi}_i) - E_{pot}(\Phi_i)$

taking into account constraints imposed by external biases (fluxes or charges)

Hamiltonian of an arbitrary circuit





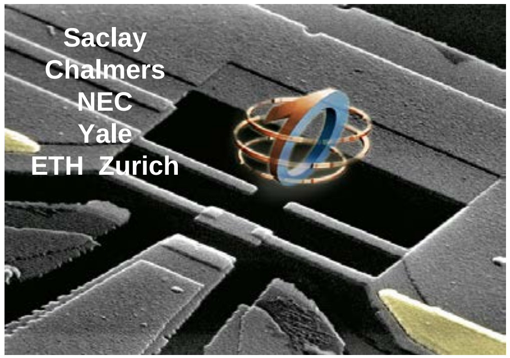
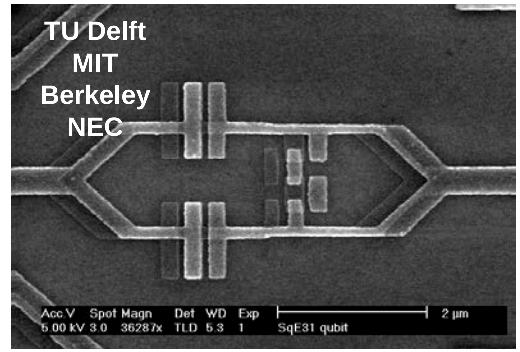
Conjugate variables : $Q_i = \dfrac{\partial L}{\partial \dot{\Phi}_i}$

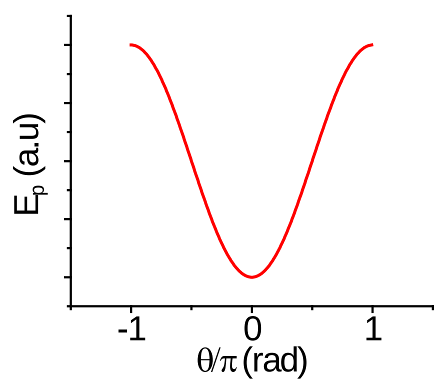
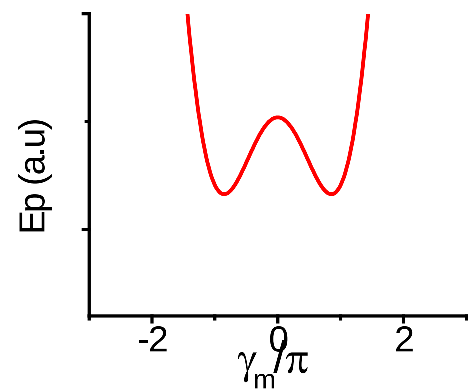
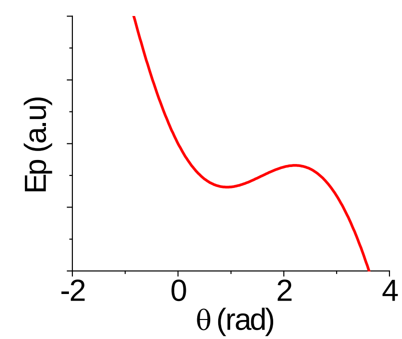
Classical Hamiltonian $H(\Phi_i, Q_i) = \sum Q_i \dot{\Phi}_i - L$

Quantum Hamiltonian $H(\hat{\Phi}_i, \hat{Q}_i)$ With $\left[\hat{\Phi}_i, \hat{Q}_i\right] = i\hbar$

or $H(\hat{\theta}_i, \hat{n}_i)$ with $\hat{n}_i = \hat{Q}_i / 2e$, $\hat{\theta}_i = \hat{\Phi}_i (2e/\hbar)$ $\left[\hat{\theta}_i, \hat{n}_i\right] = i$

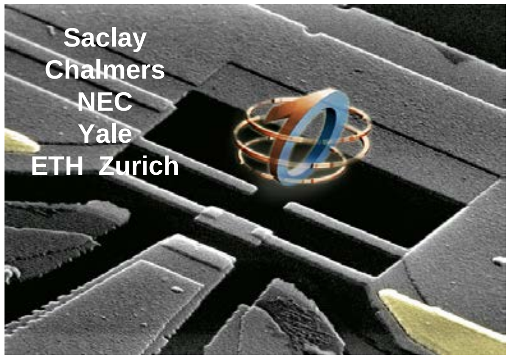
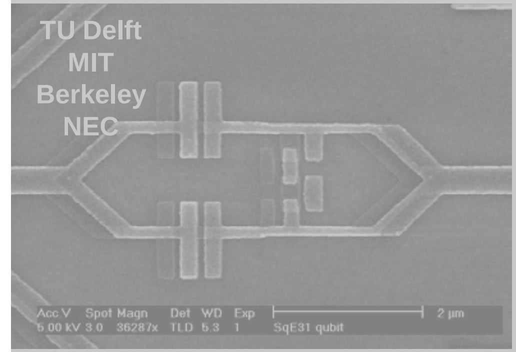
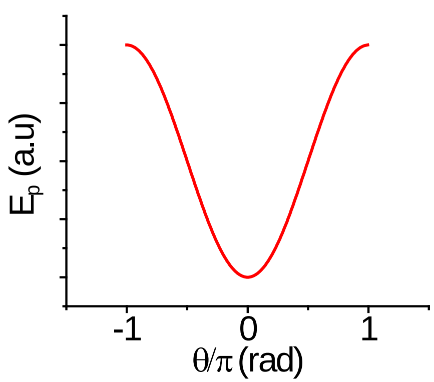
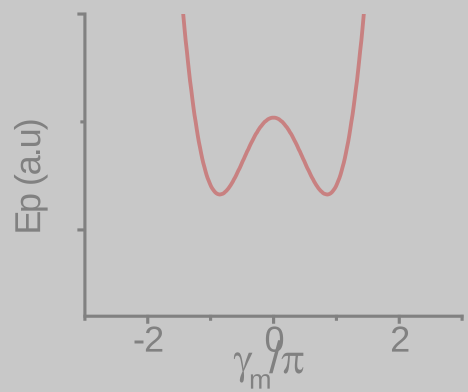
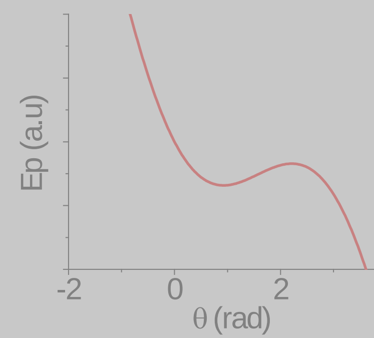
Different types of qubits

Cooper-pair boxes	Flux qubits	Phase qubits
		

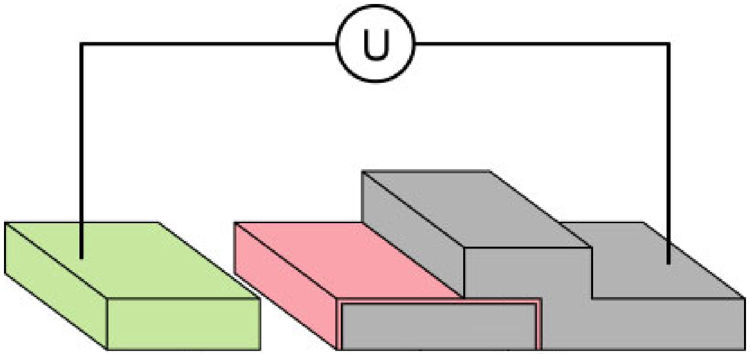
Junctions sizes	.01 to 0.04 μm^2	.04μm^2	100μm^2
Shape Of the Potential Energy			

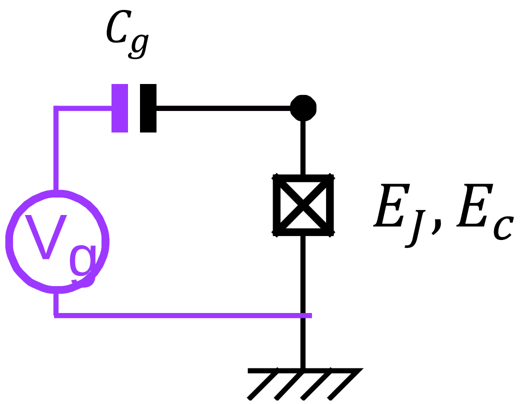
I.2) Cooper-Pair Box

Different types of qubits

	Cooper-pair boxes	Flux qubits	Phase qubits
			
Junctions sizes	.01 to 0.04 μm^2	.04μm^2	100μm^2
Shape Of the Potential Energy			

The Cooper-Pair Box





1 degree of freedom $\left[\hat{\theta}, \hat{N}\right] = i$

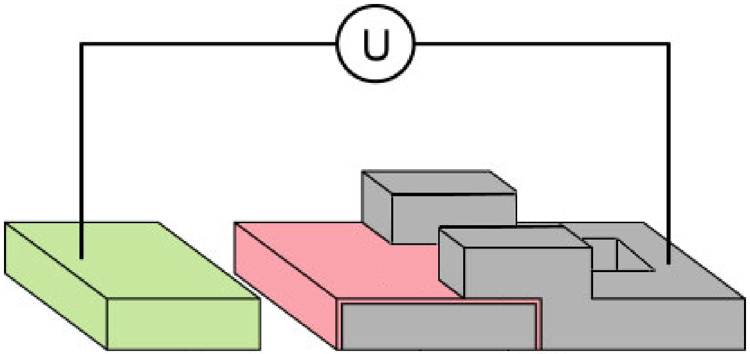
1 knob

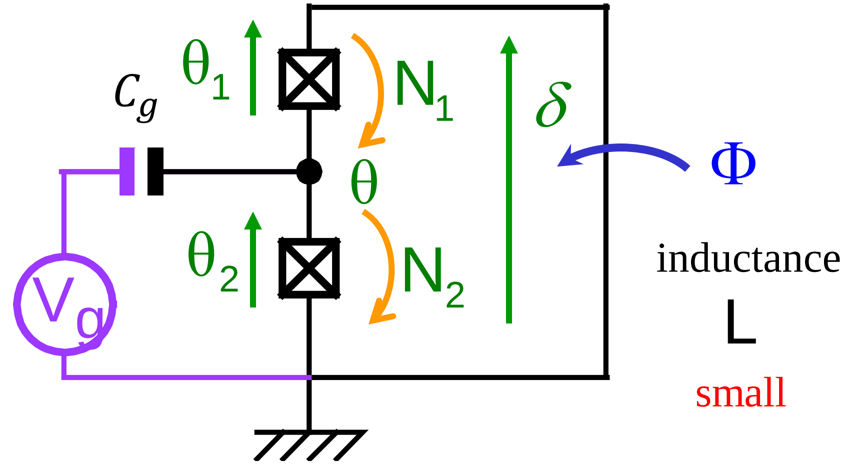
$E_c = (2e)^2/2C$ charging energy

$N_g = C_g V_g / 2e$ gate charge

$$\hat{H} = E_C(\hat{N} - N_g)^2 - E_J \cos\hat{\theta}$$

The split CPB





2 d° of freedom

$$[\hat{\theta}_1, \hat{N}_1] = i$$

$$[\hat{\theta}_2, \hat{N}_2] = i$$

or

$$\left[\hat{\theta} = \frac{\hat{\theta}_2 - \hat{\theta}_1}{2}, \hat{N} = \hat{N}_1 - \hat{N}_2\right] = i$$

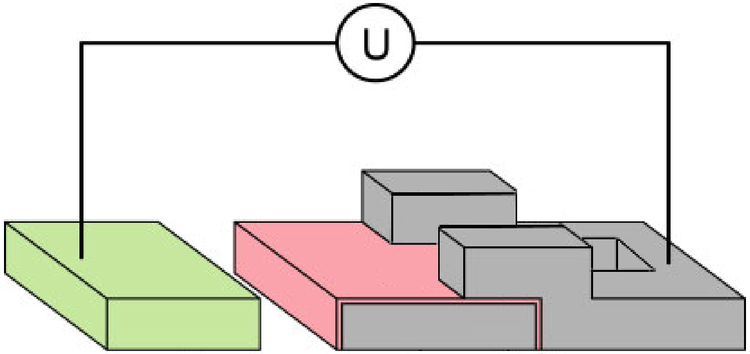
$$\left[\hat{\delta} = \hat{\theta}_1 + \hat{\theta}_2, \hat{K} = \frac{\hat{N}_1 + \hat{N}_2}{2}\right] = i$$

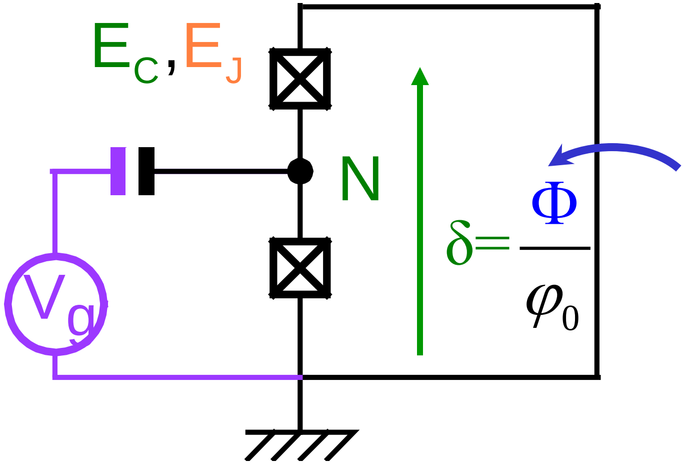
2 knobs

$$\hat{H} = E_C(\hat{N} - N_g)^2 - E_J \cos\frac{\cancel{\hat{\delta}}}{2}\cos\hat{\theta} + \cancel{\frac{(\Phi - \varphi_0\hat{\delta})^2}{2L}}$$

$$L \ll \phi_0^2 / E_J$$

The split CPB





1 d° of freedom $\left[\hat{\theta}, \hat{N}\right] = i$

2 knobs

$$\hat{H} = E_C(\hat{N} - N_g)^2 - E_J \cos\frac{\delta}{2} \cos\hat{\theta}$$

tunable E_J

Energy levels of the CPB

$$\hat{H}(N_g,\Phi) = E_C(\hat{N}-N_g)^2 - E_J(\Phi)\cos\hat{\theta} \longrightarrow \left\{E_k(N_g,\Phi), \left|\psi_k\right\rangle(N_g,\Phi)\right\}$$

Solve **either** in charge basis |N> $(N \in \mathbb{N})$ $\left|\psi_k\right\rangle = \sum_N c_{k,N}\left|N\right\rangle$

$$\hat{H} = E_C(N-N_g)^2\left|N\right\rangle\left\langle N\right| - \frac{E_J}{2}\sum_{N\in\square}\left(\left|N+1\right\rangle\left\langle N\right| + \left|N\right\rangle\left\langle N+1\right|\right)$$

Diagonalize

$$\begin{pmatrix} \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & E_C(-1-N_g)^2 & -E_J/2 & 0 & \cdots \\ \cdots & -E_J/2 & E_C(0-N_g)^2 & -E_J/2 & \cdots \\ \cdots & 0 & -E_J/2 & E_C(1-N_g)^2 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix}_{\ldots\left|N=-1\right\rangle\left|N=0\right\rangle\left|N=1\right\rangle\ldots}$$

Energy levels of the CPB

$$\hat{H}(N_g,\Phi) = E_C(\hat{N} - N_g)^2 - E_J(\Phi)\cos\hat{\theta} \longrightarrow \left\{E_k(N_g,\Phi), \left|\psi_k\right\rangle(N_g,\Phi)\right\}$$

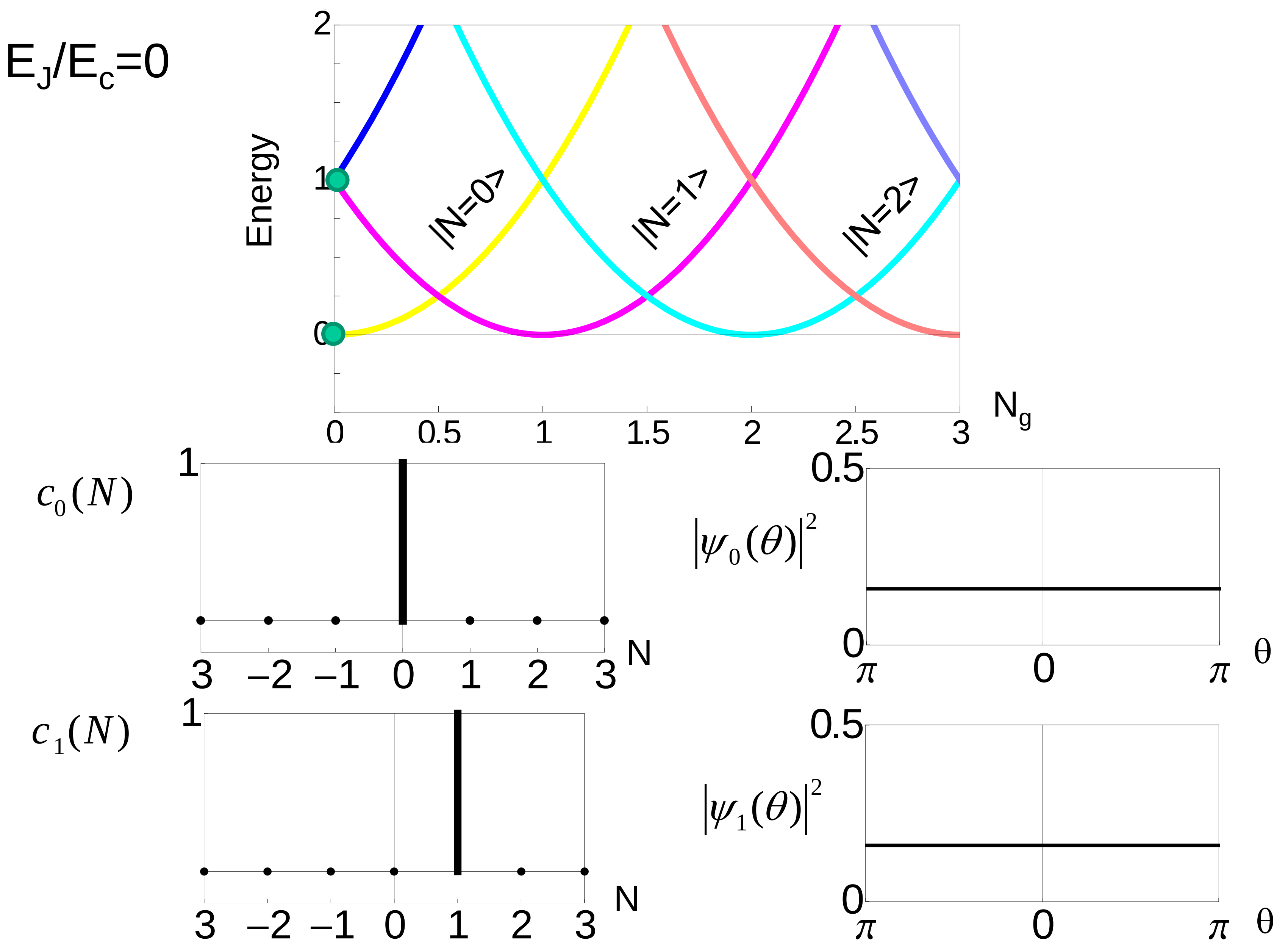
… **or** in phase basis |θ> $\left(\theta \in [0, 2\pi]\right)$ $\left|\psi_k\right\rangle = \int_0^{2\pi} d\theta \psi_k(\theta) \left|\theta\right\rangle$

$$\hat{H}(N_g,\Phi) = E_C\left(\frac{1}{i}\frac{\partial}{\partial\theta} - N_g\right)^2 - E_J(\Phi)\cos\hat{\theta}$$

Solve Mathieu equation

$$E_C\left(\frac{1}{i}\frac{\partial}{\partial\theta} - N_g\right)^2 \psi_k(\theta) - E_J(\Phi)\cos\theta\,\psi_k(\theta) = E_k \psi_k(\theta)$$

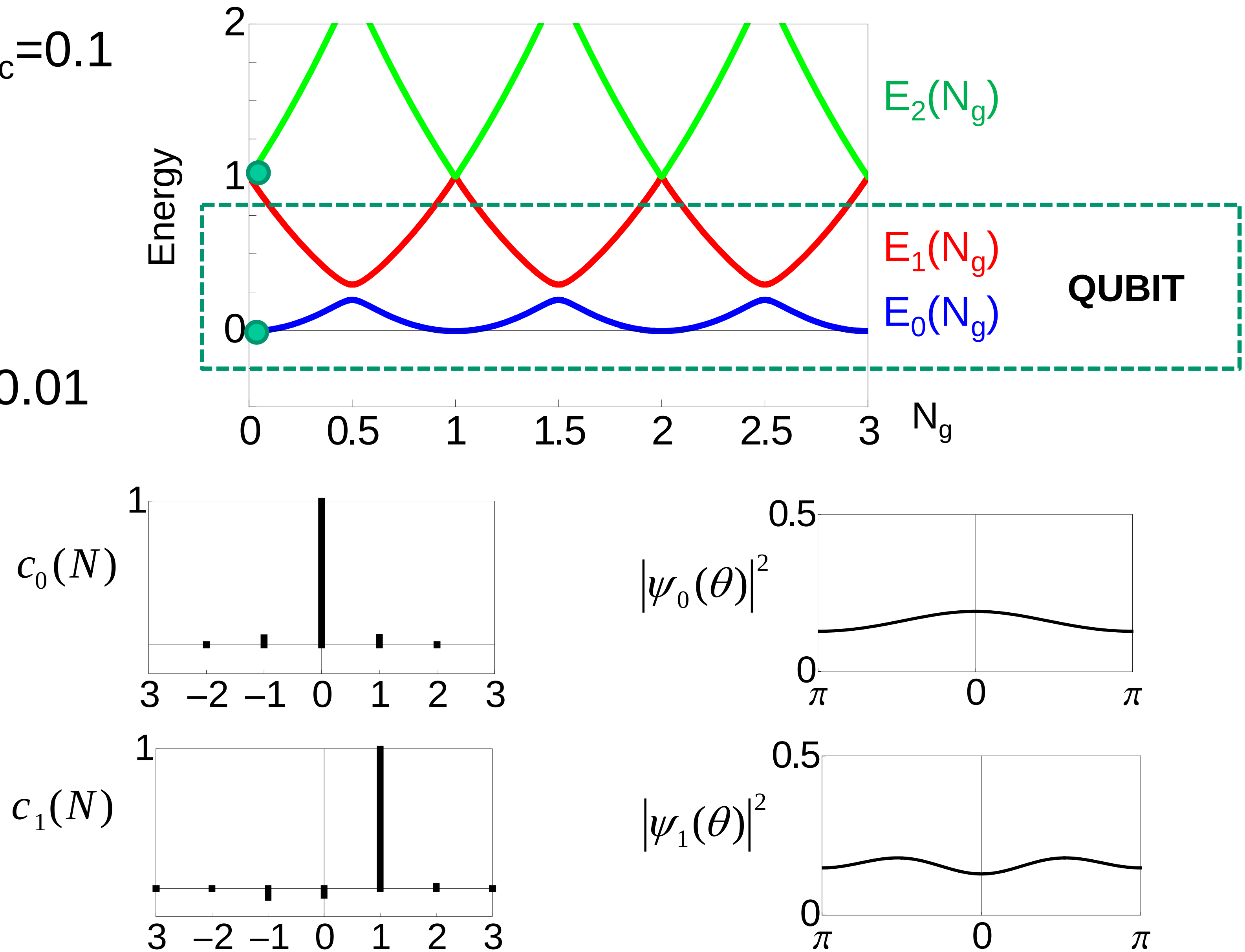
Two simple limits : (1) $E_J(\Phi) \ll E_C$ (charge regime)



Two simple limits : (1) $E_J(\Phi) \ll E_C$ (charge regime)

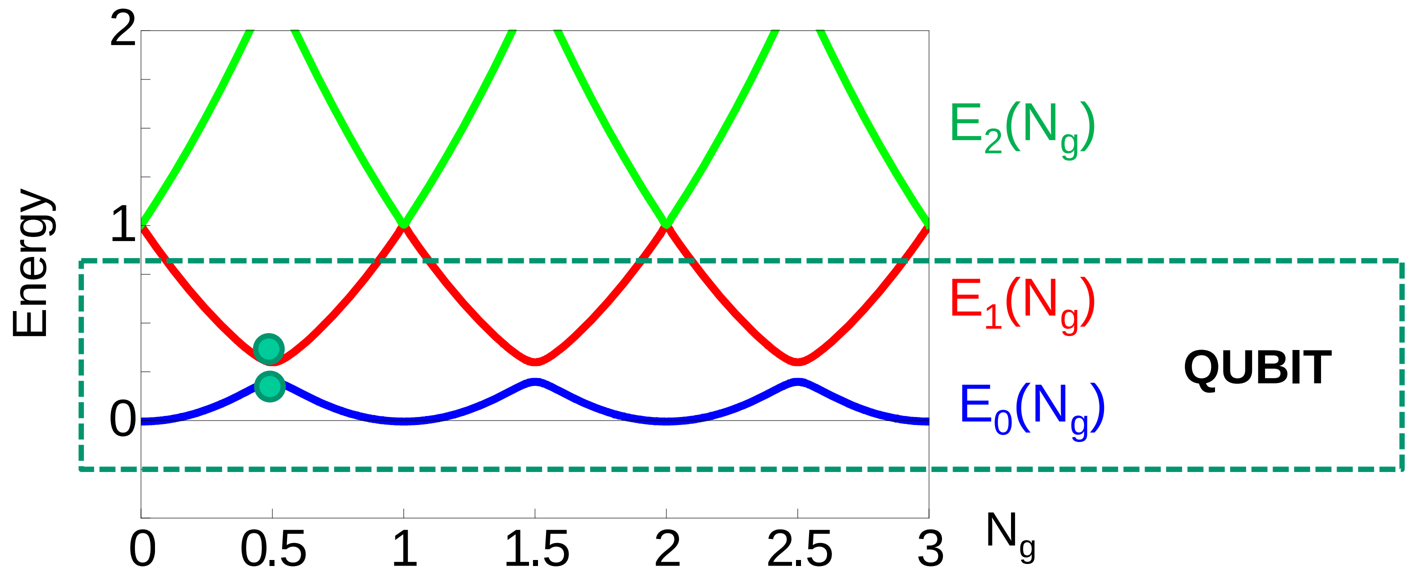
$E_J/E_c=0.1$

$N_g=0.01$

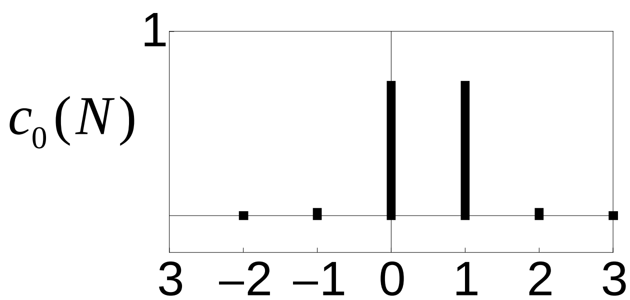


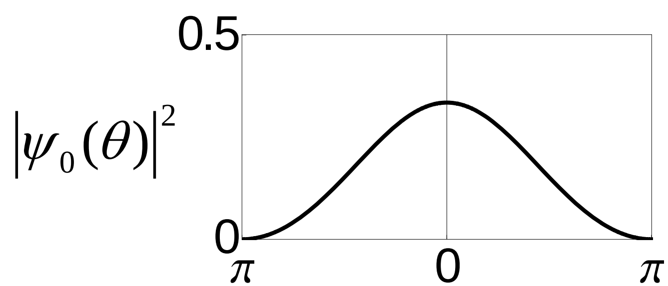
Two simple limits : (1) $E_J(\Phi) \ll E_C$ (charge regime)

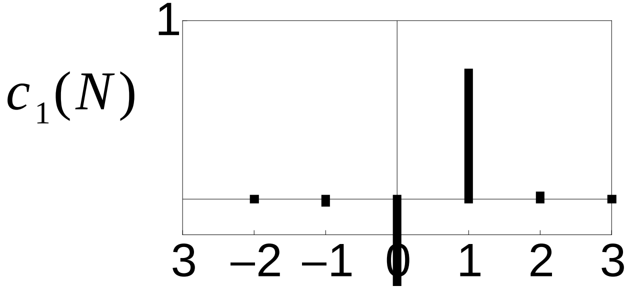
$E_J/E_c=0.1$

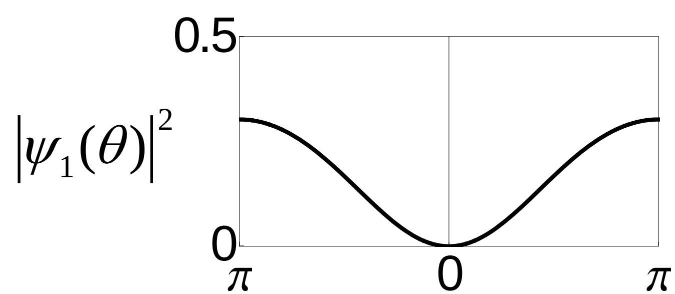


$N_g=0.5$

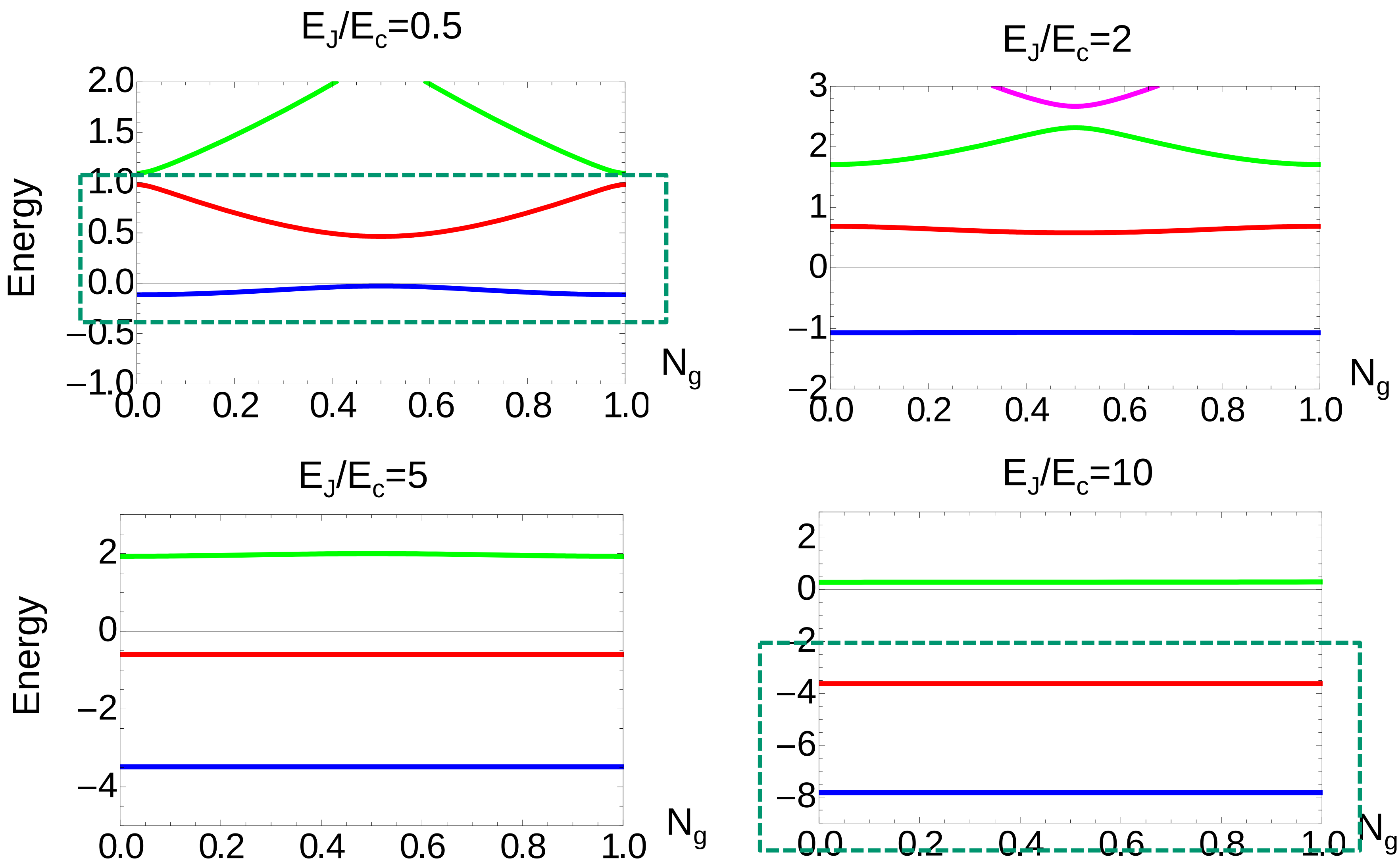








From $E_J(\Phi) \ll E_C$ to $E_J(\Phi) \gg E_C$

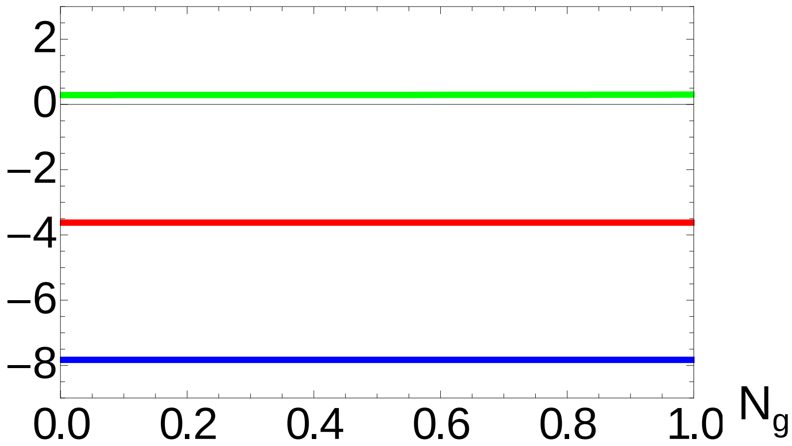


STILL A QUBIT !

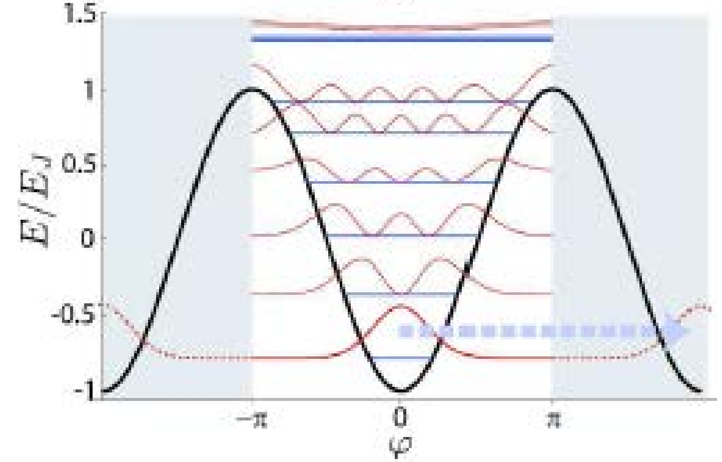
Two simple limits : (2) $E_J(\Phi) \gg E_C$ (phase regime)

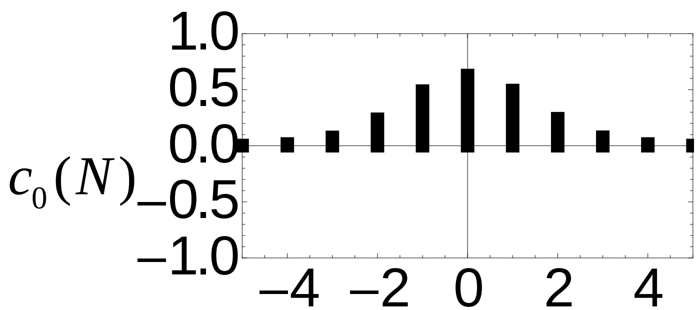
Transmon qubit

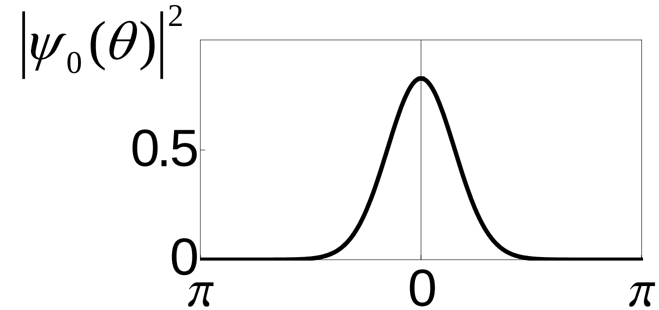
$E_J/E_c=10$

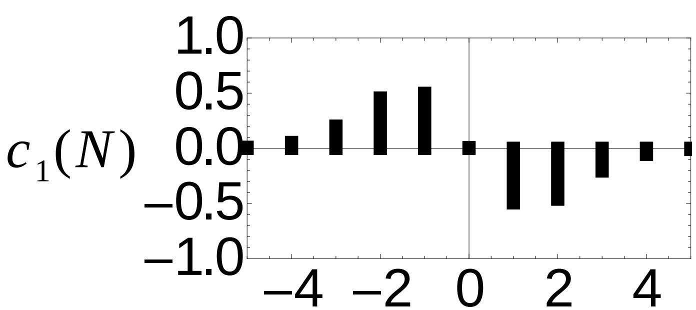


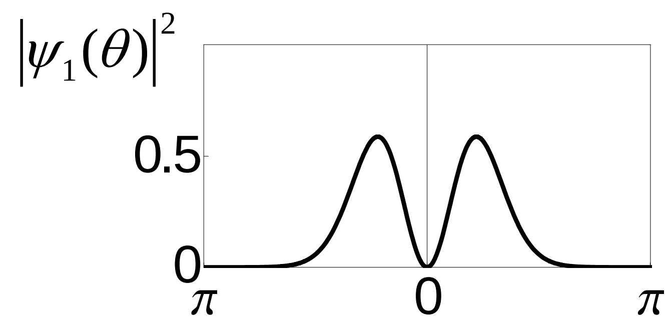
J. Koch et al., PRA (2008)





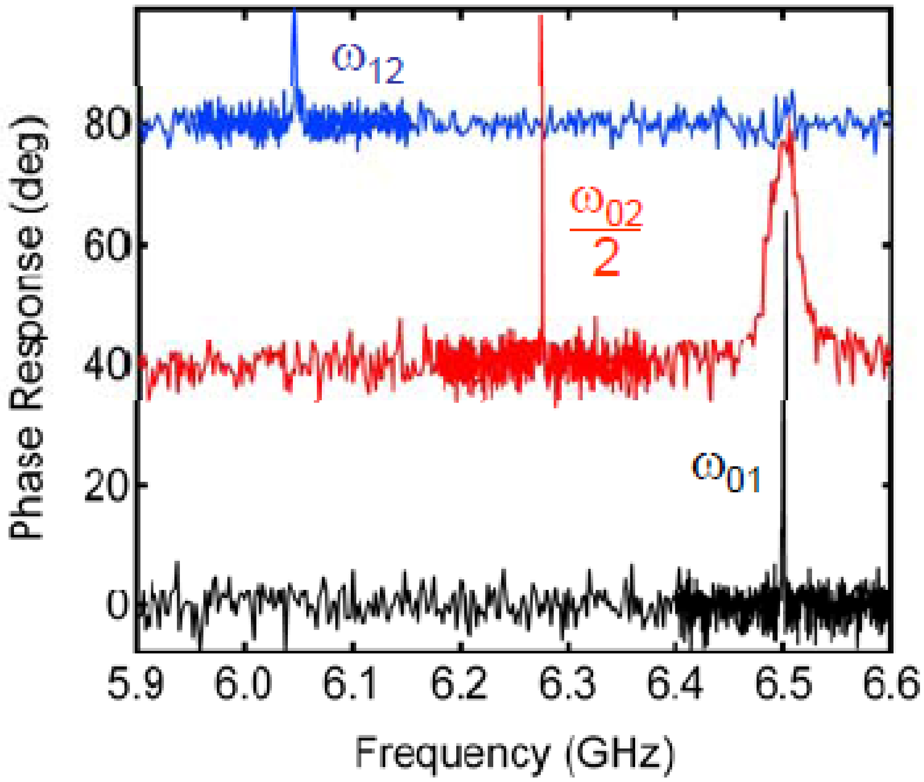


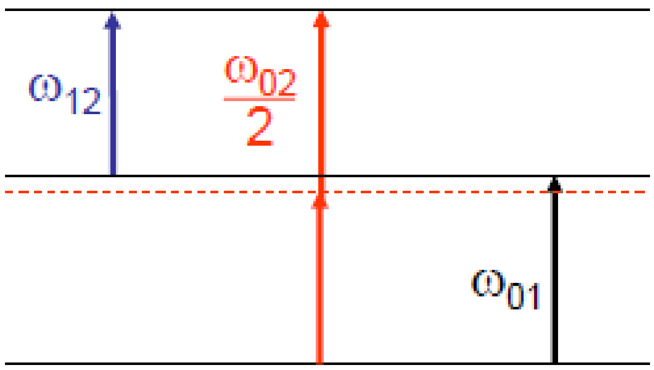




Experimental spectrum of a transmon

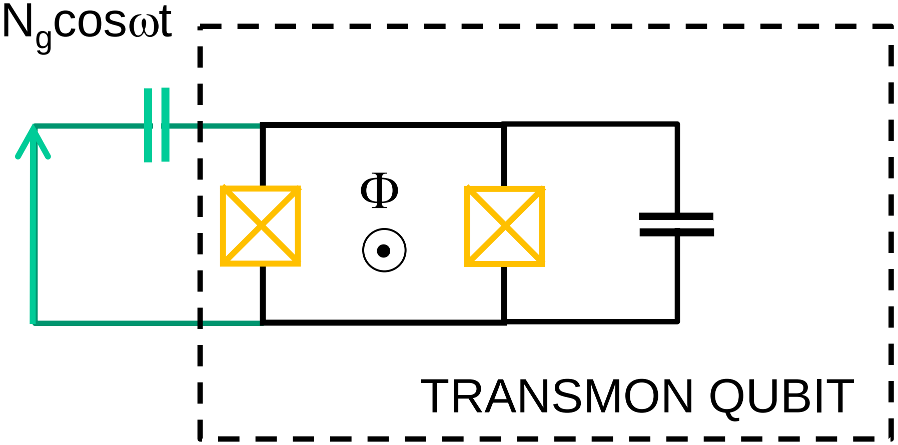
J. Schreier et al., PRB (2008)





One-qubit gates

$N_g(t)=\Delta N_g \cos\omega t$



$$\hat{H} = E_C\left(\hat{N} - N_g(t)\right)^2 - E_J\cos\hat{\theta}$$

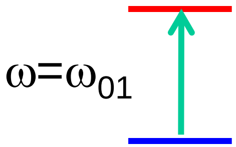
$$\hat{H} = \underbrace{E_C\hat{N}^2 - E_J\cos\hat{\theta}}_{\text{transmon}} - \underbrace{2E_C\Delta N_g\cos\omega t\hat{N}}_{\text{drive}}$$

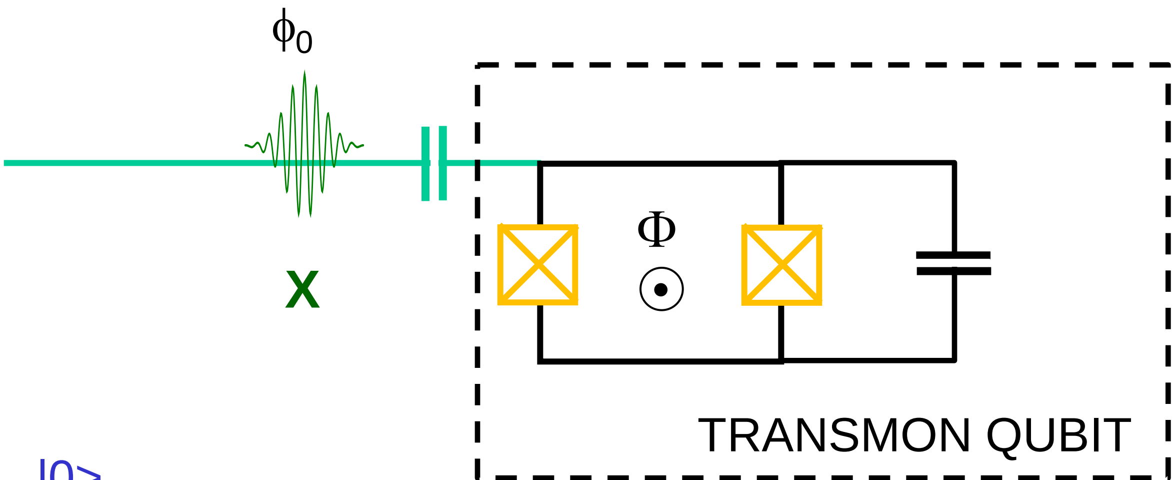
Two-level approximation

$$= -\hbar\frac{\omega_{01}(\Phi)}{2}\sigma_z \qquad = -\hbar\frac{\omega_{01}(\Phi)}{2}\sigma_z$$

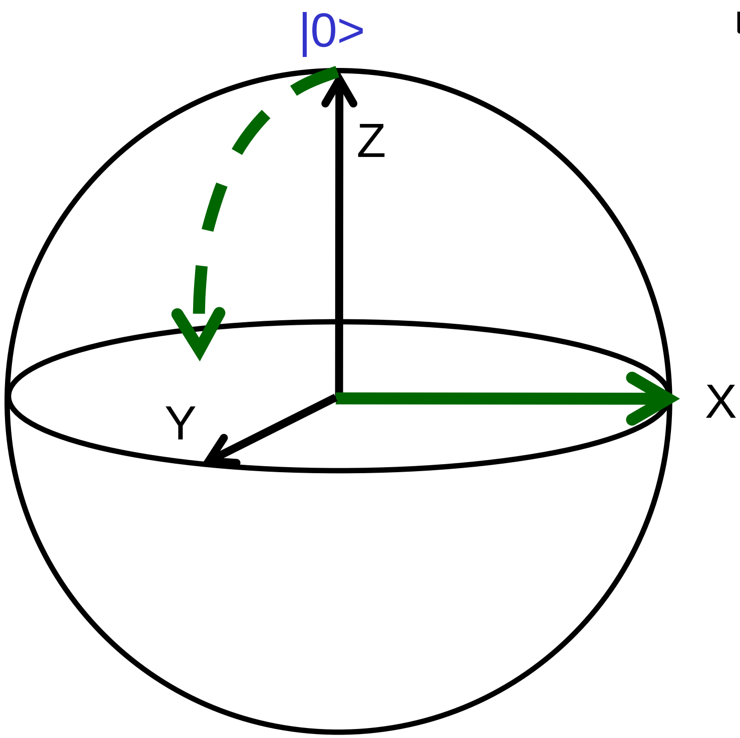
$$\hbar\Omega_R = 2E_C\langle 0|\hat{N}|1\rangle\Delta N_g$$

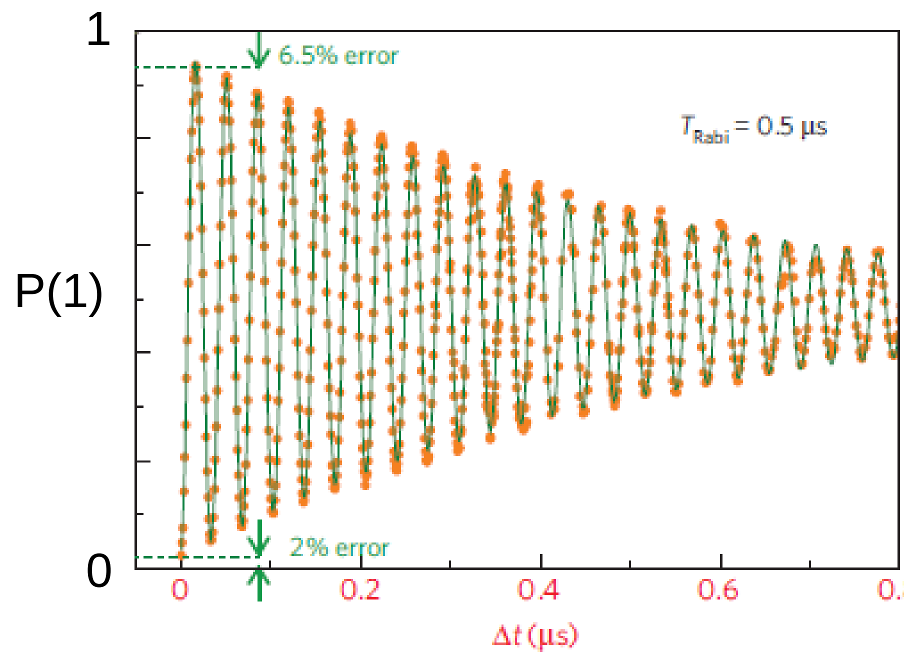
One-qubit gates





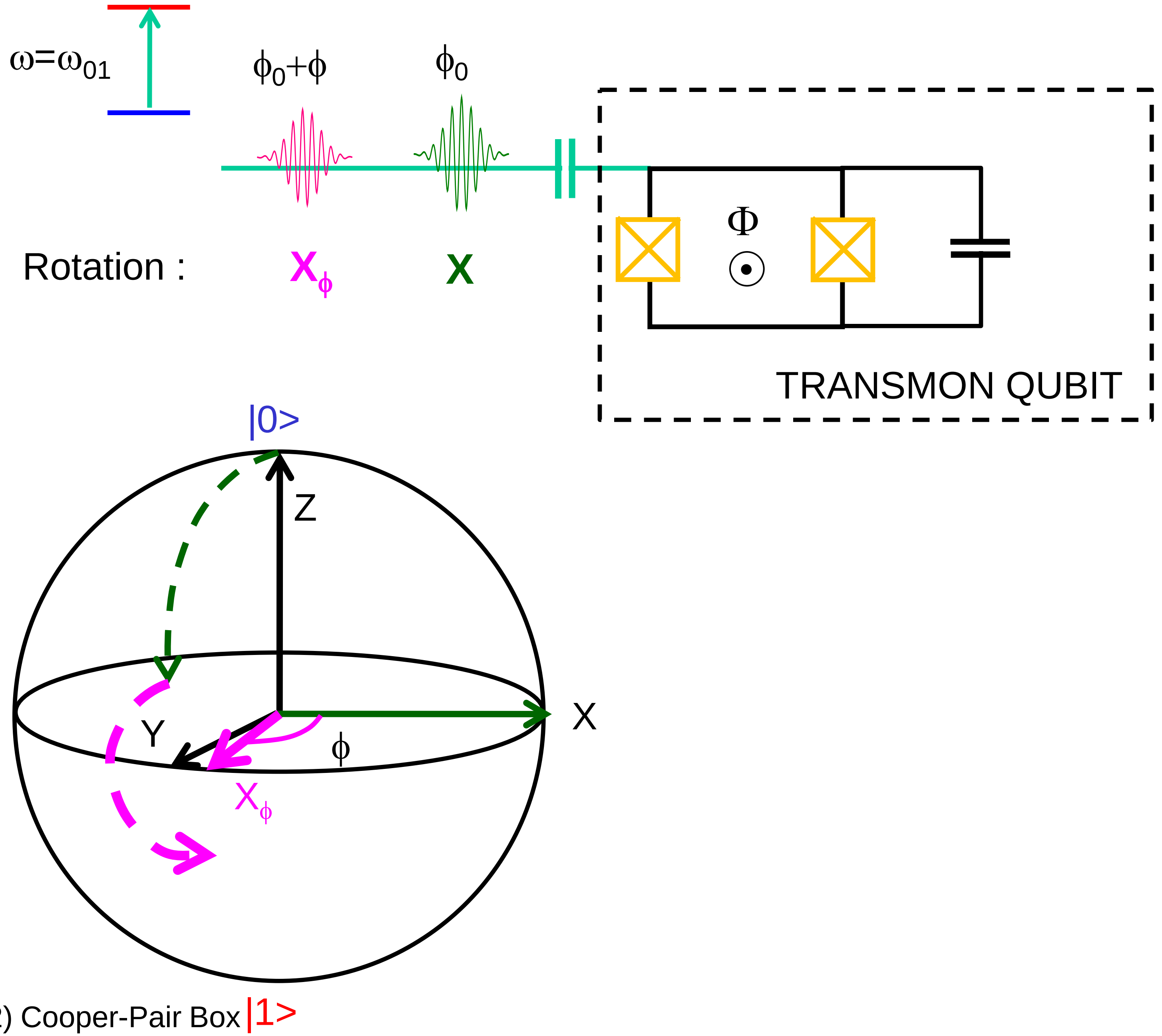
Rotation : **X**



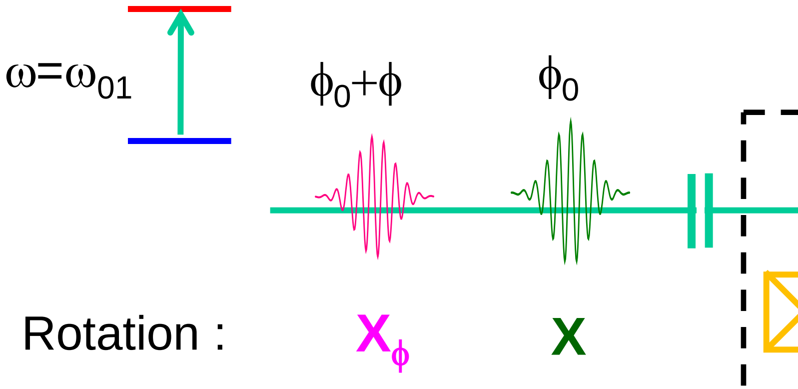


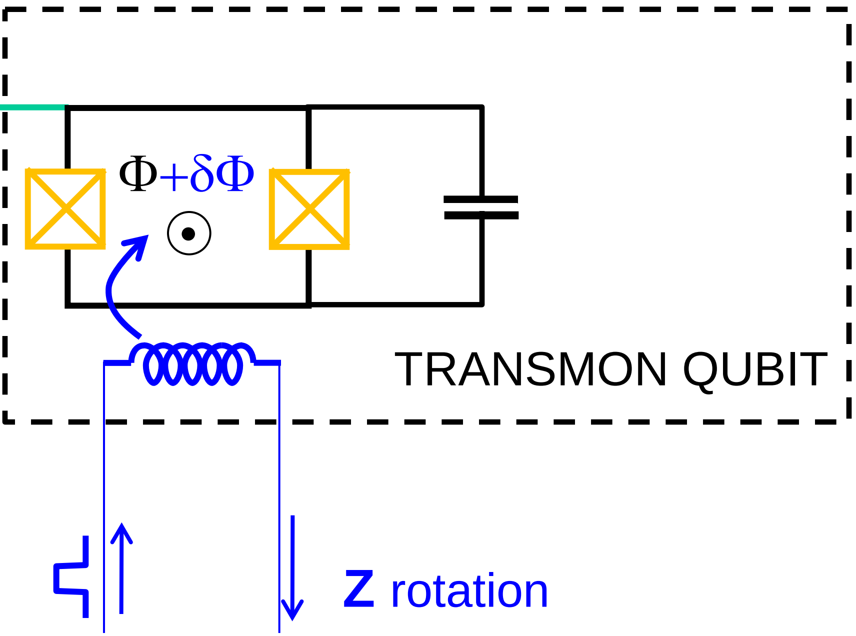
F. Mallet et al., Nature Phys. (2009)

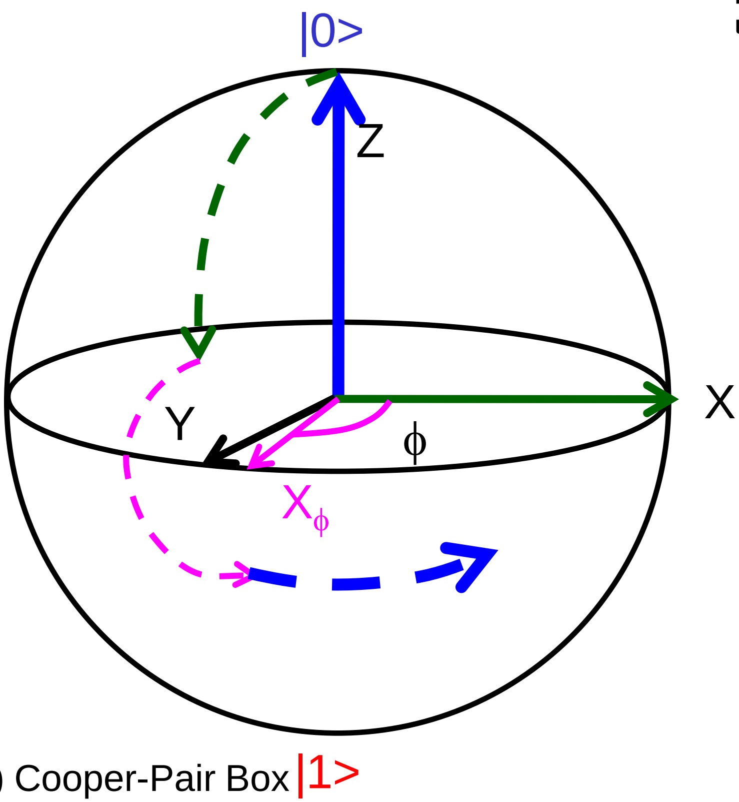
One-qubit gates



One-qubit gates





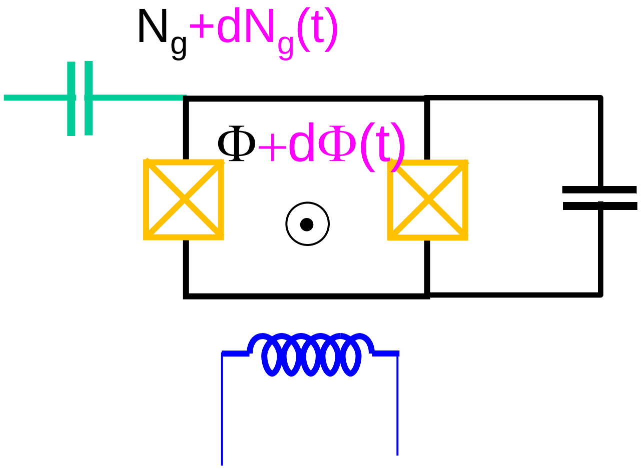


All rotations on Bloch sphere

Fidelity ? >99.9%

R. Barends et al., Nature (2014)

Decoherence



Noise in Hamiltonian parameters

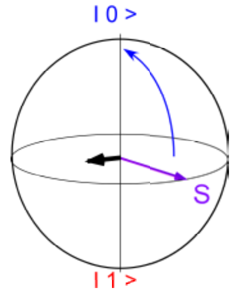
⟹ **DECOHERENCE**

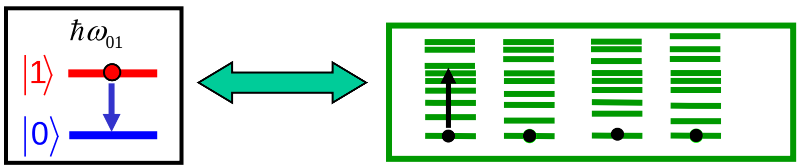
MAJOR OBSTACLE TO QUANTUM COMPUTING

Decoherence in superconducting qubits

(Ithier et al., PRB 72, 134519, 2005)

Relaxation
(Spontaneous emission)



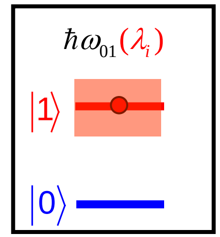


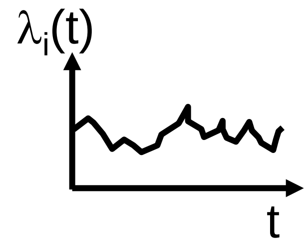
$$D_{\lambda,\perp} = \frac{1}{\hbar}\langle 0|\frac{\partial H}{\partial \lambda}|1\rangle$$

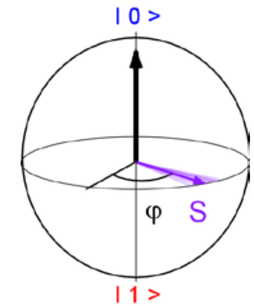
$$T_1^{-1} = \Gamma_1 = \frac{\pi}{2} D_{\lambda,\perp}^2 S_\lambda(\omega_{01})$$

environmental **density of modes** at qubit frequency

Pure dephasing







$$\alpha|0\rangle + \beta e^{i\varphi(t)}|1\rangle$$

$$f_\varphi(t) \equiv \left\langle e^{i\varphi(t)} \right\rangle \approx e^{-\Gamma_2 t}$$

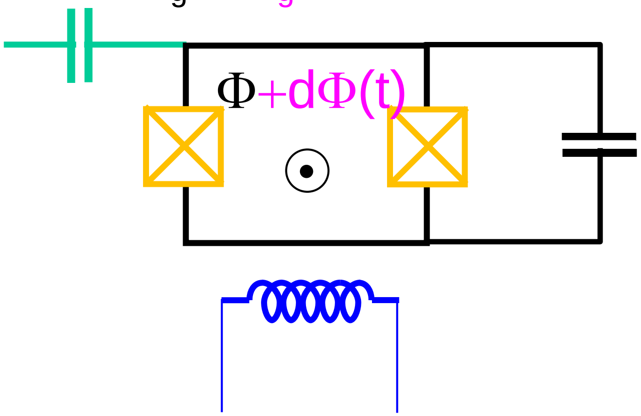
$$T_2^{-1} = \Gamma_2 = \frac{\Gamma_1}{2} + \Gamma_\varphi$$

$$\Gamma_\varphi = \pi D^2_{\lambda,z} S_\lambda(0)$$

$$D_{\lambda,z} = \frac{\partial \omega_{01}}{\partial \lambda}$$

Low-frequency noise

Decoherence

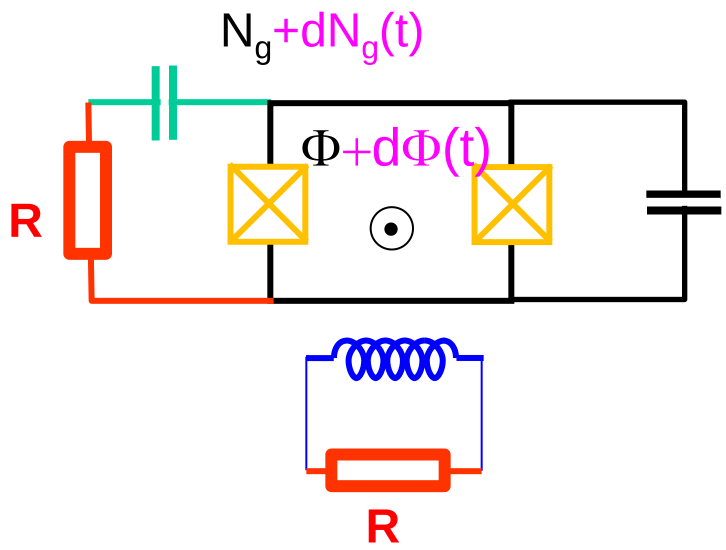


Noise in Hamiltonian parameters

⟹ **DECOHERENCE**

Origin of the noise ???

Decoherence



Noise in Hamiltonian parameters

⟹ **DECOHERENCE**

Origin of the noise ???

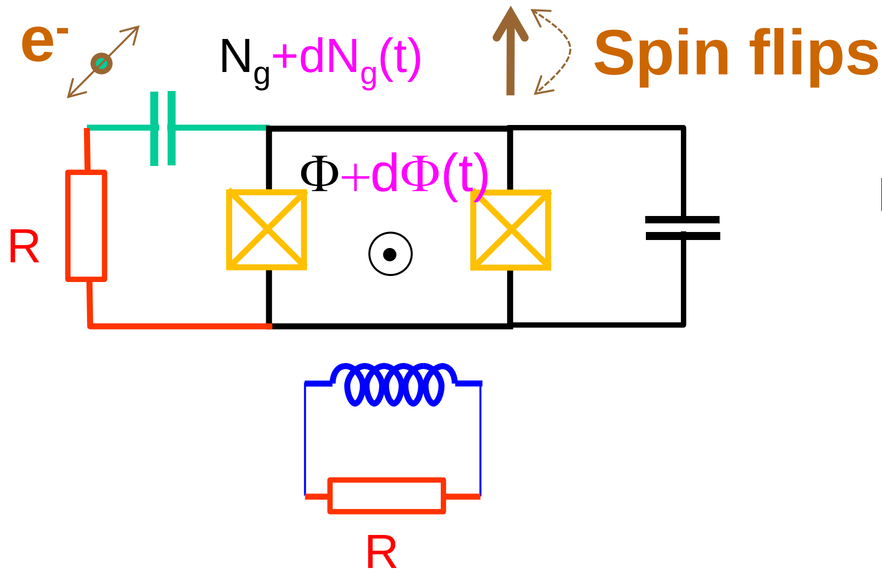
1) **ELECTROMAGNETIC**

→ Low-frequency : Johnson-Nyquist due to thermal noise

→ High-frequency : spontaneous emission (quantum noise)

≈ Under control

Decoherence



Noise in Hamiltonian parameters

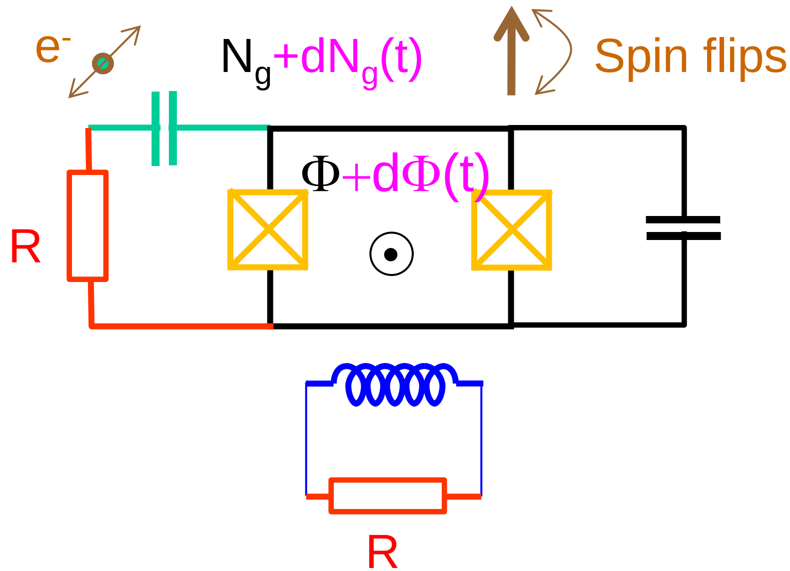
⟹ **DECOHERENCE**

Origin of the noise ???

2) MICROSCOPIC

	Charge noise	Flux noise
Low-frequency noise	$S_{N_g}(\omega) \approx (10^{-3})^2/\omega$	$S_{\Phi}(\omega) \approx (10^{-6}\Phi_0)^2/\omega$

Decoherence



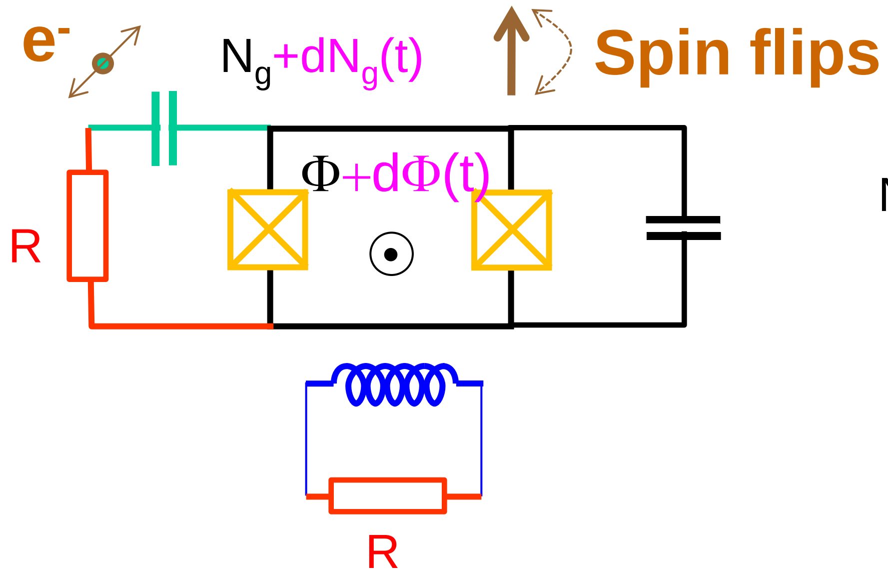
Noise in Hamiltonian parameters

⟹ **DECOHERENCE**

Origin of the noise ???

2) MICROSCOPIC	Charge noise	Flux noise
Low-frequency noise	$S_{N_g}(\omega) \approx (10^{-3})^2/\omega$	$S_{\Phi}(\omega) \approx (10^{-6}\Phi_0)^2/\omega$
CPB in charge regime	$T_2 \approx 10 - 100\text{ns}$	$T_2 \approx 1 - 100\mu s$
Transmon	$T_2 \approx 10 - 100\text{ms}$	$T_2 \approx 1 - 100\mu s$

Decoherence



Noise in Hamiltonian parameters

⟹ **DECOHERENCE**

Origin of the noise ???

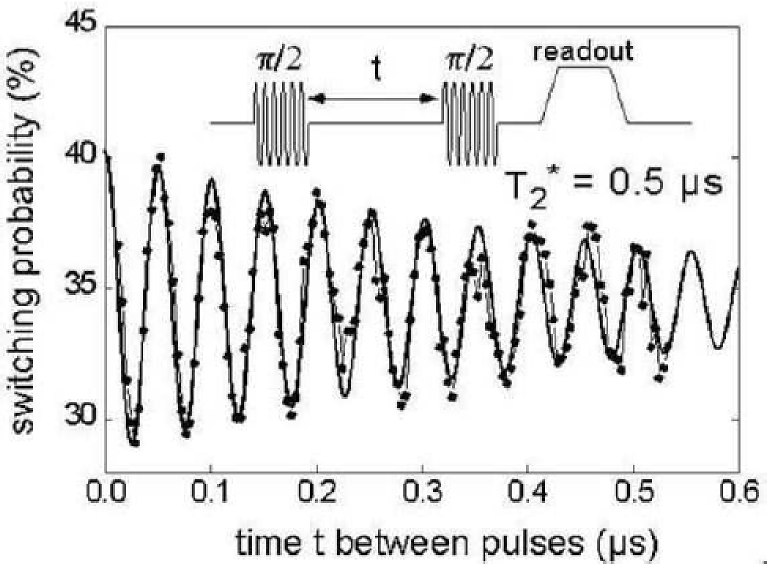
2) MICROSCOPIC

High-frequency « noise » (or equivalently « losses ») responsible for finite T1

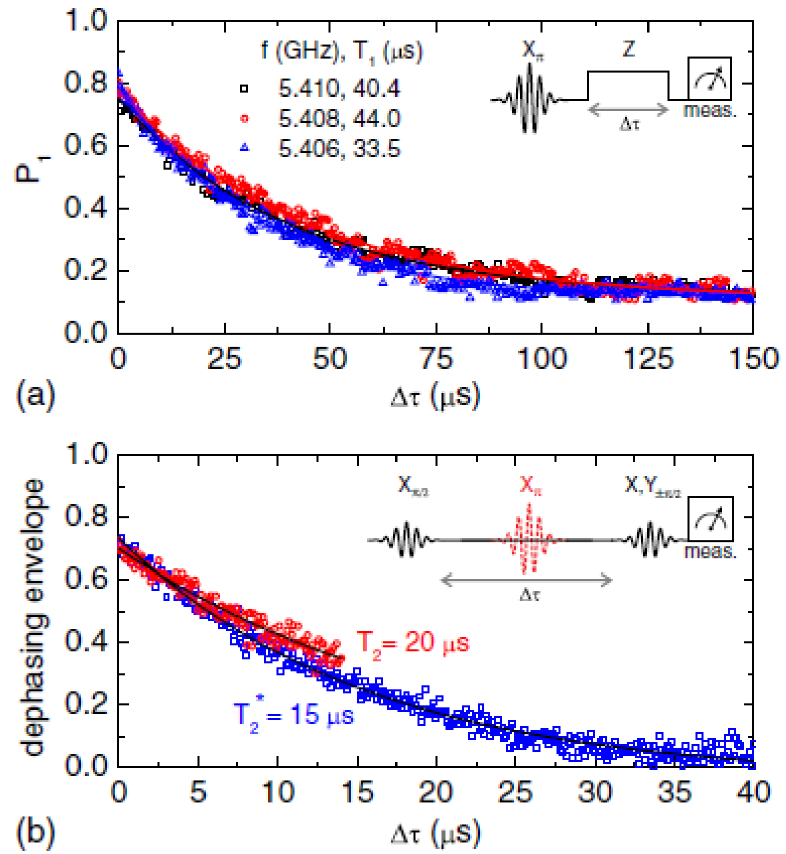
- Imperfect dielectrics, causing microwave losses
- Magnetic vortices trapped in the superconducting thin films
- Unpaired electrons (« Quasiparticles ») in the superconductors

Coherence times : measurements

1999 (Nakamura et al., Nature):
$T_1 \approx T_2 \approx$ 1ns



2002 : (« Quantronium experiment »)
$T_1 = 2\mu s$, T_2^*=500ns



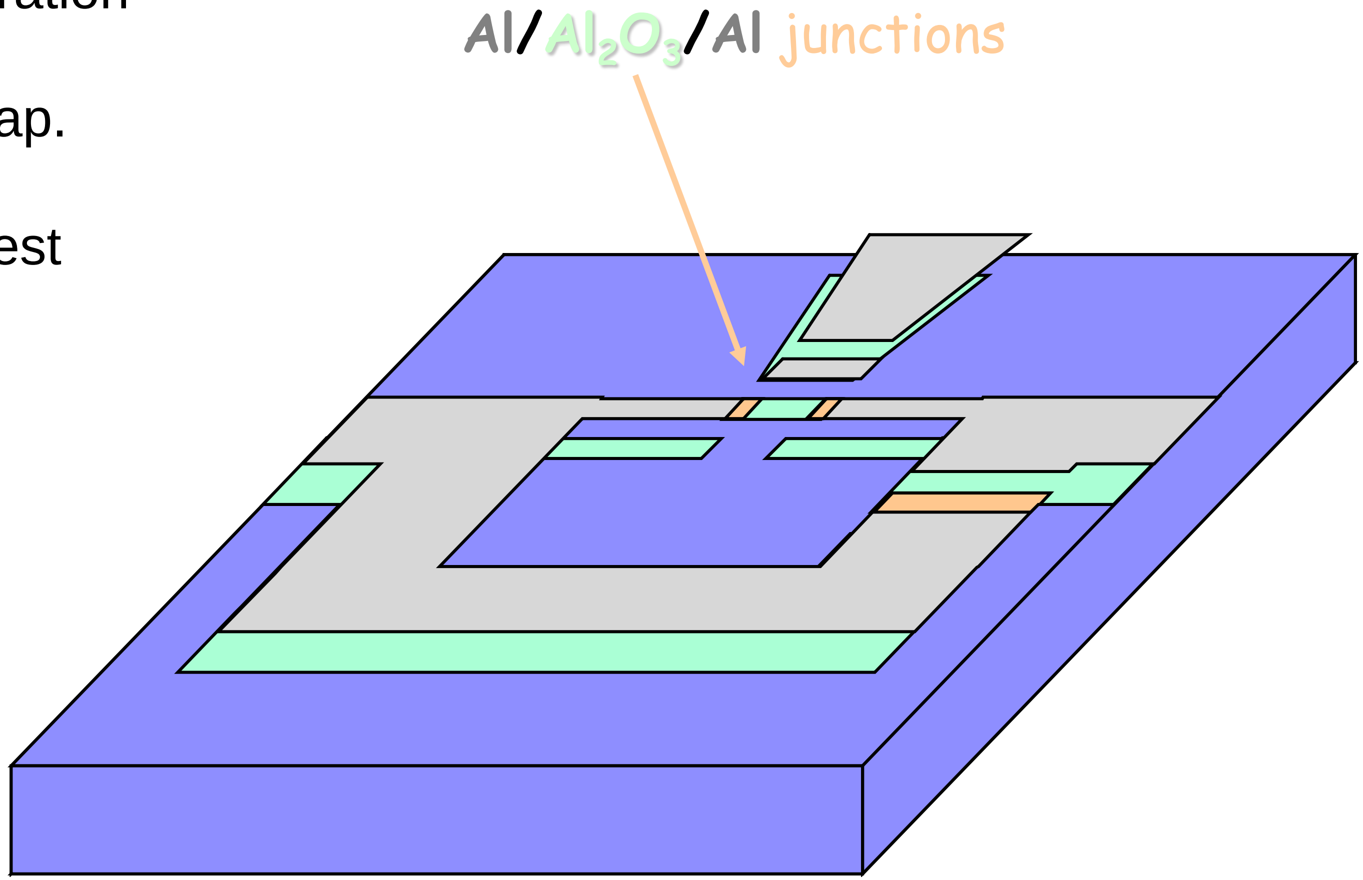
2013 (Barends et al., PRL)
$T_1 = 30 - 100\mu s$, $T_2 = 1 - 30\mu s$

ULTIMATE LIMITS ON COHERENCE TIMES UNKNOWN YET

Fabrication techniques

small junctions → e-beam lithography

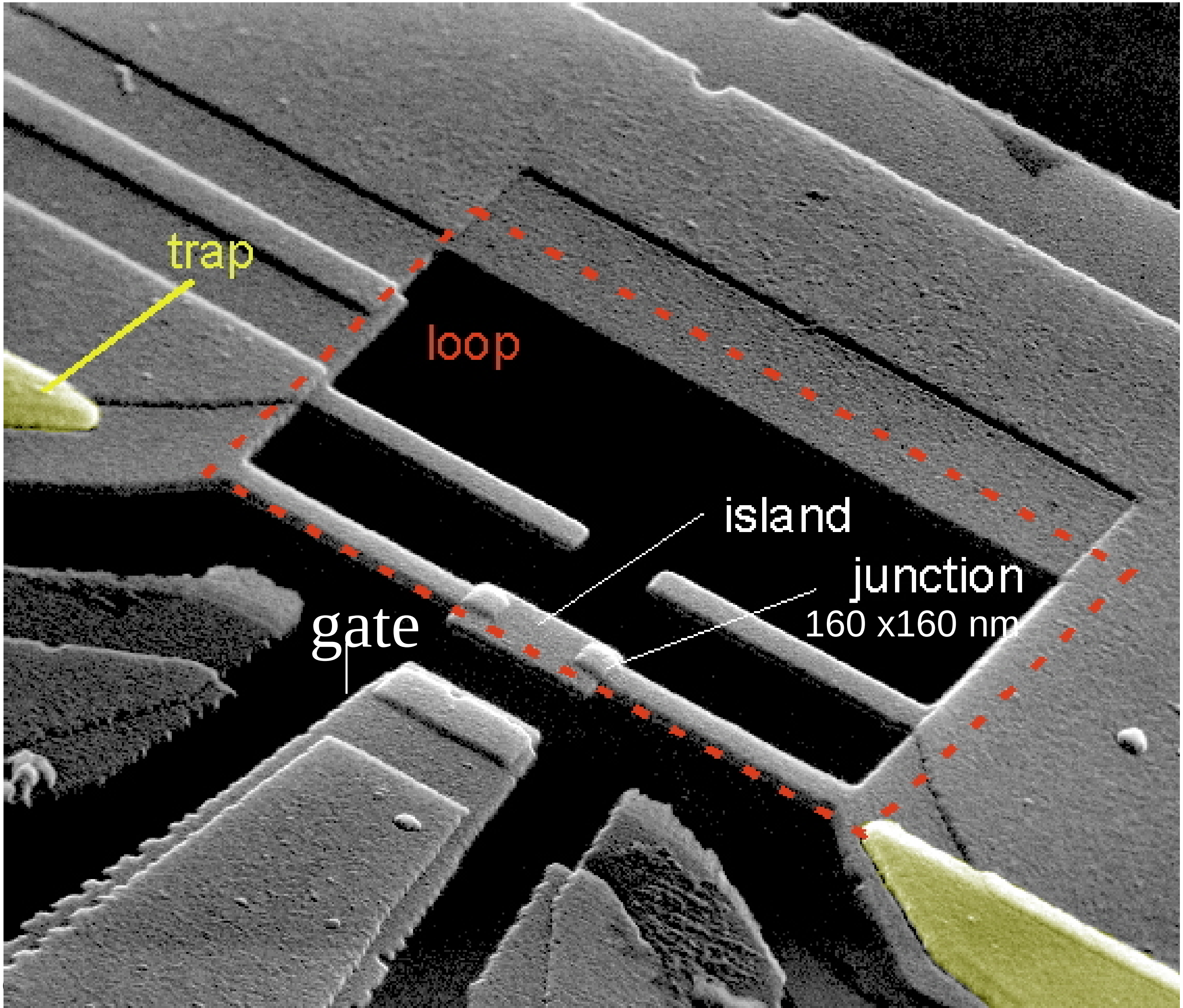
1) e-beam patterning
2) development
3) first evaporation
4) oxidation
5) second evap.
6) lift-off
7) electrical test



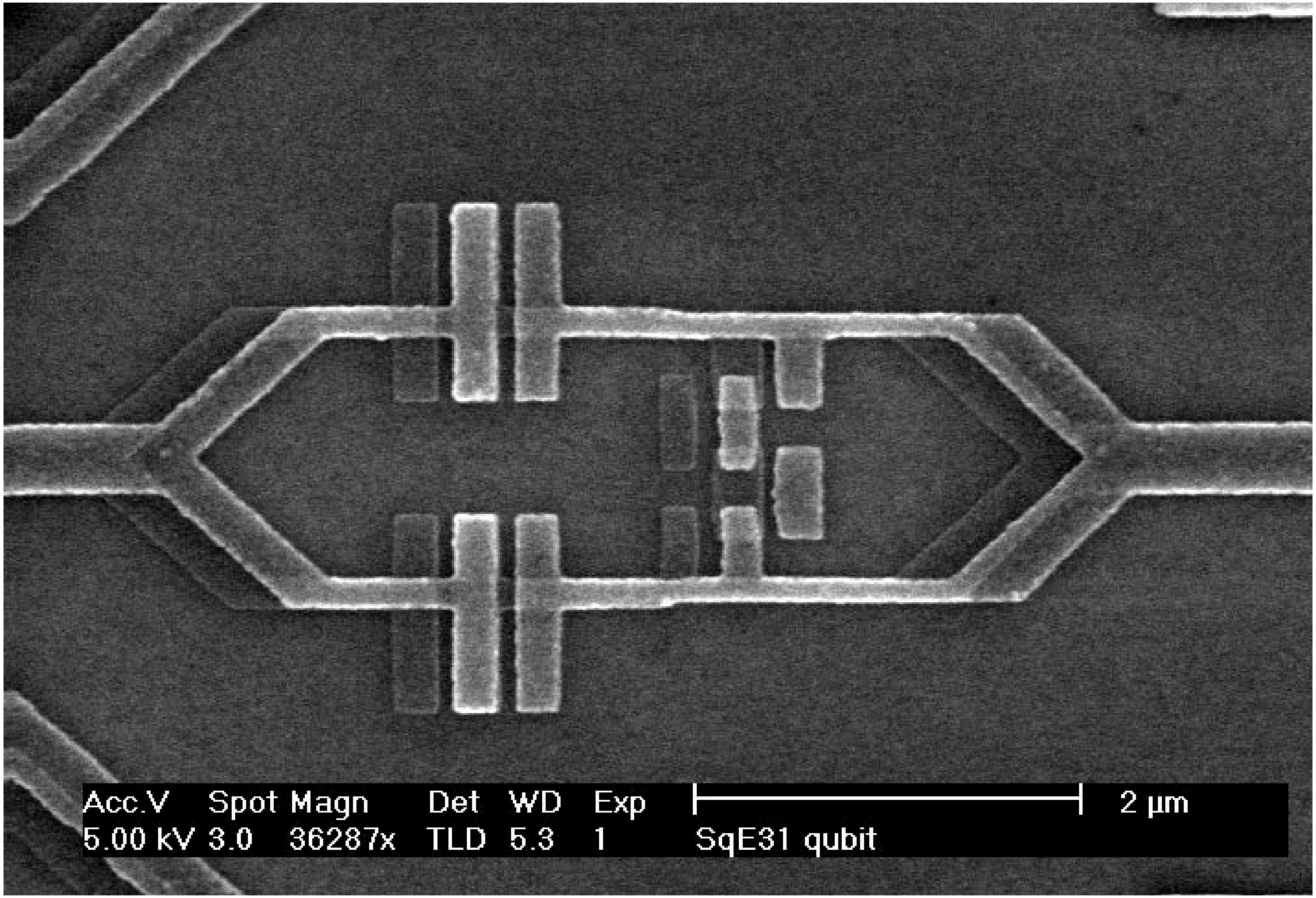
SiO_2

small junctions → Multi angle shadow evaporation

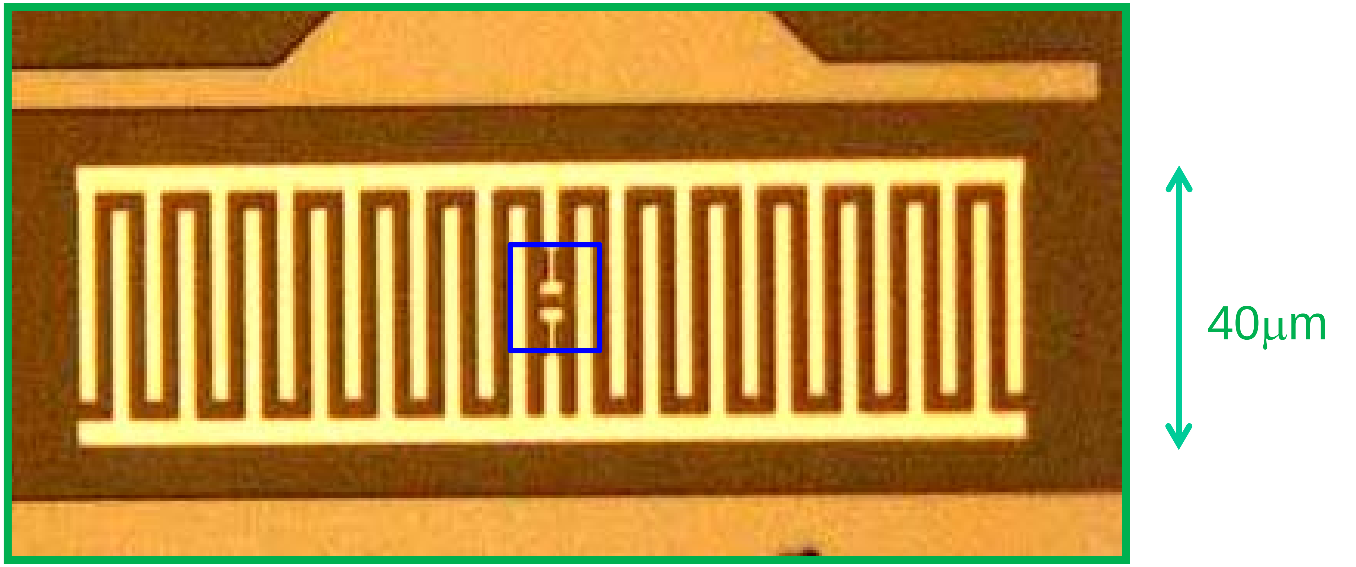
QUANTRONIUM (Saclay group)

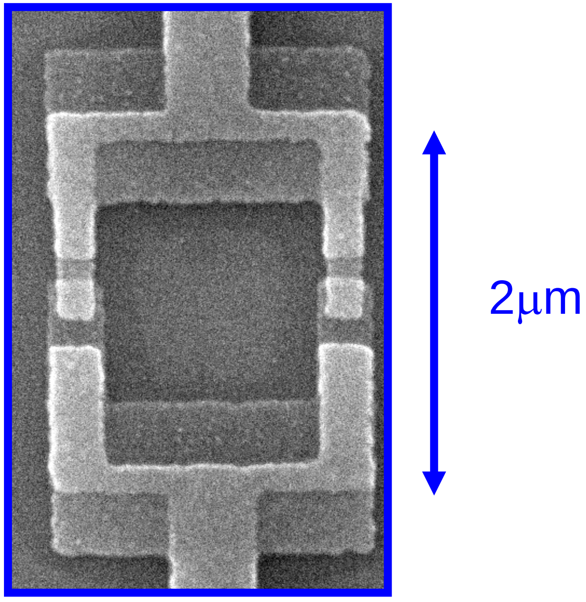


FLUX-QUBIT (Delft group)



TRANSMON QUBIT (Saclay group)





END OF FIRST LECTURE