Instituto de Física
Universidade Federal Fluminense

Entangled structures in classical and quantum optics

Antonio Zelaquett Khoury

Outline

Lecture 1:

Optical vortices as entangled structures in classical and quantum optics

Lecture 2:

Quantum-like simulations and the role of quantum inequalities

Lecture 3:

Quantization of the electromagnetic field

Lecture 4:

Vector beam quantization and the unified framework

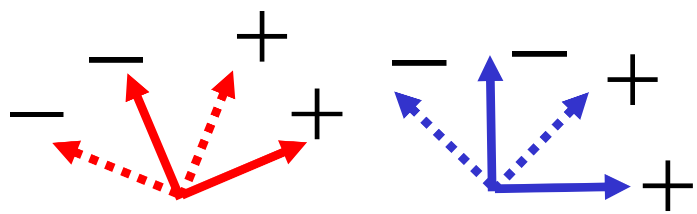
Outline

Lecture 2:

Quantum-like simulations and the role of quantum inequalities

Bell-like inequality for spin-orbit modes

Bell measurement



$$\sigma_A(\theta) = \pm 1 \qquad \sigma_B(\phi) = \pm 1$$

$$\sigma_Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = |0\rangle\langle 0| - |1\rangle\langle 1|$$

$$\sigma_X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = |0\rangle\langle 1| - |1\rangle\langle 0|$$

$$\sigma(\alpha) = \cos\alpha\,\sigma_Z + \sin\alpha\,\sigma_X$$

$$M(\theta,\phi) = \langle \sigma_A(\theta) \otimes \sigma_B(\phi) \rangle = P_{++} + P_{--} - P_{+-} - P_{-+}$$

$$S = M\left(\frac{\pi}{8}, 0\right) + M\left(\frac{\pi}{8}, \frac{\pi}{4}\right) - M\left(\frac{3\pi}{8}, 0\right) + M\left(\frac{3\pi}{8}, \frac{\pi}{4}\right)$$

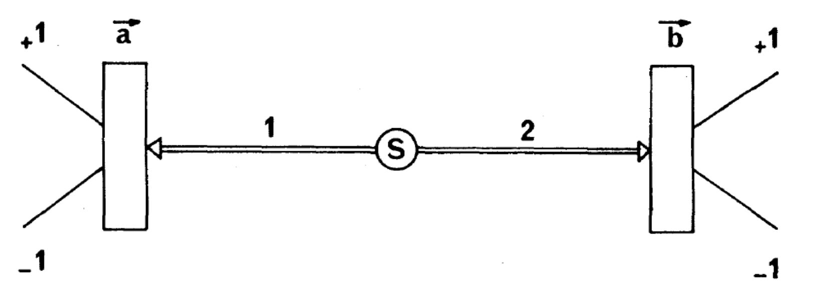
Clauser-Horne-Shimony-Holt

$|S| < 2 \rightarrow$ classical

$|S| > 2 \rightarrow$ quantum

$S = 2\sqrt{2} \rightarrow \mathrm{Bell-like} \rightarrow$ quantum bound

A. Aspect, P. Grangier, and G. Roger
Phys. Rev. Lett. **49**, 91 (1982)



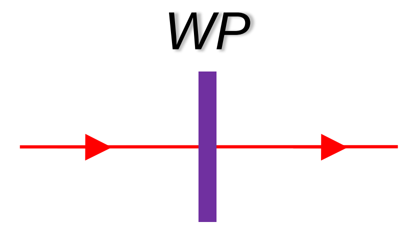
Spin-orbit rotated basis

$$\mathbf{E}(\mathbf{r}) = E_0 \boldsymbol{\Psi}(\mathbf{r})$$

$$\boldsymbol{\Psi} = \alpha \psi_H(\mathbf{r})\hat{\mathbf{e}}_H + \beta \psi_H(\mathbf{r})\hat{\mathbf{e}}_V + \gamma \psi_V(\mathbf{r})\hat{\mathbf{e}}_H + \delta \psi_V(\mathbf{r})\hat{\mathbf{e}}_V$$

Rotated Basis

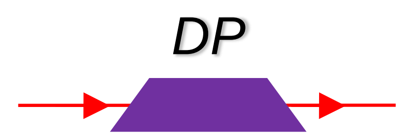
$$\left.\begin{aligned} \hat{\mathbf{e}}_{\theta+} &= \cos\theta\, \hat{\mathbf{e}}_V + \sin\theta\, \hat{\mathbf{e}}_H \\ \hat{\mathbf{e}}_{\theta-} &= \sin\theta\, \hat{\mathbf{e}}_V - \cos\theta\, \hat{\mathbf{e}}_H \end{aligned}\right\} \text{Wave plates}$$



$$\hat{\mathbf{e}}_{\theta+} \to \hat{\mathbf{e}}_H$$

$$\hat{\mathbf{e}}_{\theta-} \to \hat{\mathbf{e}}_V$$

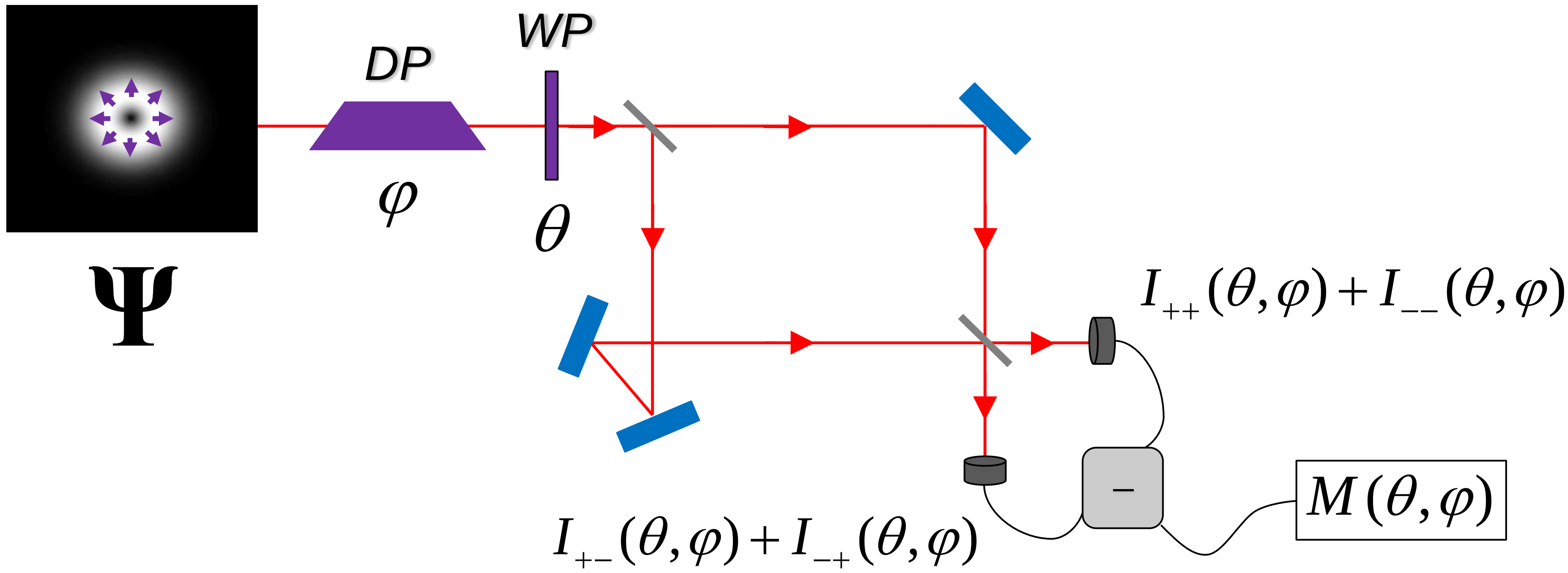
$$\left.\begin{aligned} \psi_{\varphi+} &= \cos\varphi\, \psi_V + \sin\varphi\, \psi_H \\ \psi_{\varphi-} &= \sin\varphi\, \psi_V - \cos\varphi\, \psi_H \end{aligned}\right\} \text{Dove prism}$$



$$\psi_{\varphi+} \to \psi_H$$

$$\psi_{\varphi-} \to \psi_V$$

Spin-orbit Bell measurement



$$I_{\pm\pm}(\theta,\varphi)=\frac{\left|\iint\left[\hat{e}^{*}_{\theta\pm}\cdot\mathbf{E}(\mathbf{r})\right]\psi^{*}_{\varphi\pm}(\mathbf{r})\,d^{2}\mathbf{r}\right|^{2}}{\left|E_{0}\right|^{2}}$$

Projected intensities

Bell-like Inequality for Spin-Orbit Separability

$$I_{\pm\pm}(\theta,\varphi) = \left| \int \left[\hat{e}^{*}_{\theta\pm} \cdot \mathbf{\Psi}(\mathbf{r}) \right] \psi^{*}_{\varphi\pm}(\mathbf{r})\, d^2\mathbf{r} \right|^2$$

Projected intensities

$$M(\theta,\varphi) = I_{++}(\theta,\varphi) + I_{--}(\theta,\varphi) - I_{+-}(\theta,\varphi) - I_{-+}(\theta,\varphi)$$

$$S = M\left(\frac{\pi}{8},0\right) + M\left(\frac{\pi}{8},\frac{\pi}{4}\right) - M\left(\frac{3\pi}{8},0\right) + M\left(\frac{3\pi}{8},\frac{\pi}{4}\right)$$

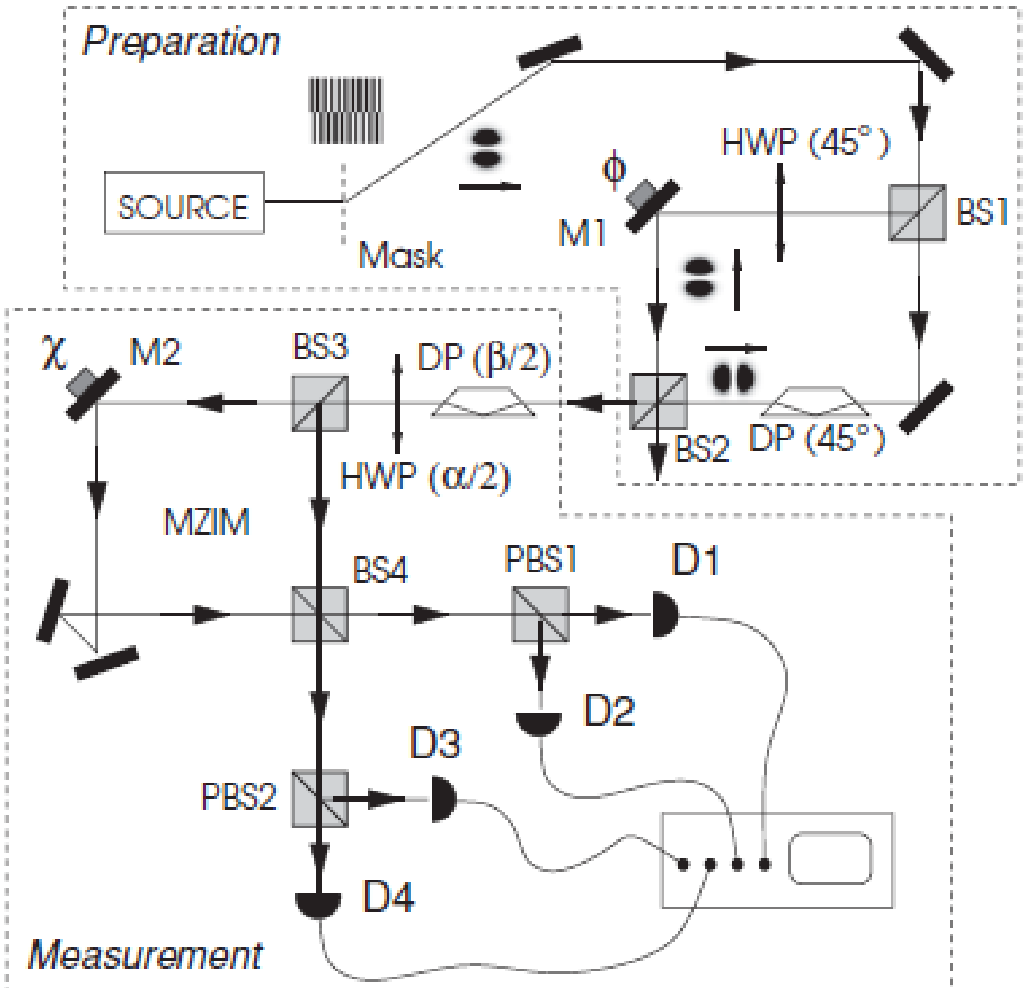
$S < 2 \rightarrow \text{separable}$

$S > 2 \rightarrow \text{nonseparable}$

$S = 2\sqrt{2} \rightarrow \text{Bell} - \text{like}$

C. V. S. Borges, M. Hor-Meyll, J. A. O. Huguenin, and A. Z. Khoury
Phys. Rev. A 82, 033833 (2010)

Bell-like Inequality for Spin-Orbit Separability



C. V. S. Borges, M. Hor-Meyll, J. A. O. Huguenin, and A. Z. Khoury
Phys. Rev. A 82, 033833 (2010)

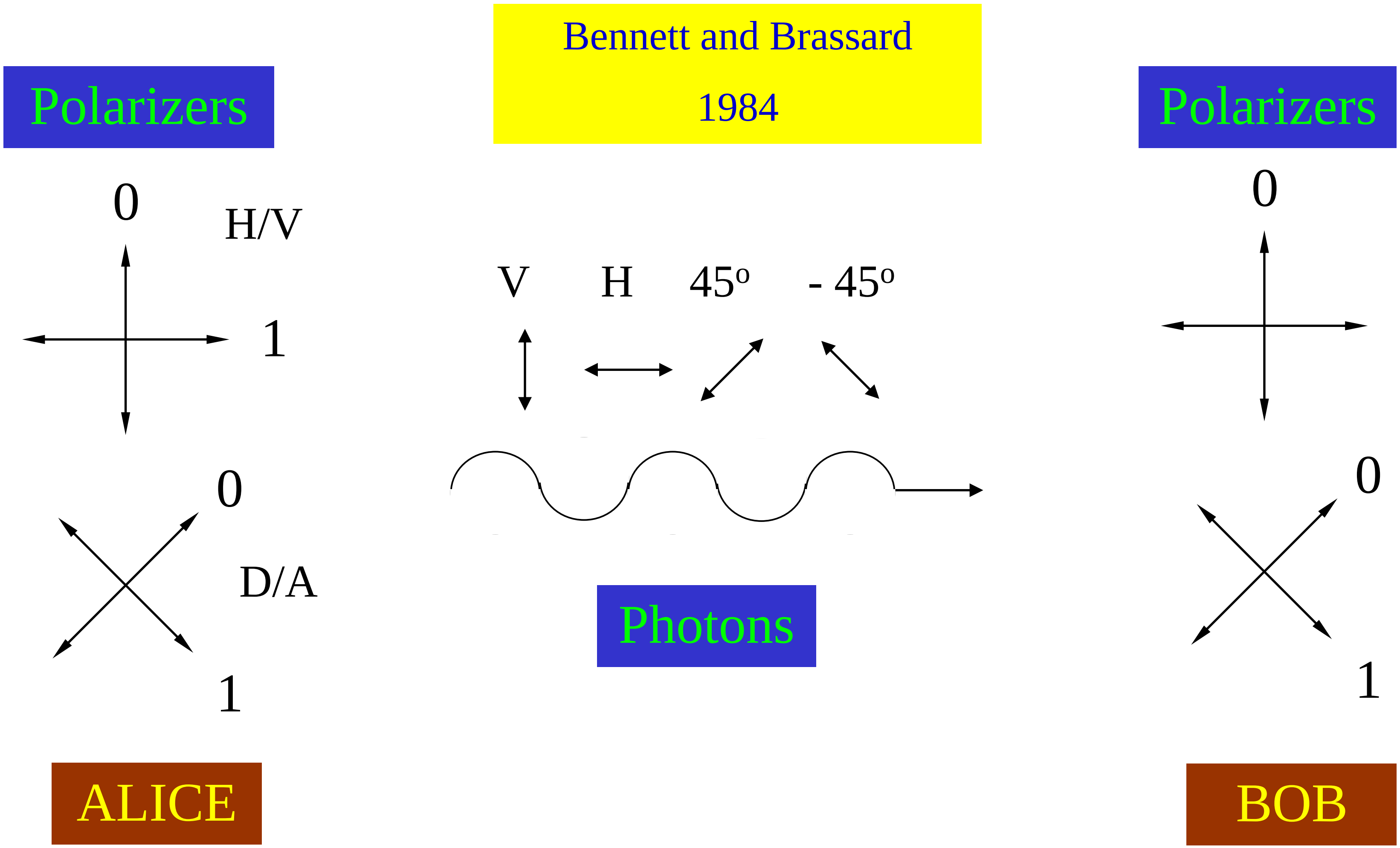
Bell-like Inequality for Spin-Orbit Separability

	Maximally Non-Separable		Separable	
	Theory	Experiment	Theory	Experiment
$M(\alpha_1, \beta_1)$	0.707	0.679	0.707	0.665
$M(\alpha_1, \beta_2)$	0.707	0.583	0.000	0.000
$M(\alpha_2, \beta_1)$	-0.707	-0.679	-0.707	-0.661
$M(\alpha_2, \beta_2)$	0.707	0.562	0.000	0.000
S	2.828	2.503	1.414	1.326

M. H. M. Passos, et al
Phys. Rev. A **98**, 062116 (2018)

Alignment-free quantum cryptography

The BB84 protocol



Alice and Bob check their basis, **but not their results !**

ALICE

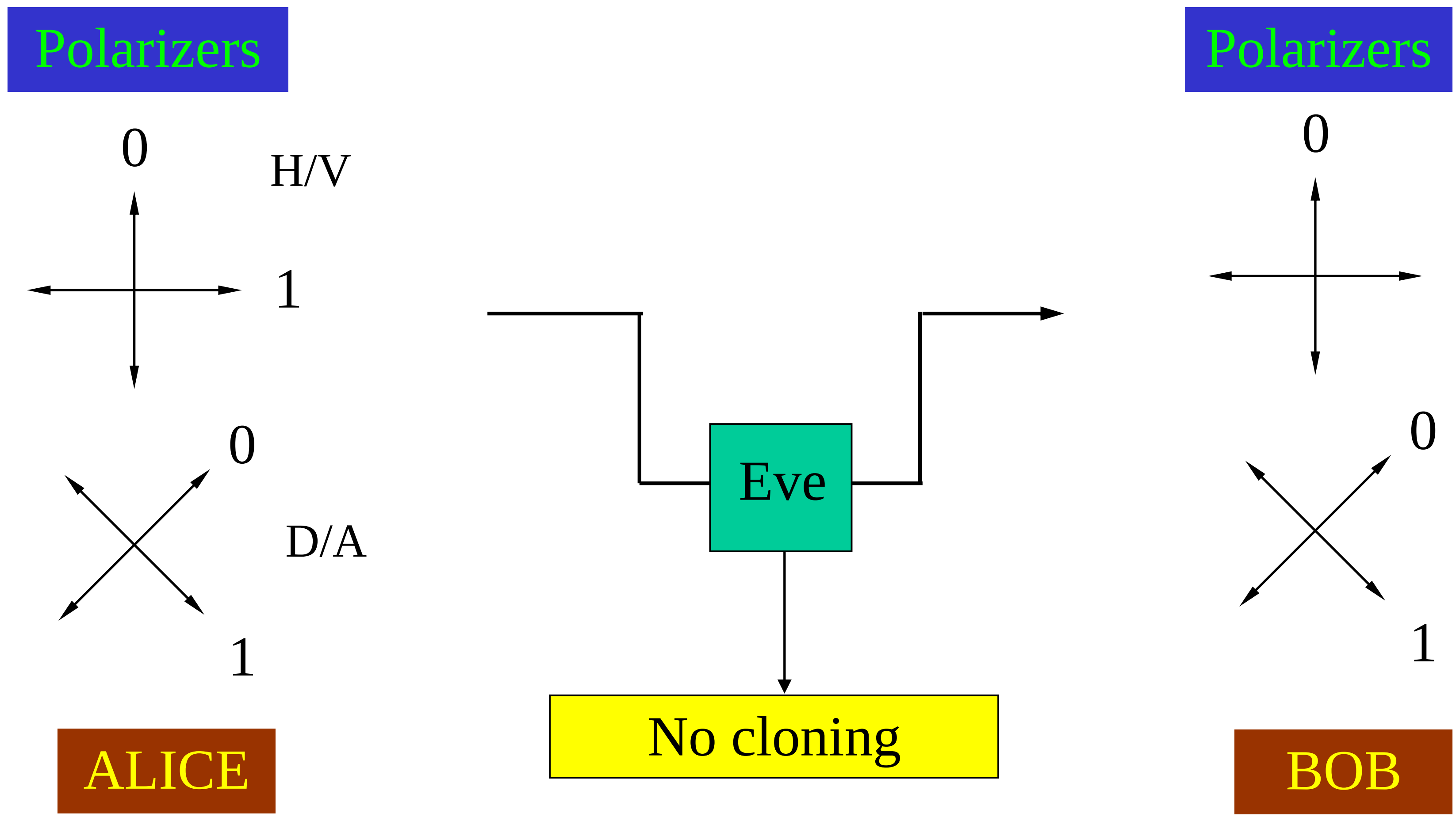
Basis	HV	+/-	+/-	+/-	HV	+/-	HV	HV	HV
Result	0	0	1	1	1	1	0	1	0

BOB

Basis	+/-	+/-	HV	+/-	HV	HV	HV	+/-	HV
Result	1	0	1	1	1	1	0	0	0

		⇧		⇧	⇧		⇧		⇧
		0		1	1		0		0

Quantum secret



- Eva introduces errors

Alice

Base	HV	+/-	+/-	+/-	HV	+/-	HV	HV	HV
Resultado	0	0	1	1	1	1	0	1	0

Bob

Base	+/-	+/-	HV	+/-	HV	HV	HV	+/-	HV
Resultado	1	0	1	0	1	1	1	0	0

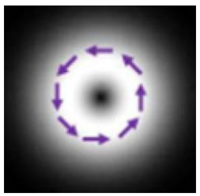
0 ?! 1 ?! 0

- Alice e Bob sacrifice **some** test bits

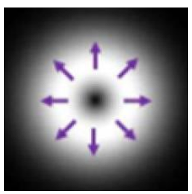
Spin-orbit entanglement

Physical Review A **77**, 032345 (2008)

Logic basis 0/1



$$|0_L\rangle = \frac{1}{\sqrt{2}}\left[\,|\rightarrow\rangle - |\uparrow\rangle\right] = \frac{1}{\sqrt{2}}\left[\,|\nearrow\rangle - |\nwarrow\rangle\right]$$



$$|1_L\rangle = \frac{1}{\sqrt{2}}\left[\,|\rightarrow\rangle + |\uparrow\rangle\right] = \frac{1}{\sqrt{2}}\left[\,|\nearrow\rangle + |\nwarrow\rangle\right]$$

Invariant under rotations ! ! ! !

Logic basis +/-

$$|\pm_L\rangle = \frac{1}{\sqrt{2}}\left[\,|0_L\rangle \pm |1_L\rangle\right]$$

BB84 without frame alignment

$$\{|0_L\rangle\ |1_L\rangle\}$$

$$\{|+_L\rangle\ |-_L\rangle\}$$

ALICE

$$|0_L\rangle\ |1_L\rangle\ |+_L\rangle\ |-_L\rangle$$

Photons

$$\{|0_L\rangle\ |1_L\rangle\}$$

$$\{|+_L\rangle\ |-_L\rangle\}$$

BOB

Robust against alignment noise !!!!

Preparation of the logic bases

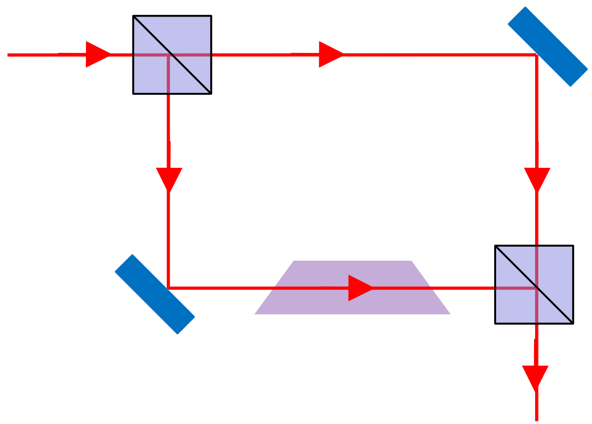
CNOT gate

$|\text{control,target}\rangle$

$|00\rangle \to |00\rangle \qquad |01\rangle \to |01\rangle$

$|10\rangle \to |11\rangle \qquad |11\rangle \to |10\rangle$

$$U_{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

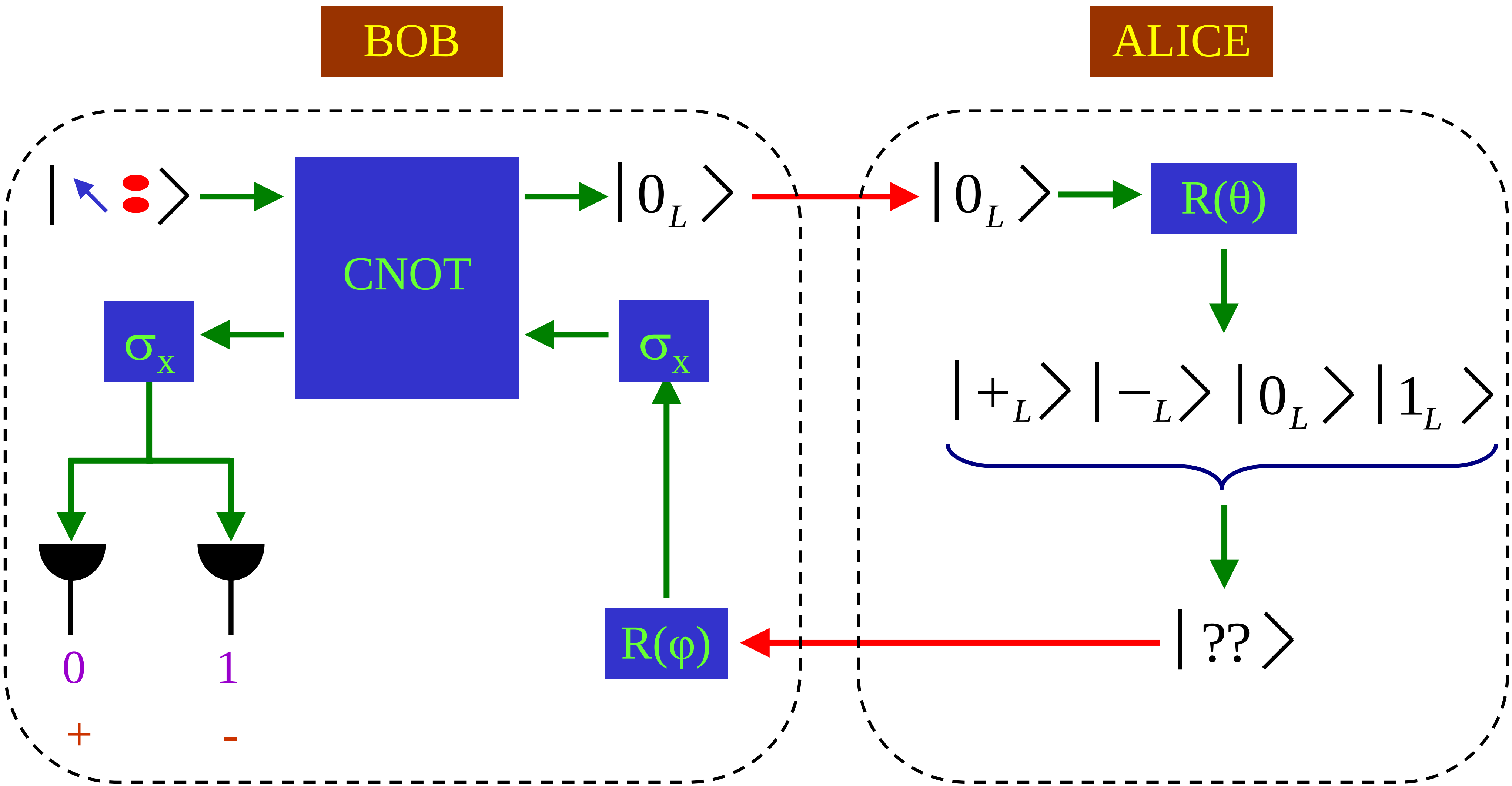


Control: POL

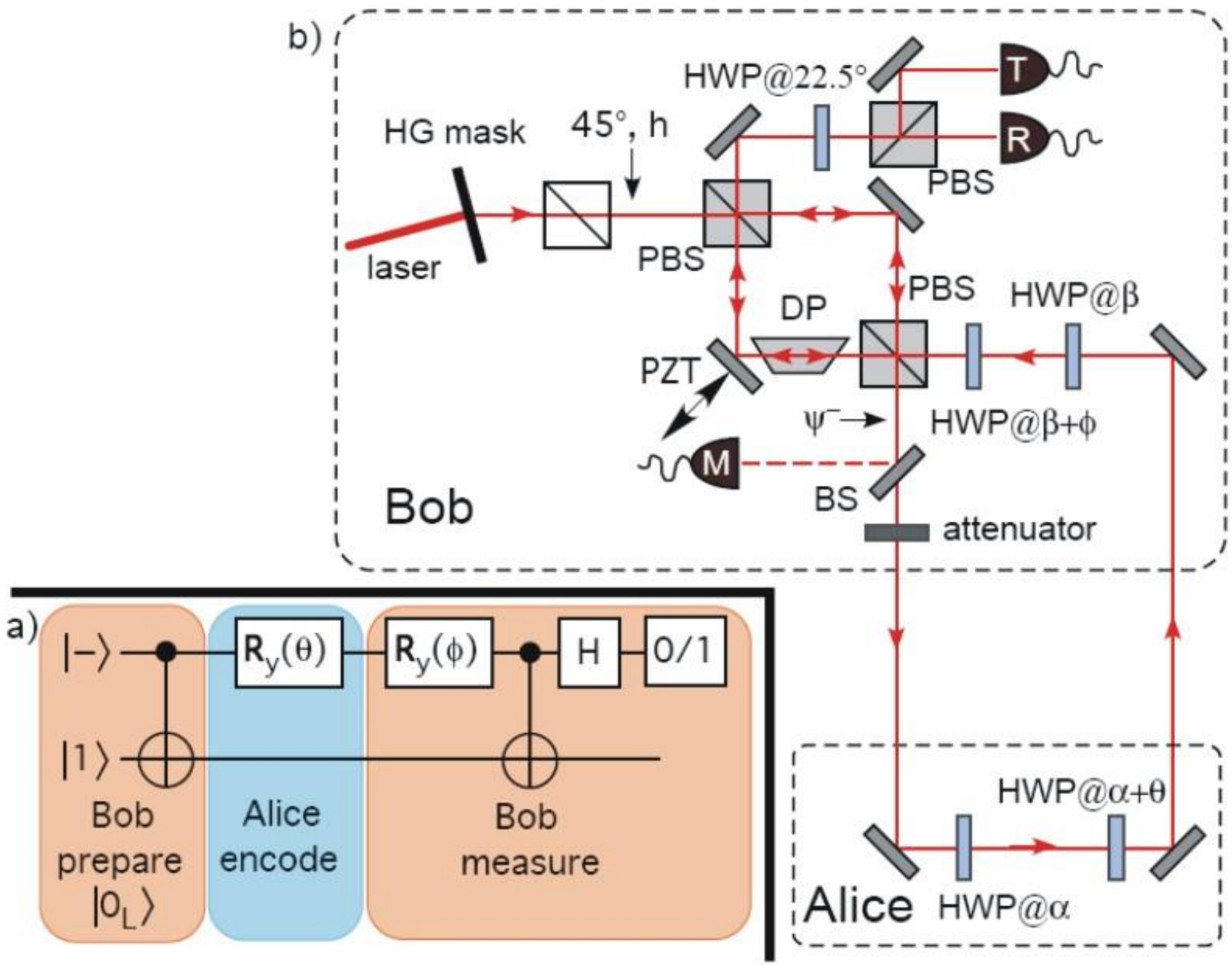
Target: TM

$$U_{CNOT}\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) \otimes |1\rangle = U_{CNOT}\left(\frac{|01\rangle - |11\rangle}{\sqrt{2}}\right) = \frac{|01\rangle - |10\rangle}{\sqrt{2}} = |0\rangle_L$$

Procedure sketch



Experimental setup

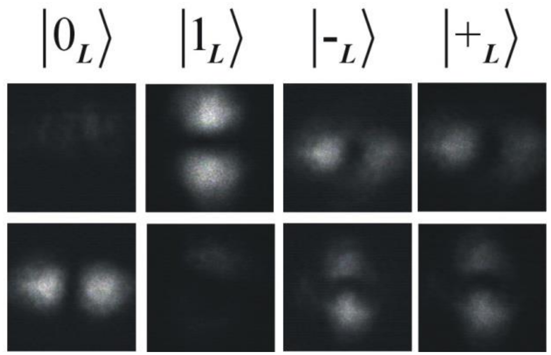


Experimental results

State sent by Alice

Bob’s detector 1

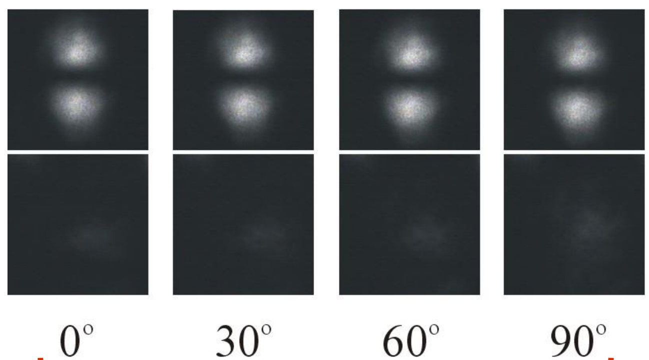
Bob’s detector 0



Bob`s detection basis:

$$\{ |0_L\rangle \ |1_L\rangle \}$$

Alice sends 1

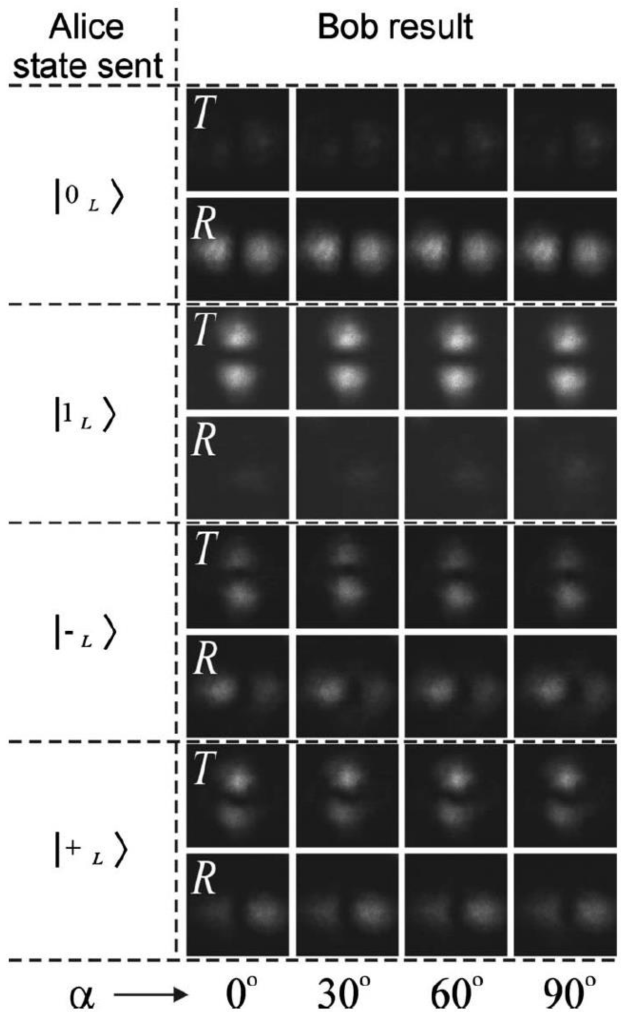


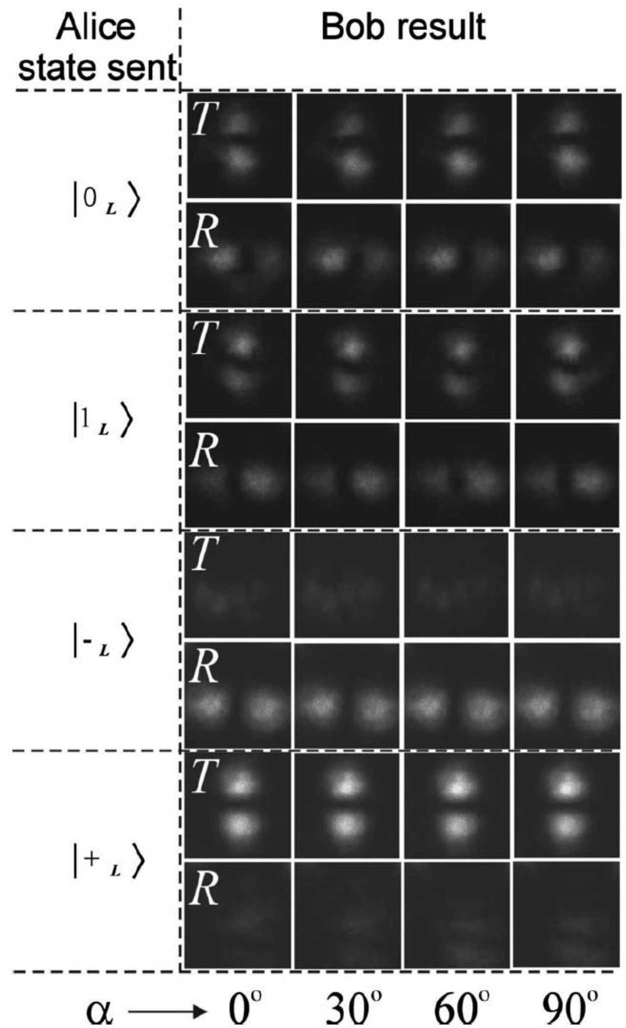
Bob’s detector 1

Bob’s detector 0

Rotation of Alice’s setup

Experimental results

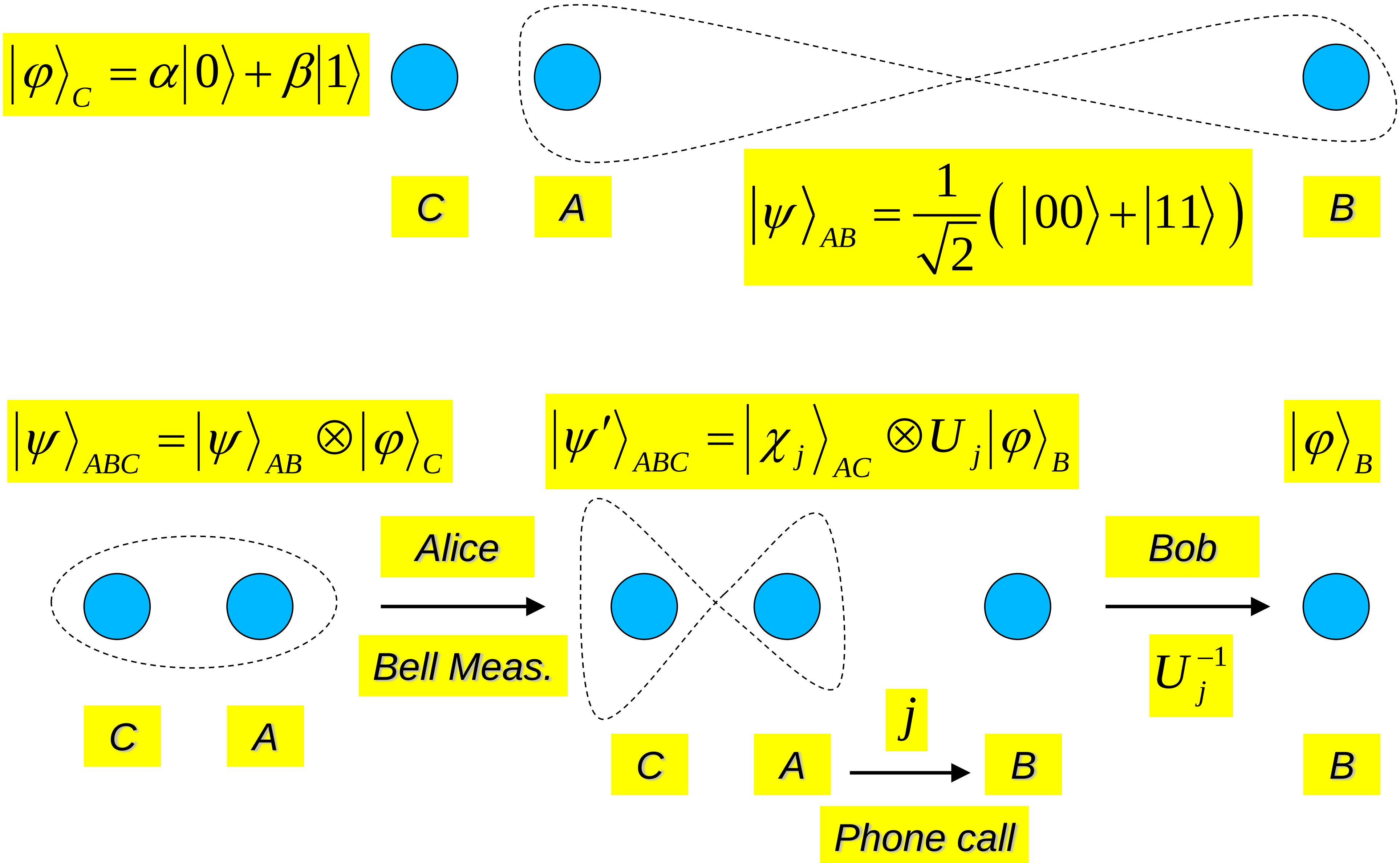




Bob`s detection basis: $\{|0_L\rangle, |1_L\rangle\}$ $\{|+_L\rangle, |-_L\rangle\}$

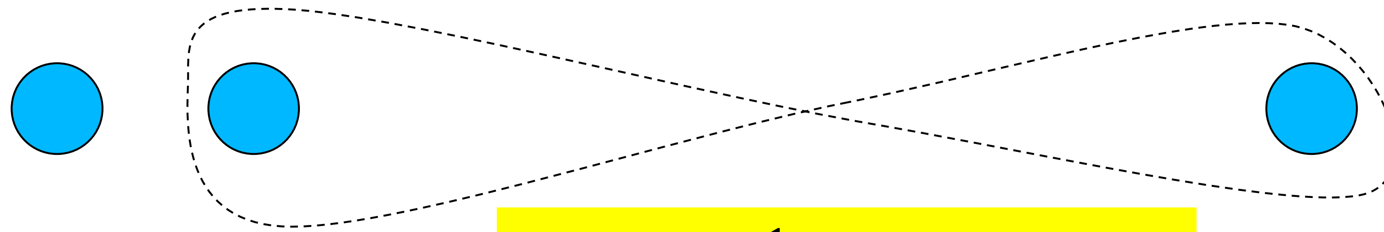
Spin-Orbit mode transfer through a teleportation-like scheme

Quantum Teleportation



Quantum Teleportation

$$|\varphi\rangle_C = \alpha|0\rangle + \beta|1\rangle$$



C A B

$$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle \right)$$

$$\begin{aligned}
|\psi\rangle_{ABC} &= |\psi\rangle_{AB} \otimes |\varphi\rangle_C \\
&= \frac{1}{\sqrt{2}}\left(\alpha|000\rangle_{ABC} + \beta|001\rangle_{ABC} + \alpha|110\rangle_{ABC} + \beta|111\rangle_{ABC}\right) \\
&= \frac{1}{\sqrt{2}}\left(\alpha|00\rangle_{AC}|0\rangle_B + \beta|01\rangle_{AC}|0\rangle_B + \alpha|10\rangle_{AC}|1\rangle_B + \beta|11\rangle_{AC}|1\rangle_B\right)
\end{aligned}$$

Bell basis for qubits

A and C

$$|\chi_1\rangle_{AC} = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle \right) \qquad |\chi_2\rangle_{AC} = \frac{1}{\sqrt{2}}\left(|00\rangle - |11\rangle \right)$$

$$|\chi_3\rangle_{AC} = \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle \right) \qquad |\chi_4\rangle_{AC} = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle \right)$$

Quantum Teleportation

$$
\begin{aligned}
|\psi\rangle_{ABC} &= |\psi\rangle_{AB} \otimes |\varphi\rangle_{C} \\
&= \frac{1}{\sqrt{2}}\left(\alpha|000\rangle_{ABC} + \beta|001\rangle_{ABC} + \alpha|110\rangle_{ABC} + \beta|111\rangle_{ABC}\right) \\
&= \frac{1}{\sqrt{2}}\left(\alpha|00\rangle_{AC}|0\rangle_{B} + \beta|01\rangle_{AC}|0\rangle_{B} + \alpha|10\rangle_{AC}|1\rangle_{B} + \beta|11\rangle_{AC}|1\rangle_{B}\right)
\end{aligned}
$$

Before AC Bell measurement

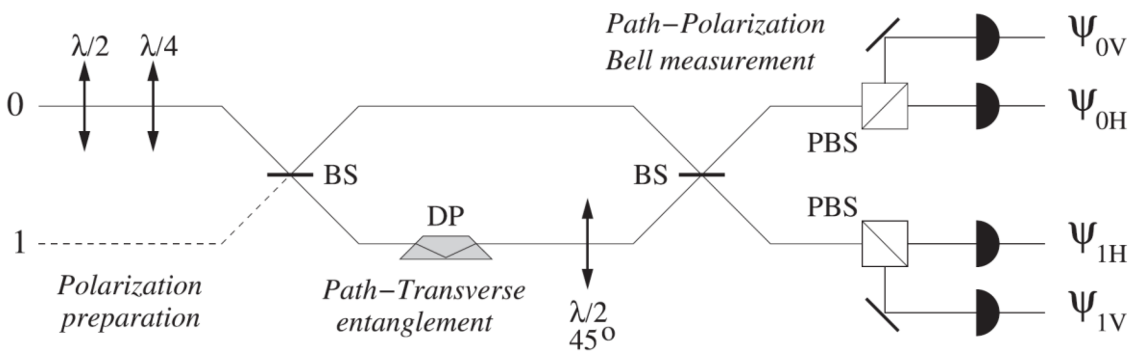
$$
\begin{aligned}
|\psi\rangle_{ABC} = &|\chi_1\rangle_{AC}\left(\alpha|0\rangle_B + \beta|1\rangle_B\right) \\
&+|\chi_2\rangle_{AC}\left(\alpha|0\rangle_B - \beta|1\rangle_B\right) \\
&+|\chi_3\rangle_{AC}\left(\beta|0\rangle_B + \alpha|1\rangle_B\right) \\
&+|\chi_4\rangle_{AC}\left(\beta|0\rangle_B - \alpha|1\rangle_B\right)
\end{aligned}
$$

After AC Bell measurement

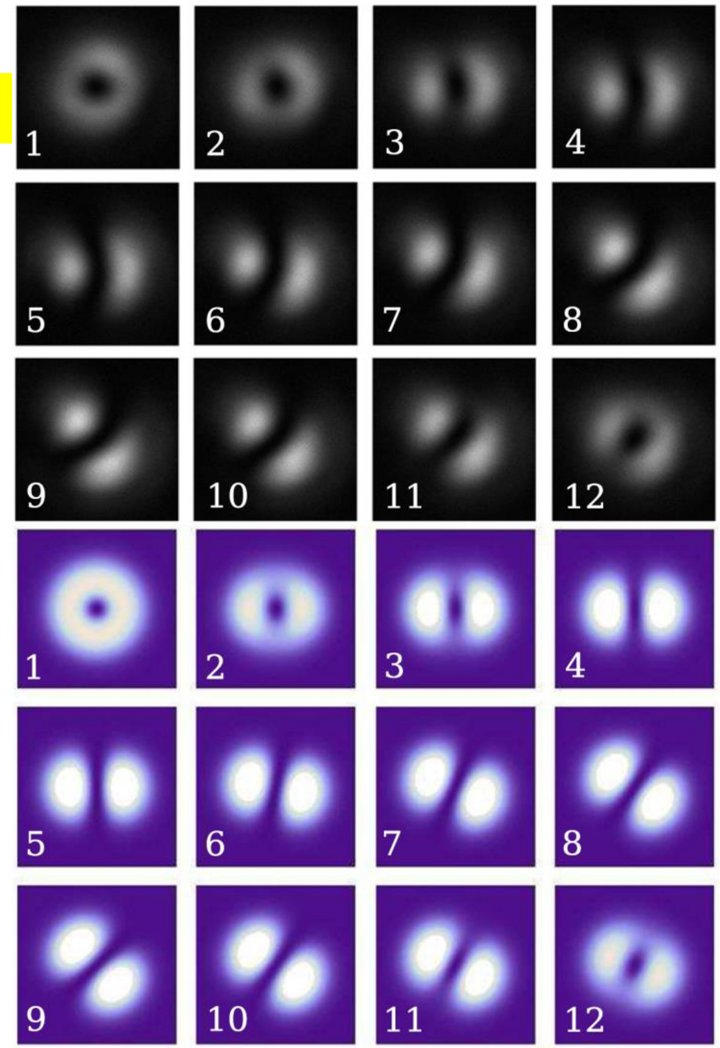
$$
\begin{aligned}
|\chi_1\rangle_{AC} &\Rightarrow \mathbf{1}|\varphi\rangle_B \\
|\chi_2\rangle_{AC} &\Rightarrow \sigma_z|\varphi\rangle_B \\
|\chi_3\rangle_{AC} &\Rightarrow \sigma_x|\varphi\rangle_B \\
|\chi_4\rangle_{AC} &\Rightarrow \sigma_z\sigma_x|\varphi\rangle_B
\end{aligned}
$$

Spin-orbit mode transfer with a teleportation protocol

B. P. Silva, M. A. Leal, C. E. R. Souza, E. F. Galvão, A. Z. Khoury, J Phys B **49**, 055501 (2016)



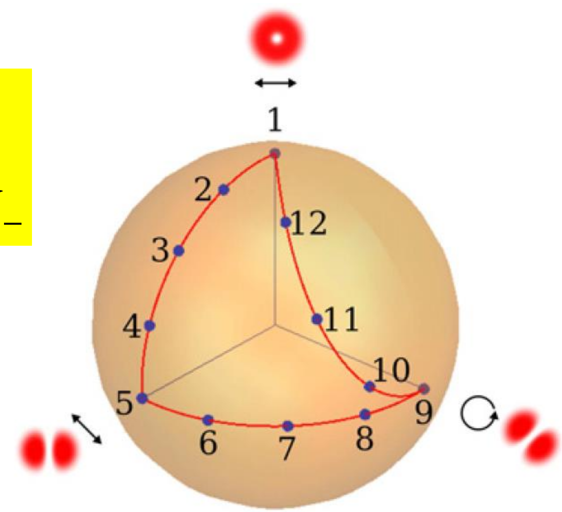
$\Rightarrow$



Workspace: $\underset{A}{\left\{e^{i\mathbf{k}_0\cdot\mathbf{r}},e^{i\mathbf{k}_1\cdot\mathbf{r}}\right\}}\otimes\underset{B}{\left\{\psi_+,\psi_-\right\}}\otimes\underset{C}{\left\{\hat{\mathbf{e}}_H,\hat{\mathbf{e}}_V\right\}}$

Input: $\hat{\boldsymbol{\varphi}}=\alpha\,\hat{\mathbf{e}}_H+\beta\,\hat{\mathbf{e}}_V$

Output: $\psi=\alpha\,\psi_++\beta\,\psi_-$



Topological phase for entangled states

Origin of the geometric phase

N. Mukunda and R. Simon, Ann. Phys. **228**, 205 (1993).

N. Mukunda and R. Simon, Ann. Phys. **228**, 269 (1993).

Discrete cycles in the Hilbert space

$$|\psi_1\rangle \to |\psi_2\rangle \to |\psi_1\rangle \Rightarrow \phi_2 = \arg\left[\langle\psi_1|\psi_2\rangle\langle\psi_2|\psi_1\rangle\right] = 0$$

$$|\psi_1\rangle \to |\psi_2\rangle \to |\psi_3\rangle \to |\psi_1\rangle \Rightarrow \phi_3 = \arg\left[\langle\psi_1|\psi_2\rangle\langle\psi_2|\psi_3\rangle\langle\psi_3|\psi_1\rangle\right]$$

$$|\psi_1\rangle \to |\psi_2\rangle \to |\psi_3\rangle \to \cdots \to |\psi_n\rangle \to |\psi_1\rangle$$

$$\phi_n = \arg\left[\langle\psi_1|\psi_2\rangle\langle\psi_2|\psi_3\rangle\langle\psi_3|\psi_4\rangle\cdots\langle\psi_n|\psi_1\rangle\right]$$

$$|\psi_j\rangle \to |\psi'_j\rangle = e^{i\theta_j}|\psi_j\rangle \Rightarrow \phi'_n = \phi_n$$

Open transformations

$$|\psi_1\rangle \to |\psi_2\rangle \to |\psi_3\rangle \to \cdots \to |\psi_n\rangle$$

$$\phi_n = \arg\left[\langle\psi_1|\psi_n\rangle\right] - \arg\left[\langle\psi_1|\psi_2\rangle\langle\psi_2|\psi_3\rangle\langle\psi_3|\psi_4\rangle\cdots\langle\psi_{n-1}|\psi_n\rangle\right]$$

Continuous Evolutions

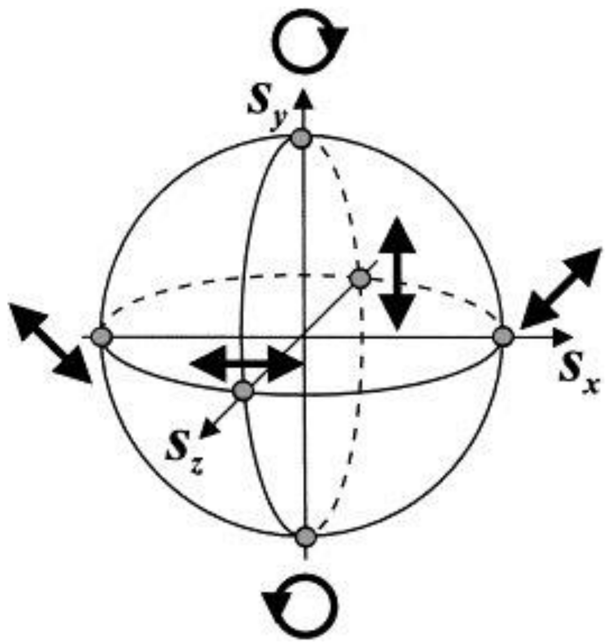
Continuous evolution : $|\psi(t)\rangle = U(t)|\psi(0)\rangle$

$$\phi_g = \arg\left(\langle\psi(0)|\psi(T)\rangle\right) + i\int_0^T \langle\psi(t)|\dot{\psi}(t)\rangle\, dt$$

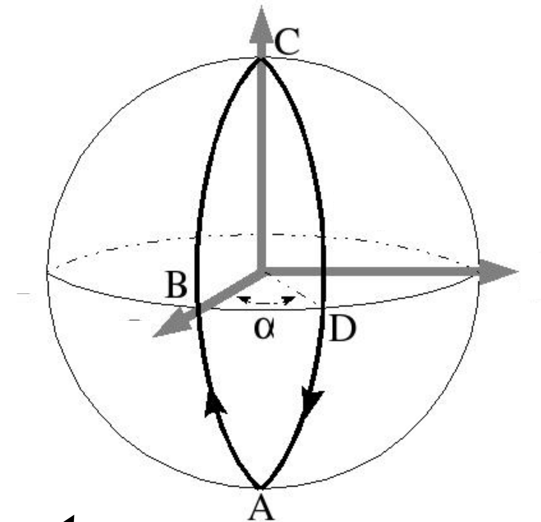
$$= \arg\left(Tr\left[\rho_0 U(T)\right]\right) + i\int_0^T Tr\left[\rho_0 U^{\dagger}(t)\dot{U}(t)\right] dt$$

$$|\psi(t)\rangle \rightarrow |\psi'(t)\rangle = e^{i\phi(t)}|\psi(t)\rangle \Rightarrow \phi_g' = \phi_g$$

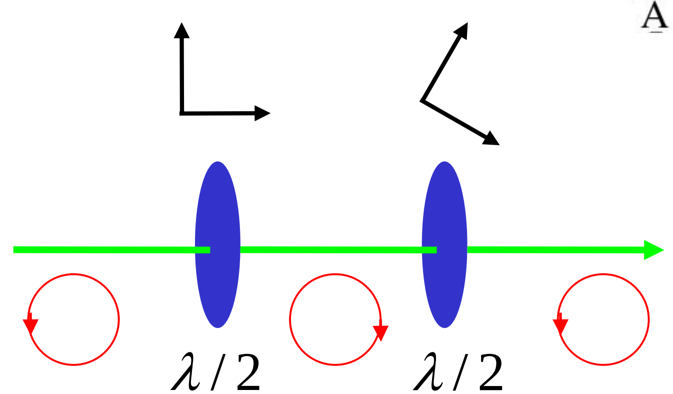
Pancharatnam Phase



Poincaré sphere for polarization modes



$$\Phi_g = -\Omega / 2 = \alpha$$



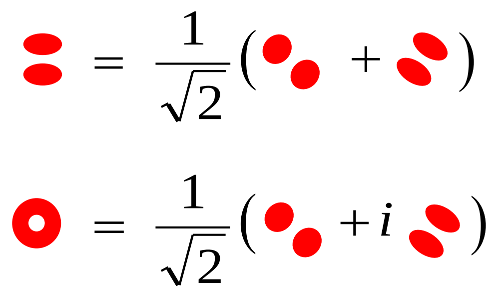
S. Pancharatnam, Proc. Indian Acad. Sci. Sect. A, V.**44**, 247 (1956)

Collected Works of S. Pancharatnam, Oxford Univ. Press, London (1975).

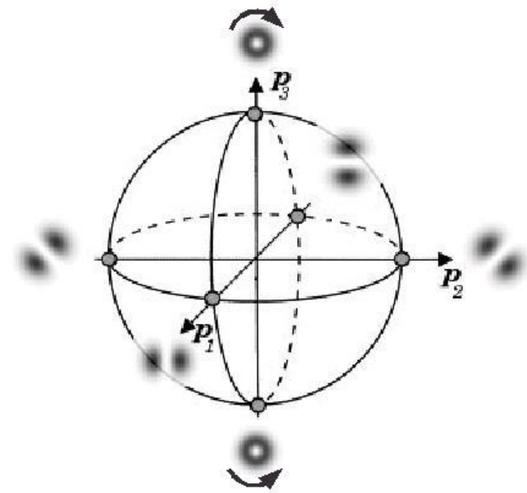
Orbital Pancharatnam Phase

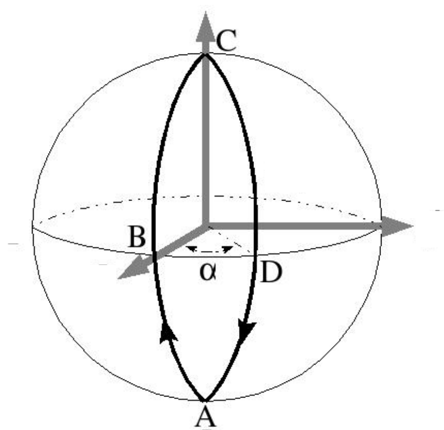
E.J. Galvez, P.R. Crawford, H.I. Sztul, M.J. Pysher,

P.J. Haglin, R.E. Williams, PRL **90**, 203901 (2003).



$\Rightarrow$

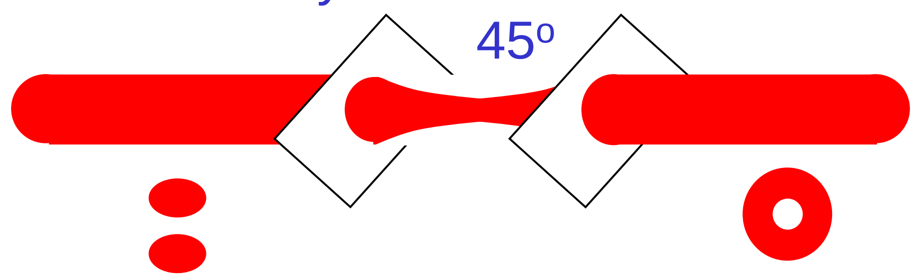


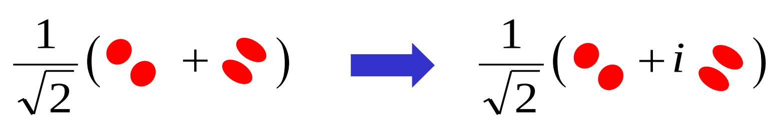


$$\phi_g = -\ \Omega/2 = \alpha$$

Astigmatic mode converter

Cylindrical lenses at 45°

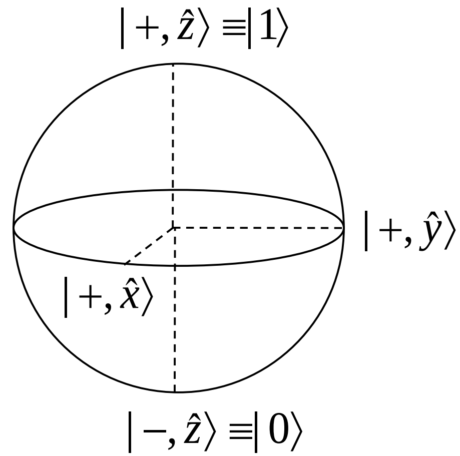




Geometric representation for two-qubit states

ONE QUBIT ⇨

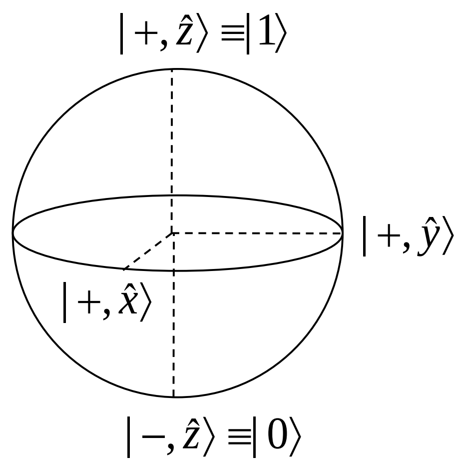
$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$



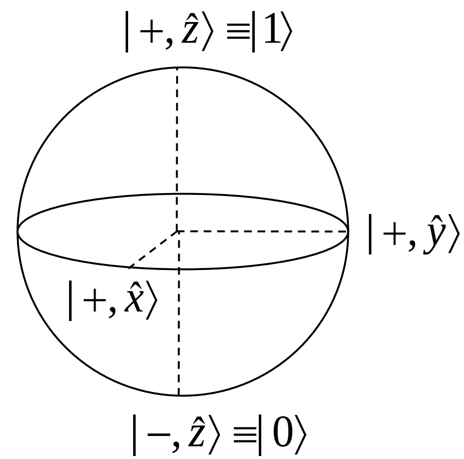
Bloch sphere

(or Poincaré sphere)

TWO QUBITS ⇨



$\otimes$



Two Bloch spheres?? Only for product states!!!

Geometric representation for two-qubit PURE states

P. Milman and R. Mosseri, Phys. Rev. Lett. **90**, 230403 (2003).

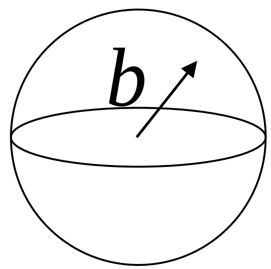
P. Milman, Phys. Rev. A **73**, 062118 (2006).

$$|\psi\rangle = \alpha\,|00\rangle + \beta\,|01\rangle + \gamma\,|10\rangle + \delta\,|11\rangle$$

Two-qubit

PURE STATES ⇨

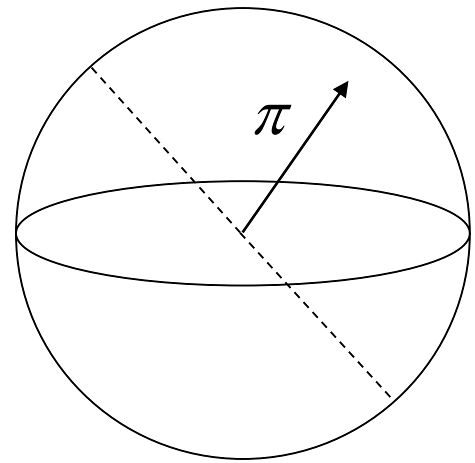
$|1\rangle\langle 1|$



$|0\rangle\langle 0|$

Bloch ball

$\otimes$



SO(3) sphere

(opposite points identified)

$$b = \sqrt{1-|C|^2}$$

$$C = 2|\alpha\delta - \beta\gamma|$$ (Concurrence)

Cyclic evolutions ⇨ $\phi_{TOT} = \phi_{dyn} + \phi_g + \phi_{top}$

$\phi_g \Rightarrow$ Bloch Ball

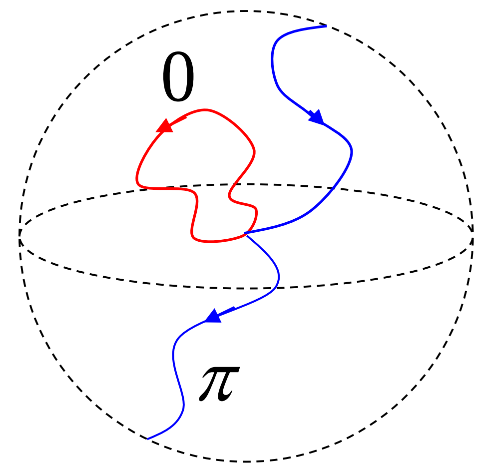
$\phi_{top} \Rightarrow$ SO(3) sphere

Topological phase for maximally entangled states

Maximally entangled state ⇨ $C = 1 \Rightarrow \rho = 0$ Bloch ball colapses to a point!!!!

$$|\psi\rangle = \alpha\,|00\rangle + \beta\,|01\rangle - \beta^*\,|10\rangle + \alpha^*\,|11\rangle$$

Cyclic evolutions preserving maximal entanglement (“Closed” trajectories) ⇨



SO(3) sphere

Two homotopy classes:

0-type trajectories ⇨ $\phi_{top} = 0 \qquad |\psi(T)\rangle = |\psi(0)\rangle$

π-type trajectories ⇨ $\phi_{top} = \pi \qquad |\psi(T)\rangle = -|\psi(0)\rangle$

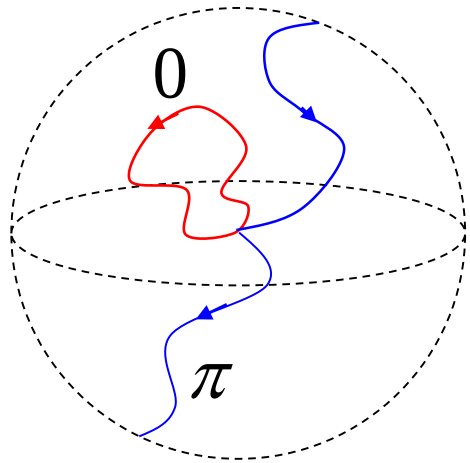
Topological phase for entangled qubits

P. Milman and R. Mosseri, PRL **90**, 230403 (2003); P. Milman, PRA **73**, 062118 (2006).

Maximally entangled state $|\psi\rangle = \alpha|00\rangle + \beta|01\rangle - \beta^*|10\rangle + \alpha^*|11\rangle$

$C = 1 \Rightarrow \rho = 0$ Bloch ball colapses to a point!!

Cyclic evolutions preserving maximal entanglement ("Closed" trajectories) ⇨



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C. E. R. Souza, J. A. O. Huguenin, P. Milman, A. Z. Khoury PRL 99, 160401 (2007)

J. Du, J. Zhu, M. Shi, X. Peng, and D. Suter PRA 76, 042121 (2007)

Topological phase for spin-orbit modes of a laser beam

C. E. Rodrigues de Souza, J. A. O. Huguenin and A. Z. Khoury

IF-UFF

P. Milman

LMPQ – Jussieu - France

Nonseparable polarization-OAM modes

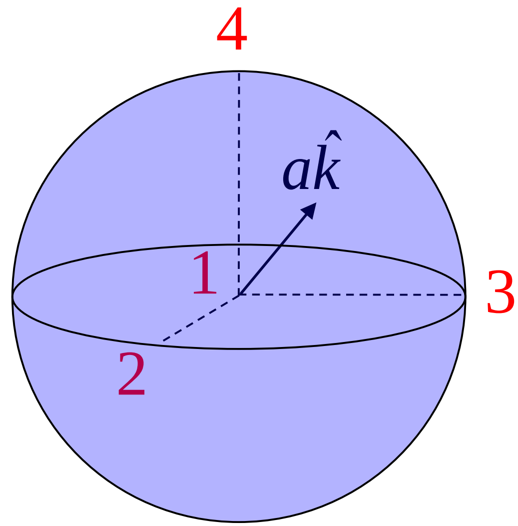
$$\mathbf{\Psi}(\mathbf{r}) = \alpha\, \psi_+(\mathbf{r})\hat{\mathbf{e}}_H + \beta\, \psi_+(\mathbf{r})\hat{\mathbf{e}}_V - \beta^*\, \psi_-(\mathbf{r})\hat{\mathbf{e}}_H + \alpha^*\, \psi_-(\mathbf{r})\hat{\mathbf{e}}_V$$

Geometric representation on the SO(3) sphere

$$\alpha = \cos(a/2) - ik_z \operatorname{sen}(a/2)$$

$$\beta = -\left(k_y + ik_x\right)\operatorname{sen}(a/2)$$

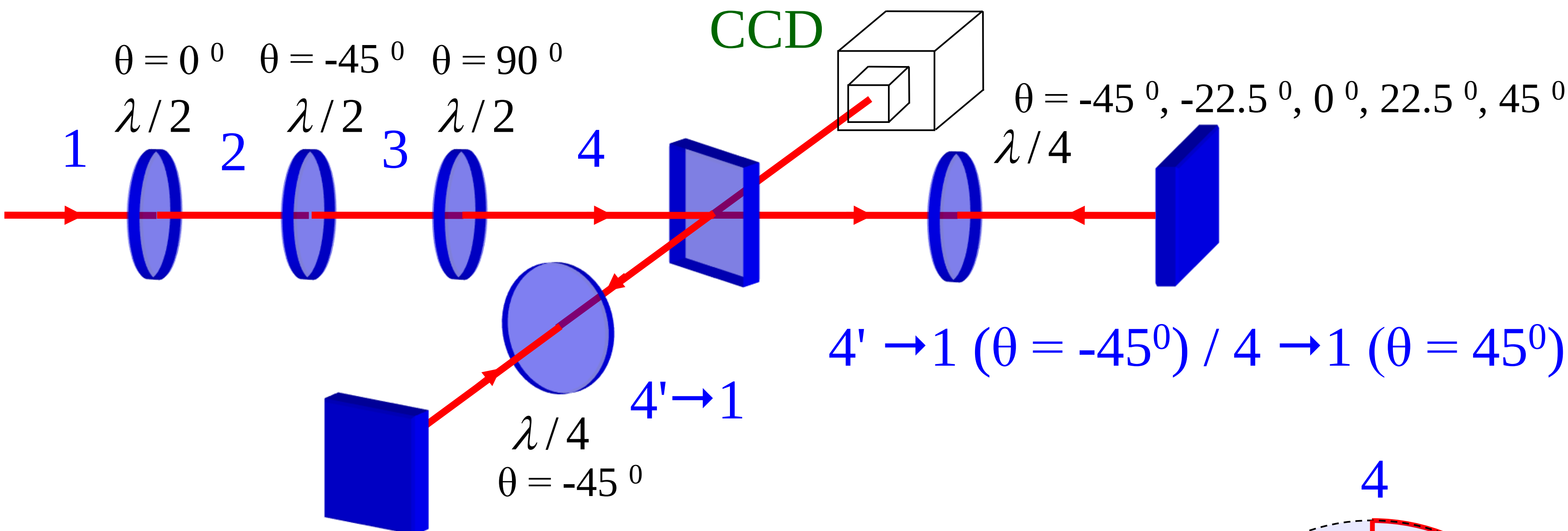
W. LiMing, Z. L. Tang, and C. J. Liao,
Phys. Rev. A 69, 064301 (2004).

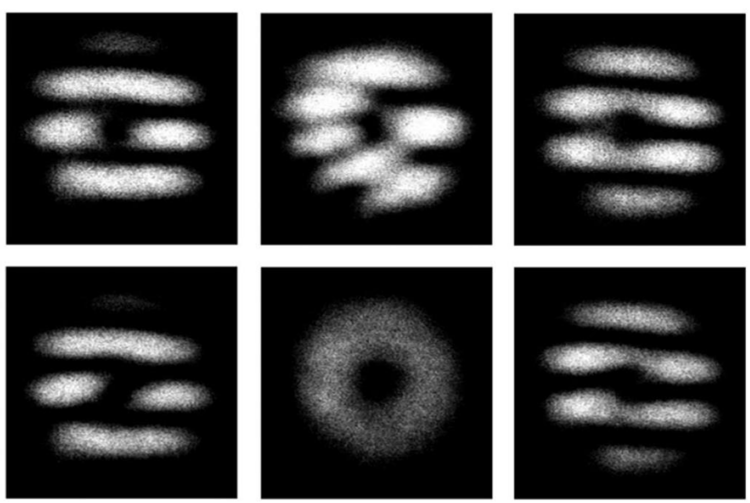


$$\mathbf{\Psi}_1(\vec{r}) = \frac{1}{\sqrt{2}}\left[\psi_+(\mathbf{r})\hat{\mathbf{e}}_H + \psi_-(\mathbf{r})\hat{\mathbf{e}}_V\right] \quad \mathbf{\Psi}_2(\mathbf{r}) = \frac{-i}{\sqrt{2}}\left[\psi_+(\mathbf{r})\hat{\mathbf{e}}_H - \psi_-(\mathbf{r})\hat{\mathbf{e}}_V\right]$$

$$\mathbf{\Psi}_3(\vec{r}) = \frac{-i}{\sqrt{2}}\left[\psi_+(\mathbf{r})\hat{\mathbf{e}}_V + \psi_-(\mathbf{r})\hat{\mathbf{e}}_H\right] \quad \mathbf{\Psi}_4(\mathbf{r}) = \frac{1}{\sqrt{2}}\left[\psi_+(\mathbf{r})\hat{\mathbf{e}}_V - \psi_-(\mathbf{r})\hat{\mathbf{e}}_H\right]$$

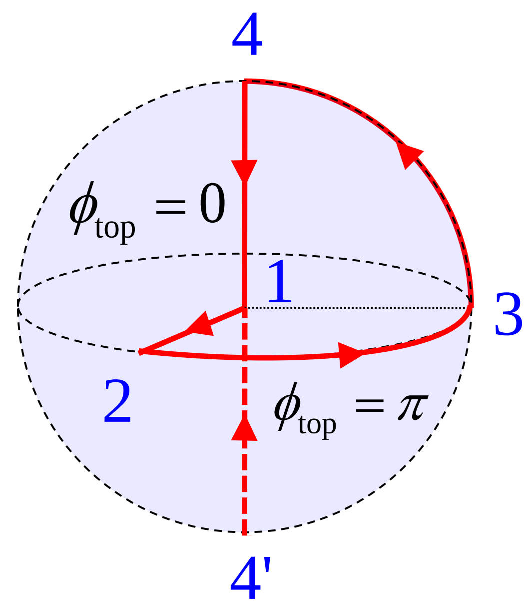
Interferometric measurement





Nonseparable

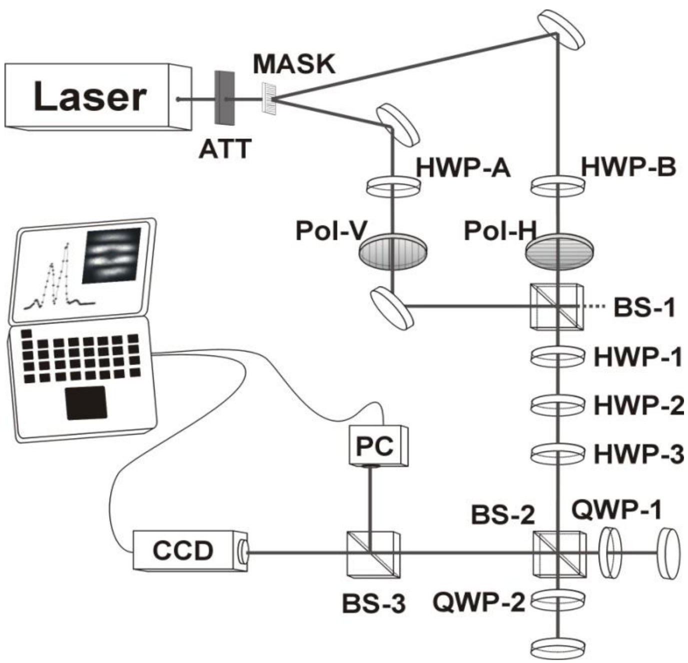
Separable

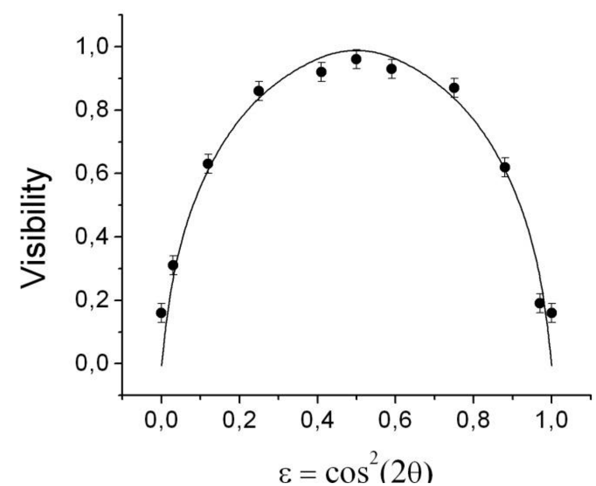


Partial separability and concurrence

Partially separable mode
$$\mathbf{E}_\varepsilon(\mathbf{r}) = E_0\left[\sqrt{\varepsilon}\,\psi_+(\mathbf{r})\hat{\mathbf{e}}_H + \sqrt{1-\varepsilon}\,\psi_-(\mathbf{r})\hat{\mathbf{e}}_V\right]$$

Interference pattern ($\theta=45^0$)
$$I(\vec{r}) = 2\left|\psi(\mathbf{r})\right|^2\left(1+\underbrace{2\sqrt{\varepsilon(1-\varepsilon)}}_{\text{CONCURRENCE}}\ \frac{2xy\sin qy}{x^2+y^2}\right)$$





C. E. R. Souza et al, PRL 99, 160401 (2007)

Conclusions

- Topological phase for spin-orbit transformations [PRL **99**, 160401 (2007)]
- Alignment free BB84 quantum cryptography [PRA **77**, 032345 (2008)]
- Bell-like inequality [PRA **82**, 033833 (2010)]
- Realization of quantum gates [Opt. Exp. **18**, 9207 (2010)]
- Quantum teleportation in the spin-orbit variables [PRA **83**, 060301 (2011)]